Dynamic Taylor Condition within an Endogenous Growth Monetary Economy

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August 10, 2011

Abstract

The paper presents an endogenous growth human capital based economy with endogenous velocity using the bank sector to produce exchange credit. It derives a dynamic Taylor Condition that occurs as the central bank stochastically supplies money. It then generates artificial data and estimates the Taylor Condition 1000 times and presents the average results. It shows that a "Taylor rule" emerges, even though the central bank is merely satisfying fiscal needs through the inflation tax. It implies that it would be spurious within this economy to associate the estimated Taylor condition as the result of central bank interest rate targeting.

Preliminary Draft: We thank Hao Hong and Ceri Davies for research assistance.

1 Introduction

Taylor and Wieland (2010) compares three central Taylor rules used in the literature, the famous one proposed by Taylor (1993) himself, the Christiano, Eichenbaum and Evans (2005) version and the Smets and Wouters (2007) version. They find that the interest rate smoothing factor such as in the later two papers is not robust, nor is the current or lagged output gap. But they conclude that

> "Some degree of robustness can be recovered by using rules without interest-rate smoothing or with GDP growth deviations from trend in place of the output gap",

and that averaging across models is a way to get the best performance.

Issues about variables identified in Taylor rules, including the Taylor "principle" of a greater than unity inflation coefficient, are addressed in this paper from a very different perspective that ususal. It might be in the realm of the perspective of McCallum (2008) in which he introduces the concept of a "well-formulated" (WF) model for considering issues such as the Taylor rule. And equally it is about the debate presented in Cochrane (2011) about the ability of Taylor rules to identify sufficiently what is the underlying inflation process.

This paper will derive a Taylor condition from a general equilibrium endogenous growth economy, thereby satisfying both the "well-formulated" McCallum concept of a Taylor rule, and the clear identification of the unique underlying inflation process on which Cochrane focuses. This combines both Taylor rules and money supply rules in a way seen relatedly in Alvarez, Lucas and Weber (2001).

A central theoretical result is that the Taylor principle, of a greater than one inflation coefficient, only holds in theory for when money velocity is greater than one. Only at the Friedman optimum of a zero nominal interest rate, along the balanced growth path (BGP) equilibrium, does this coefficient fall to one, as in the Fisher equation of interest rates. For BGP nominal interest rates greater than one, velocity is greater than one as exchange credit produced in a banking sector is used to avoid the inflation tax. In addition, a velocity term enters the Taylor condition again as long as the BGP nominal interest rate is above the Friedman optimum of zero, in a way reminiscent of McCallum's (1999) emphasis on using money demand in a reduced "central bank policy model", and the bias that can result without it.

The paper then calibrates the economy and estimates the derived Taylor equilibrium condition using an average of 1000 series of artificially simulated data. Estimation results are presented for the Taylor condition using four different data filters for comparison, and using both OLS and instrumental variables. In addition to the correct Taylor condition, two counter-theoretical Taylor conditions that arbitrarily drop some variables are also estimated to show how missing variable bias misspecification may manifest itself when the data used is still the artificially simulated data of the original model. Results show the successful estimation of the Taylor condition in terms of the Taylor principle on the inflation rate being robust in both theory and in the estimation. In addition all other coefficients are significant and of the correct sign as in theory, with some striking closeness of most of the parameters to the theoretical values. When velocity is excluded from the estimation as in a standard Taylor rule estimation, the estimation results based on the model's simulated data find an inflation coefficient below one.

The forward looking interest rate term is robustly significant in the correct estimated model specification for band pass filters. In our model this corresponds to the interest smoothing term that is discussed as not robustly significant in Taylor and Wieland (2010).

Yet, when we used the "wrong" but traditional "output gap" term, as an experiment, or a traditional Taylor rule specification without velocity, the interest smoothing term is indeed mostly not significant. And regarding the output gap measure per se as in the above Taylor and Wieland quote, our "measure" in the derived Taylor condition differs as it is the deviation of consumption growth from the BGP "trend" rate; and this is robustly significant. Alternatively we show theoretically how the output growth deviation from the BGP rate can enter the Taylor condition instead of consumption, but only with an additional term involving investment growth. Further we counter-theoretically include estimation of the model with just one change: incorrectly using output growth deviation instead of consumption growth deviation; this weakens the robustness of the model's results.

Besides the interesting role of money velocity growth, the growth in "productive hours worked" also enters our equilibrium endogenous growth Taylor condition. This variable is a combination of what can be construed as the labor force participation rate plus the fraction of time spent increasing productivity; and it shows robust significance in estimation. This is not inconsistent with the essence of what Canova et al. (2009) writes:

> "...the standard per-capita hours series displays long but essentially stationary cycles. These cycles are of longer duration than those considered in the business cycle literature and may reflect, for example, demographics, trends in labor market participation or R&D activities. This paper argues that disregarding them (as one would do by taking hours in levels) or by taking a rough short cut (as one would do by differencing the series) leads to misspecification, efficiency losses, and potentially uninterpretable

results."

Estimating the derived Taylor Condition 1000 times from the artificial data, the "Taylor rule" emerges even though the central bank is merely satisfying fiscal needs through the inflation tax. The Taylor principle fails when significant changes from the model's Taylor condition are imposed. This implies the central point of the paper: that it would be spurious within this economy to associate the estimated Taylor condition as the result of central bank interest rate targeting, since in the model the central bank merely stochastically prints money.[1] Second, failure of the so-called Taylor principle in some estimations may be a simple result of model misspecification. Further, within the standard flexible price world of this paper's economy, the Taylor principle holds because of a higher than Friedman optimal money supply growth rate, inflation rate, and velocity of money, rather than being due to an actively aggressive interest rate targeting policy by a central bank.

2 Stochastic Endogenous Growth with Banking

The representative agent economy is as in Benk et al (2008, 2010) but with a decentralized the bank sector that produces credit as in Gillman and Kejak (2011). By combining the business cycle with endogenous growth, stationary inflation lowers the output growth rate as supported empirically for example in Gillman, Harris and Matyas (2004) and Karanasos (2006). Further money supply shocks can cause inflation at low frequencies as in Haug and Dewald (2011), and as supported in Sargent and Surico (2008, 2011), which can lead to output growth effects if the shocks are persistent and repeated. This allows shocks over the business cycle to cause changes in growth

[1] We are indebted for this approach to a suggestion made by Warren Weber; see also Alvarez et al. (2001).

rates and in stationary ratios. The shocks to the goods sector productivity and the money supply growth rate are standard, while the third shock to the credit sector productivity exists by virtue of the model's endogeneity of money velocity via the production function used extensively in the financial intermediation microeconomics literature starting with Clark (1985).

The shocks occur at the beginning of the period, observed by the consumer before the decision process, and follow a vector first-order autoregressive process. For goods sector productivity, z_t, the money supply growth rate, u_t, and bank sector productivity, v_t :

$$Z_t = \Phi_Z Z_{t-1} + \varepsilon_{Zt}, \tag{1}$$

where the shocks are $Z_t = [z_t \; u_t \; v_t]'$, the autocorrelation matrix is $\Phi_Z = diag\{\varphi_z, \varphi_u, \varphi_v\}$ and $\varphi_z, \varphi_u, \varphi_v \in (0,1)$ are autocorrelation parameters, and the shock innovations are $\varepsilon_{Zt} = [\epsilon_{zt} \; \epsilon_{ut} \; \epsilon_{vt}]' \sim N(\mathbf{0}, \mathbf{\Sigma})$. The general structure of the second-order moments is assumed to be given by the variance-covariance matrix $\mathbf{\Sigma}$. These shocks affect the economy as described below.

2.1 Consumer Problem

A representative consumer has expected lifetime utility from consumption of goods, c_t, and leisure, x_t; with $\beta \in (0,1)$ and $\theta > 0$, this is given by

$$U = E_0 \sum_{t=0}^{\infty} \beta \frac{(c_t x_t^{\psi})^{1-\theta}}{1-\theta}. \tag{2}$$

Output of goods, y_t, and increases in human capital, are produced with physical capital and effective labor each in Cobb-Douglas fashion; the bank sector produces exchange credit using labor and deposits as inputs. Let s_{Gt} and s_{Ht} denote the fractions of physical capital that the agent uses in the goods production (G) and human capital investment (H), whereby

$$s_{Gt} + s_{Ht} = 1. \tag{3}$$

The agent allocates a time endowment of one amongst leisure, x_t, labor in goods production, l_t, time spent investing in the stock of human capital, n_t, and time spent working in the bank sector, denoted by f_t:

$$l_t + n_t + f_t + x_t = 1. \tag{4}$$

Output of goods can be converted into physical capital, k_t, without cost and so is divided between consumption goods and investment, denoted by i_t, net of capital depreciation. Thus, the capital stock used for production in the next period is given by:

$$k_{t+1} = (1 - \delta_k)k_t + i_t = (1 - \delta_k)k_t + y_t - c_t. \tag{5}$$

The human capital investment is produced using capital $s_{Ht}k_t$ and effective labor $n_t h_t$:

$$H(s_{Ht}k_t, n_t h_t) = A_H(s_{Ht}k_t)^{1-\eta}(n_t h_t)^{\eta}. \tag{6}$$

And the human capital flow constraint is:

$$h_{t+1} = (1 - \delta_h)h_t + H(s_{Ht}k_t, n_t h_t). \tag{7}$$

With w_t and r_t denoting the real wage and real interest rate, the consumer receives nominal income of wages and rents, $P_t w_t\,(l_t + f_t)\,h_t$ and $P_t r_t\,(s_{Gt} + s_{Qt})\,k_t$, a nominal transfer from the government, T_t, and dividends from the bank.

The consumer buys shares in the bank by making deposits of income at the bank. Each dollar deposited buys one share at a fixed price of one, and the consumer receives the residual profit of the bank as dividend income in proportion to the number of shares (deposits) owned. Denoting the real quantity of deposits by d_t, and the dividend per unit of deposits as R_{Qt}, the consumer receives a nominal dividend income of $P_t R_{Qt} d_t$. The consumer also pays to the bank a fee for credit services, whereby one unit of credit service is required for each unit of credit that the bank supplies the consumer for use in buying goods. With P_{Qt} denoting the nominal price of each unit of credit, and q_t the real quantity of credit that the consumer can use in exchange, the consumer pays $P_{Qt} q_t$ in credit fees.

With other expenditures on goods, of $P_t c_t$, and physical capital investment, $P_t k_{t+1} - P_t(1-\delta_k)k_t$, and on investment in cash for purchases, of $M_{t+1} - M_t$, and in nominal bonds $B_{t+1} - B_t(1+R_t)$, the consumer's budget constraint is

$$\begin{aligned} & P_t w_t \left(l_t + f_t\right) h_t + P_t r_t s_{Gt} k_t + P_t R_{Qt} d_t + T_t \\ \geq \ & P_{Qt} q_t + P_t c_t + P_t k_{t+1} - P_t(1-\delta_k)k_t + M_{t+1} - M_t \\ & + B_{t+1} - B_t(1+R_t). \end{aligned} \tag{8}$$

The consumer can purchase the goods by using either money M_t or credit services. With the lump sum transfer of cash T_t coming from the government at the beginning of the period, and with money and credit equally usable to buys goods, the consumer's exchange technology is

$$M_t + T_t + P_t q_t \geq P_t c_t. \tag{9}$$

Since all cash comes out of deposits at the bank, and credit purchases are paid off at the end of the period out of the same deposits, the total deposits are equal to consumption. This gives the constraint that

$$d_t = c_t. \tag{10}$$

Given k_0, h_0, and the evolution of M_t $(t \geq 0)$ as given by the exogenous monetary policy in equation (18) below, the consumer maximizes utility subject to the budget,exchange and deposit constraints (8)-(10).

2.2 Banking Firm Problem

The bank produces credit that is available for exchange at the point of purchase. The bank determines the amount of such credit by maximizing its dividend profit subject to the labor and deposit costs of producing the credit. The production of credit uses a constant returns to scale technology with effective labor and deposited funds as inputs. In particular, with $A_Q > 0$ and $\gamma \in (0,1)$,

$$q_t = A_F e^{v_t} \left(f_t h_t\right)^{\gamma} d_t^{1-\gamma}, \tag{11}$$

where $A_F e^{v_t}$ is the stochastic factor productivity.

Subject to the production function in equation (11), the bank maximizes profit Π_{Ft} with respect to the labor f_t and deposits d_t:

$$\Pi_{Ft} = P_{Ft} q_t - P_t w_t f_t h_t - P_t R_{Ft} d_t. \tag{12}$$

Equilibrium implies that

$$\left(\frac{P_{Ft}}{P_t}\right) \gamma A_F e^{v_t} \left(\frac{f_t h_t}{d_t}\right)^{\gamma-1} = w_t; \tag{13}$$

$$\left(\frac{P_{Ft}}{P_t}\right) (1-\gamma) A_F e^{v_t} \left(\frac{f_t h_t}{d_t}\right)^{\gamma} = R_{Ft}. \tag{14}$$

These indicate that the marginal cost of credit, $\left(\frac{P_{Ft}}{P_t}\right)$, is equal to the marginal factor price divided by the marginal factor product, or $\frac{w_t}{\gamma A_F e^{v_t} \left(\frac{f_t h_t}{d_t}\right)^{\gamma-1}}$, and that the zero profit dividend yield paid on deposits is equal to the fraction of the marginal cost given by $\left(\frac{P_{Ft}}{P_t}\right) (1-\gamma) \left(\frac{q_t}{d_t}\right)$.

2.3 Goods Producer Problem

The firm maximizes profit given by $y_t - w_t l_t h_t - r_t s_{Gt} k_t$, subject to a standard Cobb-Douglas production function in effective labor and capital:

$$y_t = A_G e^{z_t} (s_{Gt} k_t)^{1-\alpha} (l_t h_t)^{\alpha}. \tag{15}$$

The first order conditions for the firm's problem yield the following expressions for the wage rate and the rental rate of capital:

$$w_t = \alpha A_G e^{z_t} \left(\frac{s_{Gt} k_t}{l_t h_t} \right)^{1-\alpha}, \tag{16}$$

$$r_t = (1-\alpha) A_G e^{z_t} \left(\frac{s_{Gt} k_t}{l_t h_t} \right)^{-\alpha}. \tag{17}$$

2.4 Government Money Supply

It is assumed that the government policy includes sequences of nominal transfers which satisfy:

$$T_t = \Theta_t M_t = (\Theta^* + e^{u_t} - 1) M_t, \qquad \Theta_t = [M_t - M_{t-1}]/M_{t-1}. \tag{18}$$

where Θ_t is the growth rate of money and Θ^* is the stationary gross growth rate of money.

2.5 Definition of Competitive Equilibrium

The representative agent's optimization problem can be written recursively as:

$$V(s) = \max_{c,x,l,n,f,s_G,q,d,k',h',M'} \{u(c,x) + \beta E V(s')\} \tag{19}$$

subject to the conditions (3) to (10), where the state of the economy is denoted by $s = (k, h, M, B; z, u, v)$ and a prime (') indicates the next-period values. A competitive equilibrium consists of a set of policy functions $c(s)$, $x(s)$, $l(s)$, $n(s)$, $f(s)$, $s_G(s)$, $q(s)$, $d(s)$, $k'(s)$, $h'(s)$, $M'(s)$, $B'(s)$ pricing functions $P(s)$, $w(s)$, $r(s)$, $R_F(s)$, $P_F(s)$ and a value function $V(s)$, such that:

(i) the consumer maximize utility, given the pricing functions and the policy functions, so that $V(s)$ solves the functional equation (19);

(ii) the goods producer maximizes profit similarly, with the resulting functions for w and r being given by equations (16) and (17);

(iii) the bank firm maximizes profit similarly in equation (12) subject to the technology of equation (11)

(iv) the goods, money and credit markets clear, in equations (8) and (15), and in (9), (18), and (11).

3 General Equilibrium Taylor Condition

The model's derived exact functional form of what we can here the "Taylor condition" closely shares many properties from the Taylor rules proposed in the literature. In this context, we show how the inclusion of endogenous growth and/or the banking sector leads to modified Taylor rules. We estimate the main Taylor condition plus several of these other rules in what are counter–theoretical based estimations. The idea emerges that a sufficiently rich flexible price economy model can provide an encompassing structure for the Taylor condition that may be able, in turn, to serve as a benchmark for optimal policy, rather than using any of a host of various rigid and flexible price models that then exogenously add in some form of a Taylor rule.

Starting from the first-order conditions of the model, we get

$$1 = \beta E_t \left\{ \frac{c_{t+1}^{-\theta} x_{t+1}^{\psi(1-\theta)}}{c_t^{-\theta} x_t^{\psi(1-\theta)}} \frac{\tilde{R}_t}{\tilde{R}_{t+1}} \frac{R_{t+1}}{\pi_{t+1}} \right\},$$

where R and π are gross rates of nominal interest and inflation, respectively, where $\tilde{R}_t$ is defined below as

$$\tilde{R}_t \equiv R_t - (1-\gamma)\left(1 - \frac{m_t}{c_t}\right)(R_t - 1),$$

and where $\frac{m_t}{c_t}$ is the normalized money demand, or called the "consumption velocity of money".

With log-linearization around the BGP, notationally first consider in general for the variable z that a lack of a time subscript indicates the BGP stationary value; $\widehat{z}_t \equiv \ln z_t - \ln z$; and $\widehat{g}_{z,t+1} \equiv \ln z_{t+1} - \ln z_t$, which is approximately the growth rate of z at time $t+1$ for small z. This then gives that

$$0 = E_t\left\{\theta \widehat{g}_{c,t+1} - \psi(1-\theta)\widehat{g}_{x,t+1} + \widehat{g}_{\tilde{R},t+1} + \widehat{\pi}_{t+1} - \widehat{R}_{t+1}\right\}$$

Rearranging with $\widehat{R}_t$ on the left-hand side, the Taylor Rule as expressed in log-deviations from the BGP is given as

$$\begin{aligned}\widehat{R}_t \;=\; & E_t\{\Omega\theta\widehat{g}_{c,t+1} - \Omega\psi(1-\theta)\widehat{g}_{x,t+1} + \Omega\widehat{\pi}_{t+1} \\ & -\frac{(1-\gamma)\left(1-\frac{m}{c}\right)}{R\left[1-(1-\gamma)\left(1-\frac{m}{c}\right)\right]}\left[\widehat{R}_{t+1} - (R-1)\frac{\frac{m}{c}}{1-\frac{m}{c}}\widehat{g}_{\frac{m}{c},t+1}\right]\right\}\end{aligned} \tag{20}$$

where

$$\begin{aligned}\Omega \;\equiv\; & \frac{R\left(1-\left[(1-\gamma)\left(1-\frac{m}{c}\right)\right]\right) + \left[(1-\gamma)\left(1-\frac{m}{c}\right)\right]}{R\left(1-\left[(1-\gamma)\left(1-\frac{m}{c}\right)\right]\right)} \\ =\; & 1 + \frac{(1-\gamma)\left(1-\frac{m}{c}\right)}{R\left[1-(1-\gamma)\left(1-\frac{m}{c}\right)\right]} \geq 1.\end{aligned}$$

The parameter Ω is important in that for example it determines whether the "Taylor principle" applies of having a coefficient greater than 1 on the expected inflation rate deviation, as discussed further below.

Consider some additional related notation that since

$$\widehat{g}_{t+1} = \ln\left(1+\overline{g}_{t+1}\right) - \ln(1+\overline{g}) \approx \overline{g}_{t+1} - \overline{g},$$

we can express the Taylor Condition in net rates and absolute differences from the BGP.

Proposition 1 *An equilibrium condition of the economy is in the form of the Taylor Rule (Orphanides, 2008) that sets deviations of the short-term nominal interest rate from some baseline path in proportion to deviations of target variables from their targets:*

$$\begin{aligned}\overline{R}_t - \overline{R} &= \Omega E_t\left(\overline{\pi}_{t+1} - \overline{\pi}\right) + \Omega\theta E_t\left(\overline{g}_{c,t+1} - \overline{g}\right) - \Omega\psi\left(1-\theta\right) E_t \overline{g}_{x,t+1} \\ &\quad - \frac{\left(1-\gamma\right)\left(1-\frac{m}{c}\right)}{R\left[1-\left(1-\gamma\right)\left(1-\frac{m}{c}\right)\right]}\left[E_t\left(\overline{R}_{t+1} - \overline{R}\right) - \left(R-1\right)\frac{\frac{m}{c}}{1-\frac{m}{c}} E_t \overline{g}_{\frac{m}{c},t+1}\right].\end{aligned} \tag{21}$$

where $\Omega \geq 1$, *and for a given* w, *then* $\frac{\partial\Omega}{\partial R} < 0$ *and* $\frac{\partial\Omega}{\partial A_F} > 0$, *and the target values are equal to the balanced growth path (BGP) equilibrium values.*[2]

Proof. Since the BGP solution for normalized money demand is

$$0 \leq \frac{m}{c} = 1 - A_F\left(\frac{\left(R-1\right)\gamma A_F}{w}\right)^{\frac{\gamma}{1-\gamma}} \leq 1,$$

then $\Omega \equiv 1 + \frac{\left(1-\gamma\right)\left(1-\frac{m}{c}\right)}{R\left[1-\left(1-\gamma\right)\left(1-\frac{m}{c}\right)\right]} > 1$ and, given w, $\frac{\partial\Omega}{\partial R} < 0$ and $\frac{\partial\Omega}{\partial A_F} > 0$.
■

For a linear production function of goods, w is the constant marginal product of labor, but more generally w is endogenous and will change; however this change in w quantitatively is small comparatively to changes in R and A_F, so that the derivatives above almost always hold true.

The fact that $\Omega > 1$ establishes the Taylor principle in theory of responding by more that one for one to changes in expected inflation rate departures from the "target", which here is the stationary balanced growth path equilibrium value. Second, this coefficient Ω

[2] This is the the Brookings project form of the Taylor rule as described in Orphanides (2008).

depends not only on the stationary nominal interest rate itself, but also on the productivity in the credity sector that allows inflation tax avoidance and that at the same time helps determine the money velocity. A lower R calls for a more aggressive response to increases in expected inflation above their target than does a higher R. And a higher productivity of financial intermediation production of exchange credit services causes a higher velocity and calls for a more aggressive response to expected inflation that is above its target.

The second two terms concerning $E_t\left(\overline{g}_{c,t+1}-\overline{g}\right)$ and $E_t\overline{g}_{x,t+1}$ are for the growth in consumption as compared to the BGP target and the growth in leisure, respectively; the latter employment growth is zero along the BGP. These two terms are the counterpart of the typical output gap. The last composite terms concerns what is typically known as interest smoothing components, involving forward looking behaviour for the nominal interest rate. These last two terms can be expressed perhaps more simply by denoting the consumption velocity of money as $V_t \equiv \frac{c_t}{m_t}$; if we also define "productive employment rate" as $l \equiv 1-x$, such that $\widehat{x}_t = -\frac{1-x}{x}\widehat{l}_t$, we can write the Taylor rule contained within the model as an equilibrium condition that has ready comparison to the literature.and writing the taylor condition as

$$\begin{aligned}\overline{R}_t-\overline{R} &= \Omega E_t\left(\overline{\pi}_{t+1}-\overline{\pi}\right)+\Omega\theta E_t\left(\overline{g}_{c,t+1}-\overline{g}\right)+\Omega\psi\left(1-\theta\right)\frac{l}{1-l}E_t\overline{g}_{l,t+1} \\ &\quad -\left(\Omega-1\right)E_t\left(\overline{R}_{t+1}-\overline{R}\right)-\Omega_V E_t\overline{g}_{V,t+1}, \qquad (22)\end{aligned}$$

again noting that 'bar' variables are net rates, and where

$$\Omega_V \equiv \frac{(R-1)}{R}\left(\frac{1-\gamma-(1-\gamma)\left(1-\frac{m}{c}\right)}{1-(1-\gamma)\left(1-\frac{m}{c}\right)}\right)=\frac{(R-1)}{R}\left(\frac{\gamma\frac{m}{c}}{\gamma+(1-\gamma)\frac{m}{c}}\right).$$

Proposition 2 *For the Taylor condition of equation (22), it is always true that* $0 \leq \Omega_V \leq 1 \leq \Omega$.

Proof.

$$\begin{aligned}
\Omega &\equiv \frac{R\left(1-(1-\gamma)\left(1-\frac{m}{c}\right)\right)+(1-\gamma)\left(1-\frac{m}{c}\right)}{R\left(1-(1-\gamma)\left(1-\frac{m}{c}\right)\right)} = 1+\frac{(1-\gamma)\left(1-\frac{m}{c}\right)}{R\left(1-(1-\gamma)\left(1-\frac{m}{c}\right)\right)}, \\
\Omega &= 1+\left(\frac{\omega}{R\,(1-\omega)}\right) \geq 1; \\
0 &\leq \omega \equiv (1-\gamma)\left(1-\frac{m}{c}\right) = (1-\gamma)\left[\gamma^{\gamma}\,(R-1)^{\gamma}\,w^{-\gamma}A_F\right]^{\frac{1}{1-\gamma}}, \\
\frac{m}{c} &= 1-A_F^{\frac{1}{1-\gamma}}\left[\frac{(R-1)\,\gamma}{w}\right]^{\frac{\gamma}{1-\gamma}} \leq 1, \\
1 &\geq 1-\gamma \geq \omega \geq 0. \\
\gamma+\omega &= \gamma+(1-\gamma)\left(1-\frac{m}{c}\right) = 1-\frac{m}{c}(1-\gamma) \leq 1. \\
&\Rightarrow \Omega_V \equiv \frac{(R-1)}{R}\left(\frac{1-\gamma-\omega}{1-\omega}\right) \leq 1. \\
&\Rightarrow 0 \leq \Omega_V \leq 1 \leq \Omega.
\end{aligned}$$

■

Note that at the Friedman (1969) optimum of $R=0$, then $\frac{m}{c}=1$, $\omega=0$, and the velocity coefficient is $\Omega_V=0$. The velocity term only matters when the nominal interest rate and inflation are away from the optimum and fluctuating. In turn, this has implications for $\Omega = 1+\left(\frac{\omega}{R(1-\omega)}\right)$, since when $R=0$, then $\Omega=1$, as in the Fisher equation of interest rates. Only as $\frac{m}{c}$ falls below one, and velocity rises above one, does the "Taylor principle" of $\Omega>1$ come into play, which of course is true for most practical experience.

Corollary 3 *Given* w, *then* $\frac{\partial\Omega}{\partial R}\geq 0$, $\frac{\partial\Omega_V}{\partial R}\geq 0$, $\frac{\partial\Omega}{\partial A_F}\geq 0$, $\frac{\partial\Omega_V}{\partial A_F}\leq 0$.

Proof. This comes directly from the definitions of parameters as given above. ■

A higher target R can be accomplished only by a higher BGP money supply growth rate. This would in turn make the inflation coefficient Ω higher and so also the consumption growth coefficient. For the "smoothing" terms, the forwards interest rate and velocity coefficients would be more negative. A higher credit productivity factor A_F, and so a higher velocity, causes a higher inflation coefficient, and a more negative response to the forward-looking interest term but a less negative coefficient on the velocity growth term.

3.1 Taylor Condition with Output Growth

It is not surprising to be targeting the growth of consumption rather than the growth in output as the latter one is more relevant to household welfare maximizers. However, the Taylor condition can be rewritten in output growth terms so as to be more comparable with standard Taylor rule specifications (in particular the speed limit versions of the output gap). For this, use the facts that

$$\begin{aligned} y_t &= c_t + i_t, \\ \widehat{y}_t &= \frac{c}{y}\widehat{c}_t + \frac{i}{y}\widehat{i}_t, \end{aligned}$$

and interpret $\widehat{i}_t$ using the fact that

$$\widehat{i}_t = \frac{k}{i}\left[\widehat{k}_t - (1-\delta)\,\widehat{k}_{t-1}\right],$$

so that the growth rate of investment can be understood as the acceleration of the growth of capital gross of depreciation. Then the rewritten Taylor condition is

$$\begin{aligned} \overline{R}_t - \overline{R} &= \Omega\theta\left[\frac{y}{c}E_t\left(\overline{g}_{y,t+1} - \overline{g}\right) - \frac{i}{c}E_t\left(\overline{g}_{i,t+1} - \overline{g}\right)\right] + \Omega\psi\,(1-\theta)\,\frac{l}{1-l}E_t\overline{g}_{l,t+1} \\ &\quad + \Omega E_t\left(\overline{\pi}_{t+1} - \overline{\pi}\right) - (\Omega - 1)\,E_t\left(\overline{R}_{t+1} - \overline{R}\right) - \Omega_V E_t\overline{g}_{V,t+1}. \qquad (23) \end{aligned}$$

The term with the investment growth rate in the formula does not appear in the standard exogenously specified Taylor rules, but here plays a role as part of what is interpreted as the output gap.

3.2 Backward Looking Taylor Condition

Mathematically the Taylor condition can also be formulated to have a "backward looking" interest rate term instead of the "forward looking" one above. This gives a similar equation that can also be estimated, by solving in terms of R_{t+1} instead of R_t.

$$\begin{aligned}\overline{R}_{t+1} - \overline{R} &= \frac{\Omega}{(\Omega-1)} E_t\left(\overline{\pi}_{t+1} - \overline{\pi}\right) + \frac{\Omega\theta}{(\Omega-1)} E_t\left(\overline{g}_{c,t+1} - \overline{g}\right) - \frac{\Omega\psi\left(\theta-1\right)\frac{l}{1-l}}{(\Omega-1)} E_t \overline{g}_{l,t+1} \\ &\quad - \frac{1}{(\Omega-1)}\left(\overline{R}_t - \overline{R}\right) - \frac{\Omega_V}{(\Omega-1)} E_t \overline{g}_{V,t}. \end{aligned} \tag{24}$$

However the coefficients here would be quite different from standard Taylor estimates. And more importantly this raises the issue that McCallum (2010) has often discussed. He argues that having a difference or differential equation in the equilibrium conditions does not mean that the equation can be interpreted as either forward looking or backward looking, as the researcher wishes. Rather the point is that the model implies that either one way or the other is unique, and the forward looking version is the long accepted rational expectations version. For example Lucas (1980, AER) suggests that the forward looking "filters" suit models of an optimizing consumer.

3.3 Credit Interpretation of the Taylor Condition

Christiano, Ilut, Motto and Rostagno (2011) have considered how the growth rate of credit might be a part of a Taylor rule:

> "Inflation is low during stock market booms, so that an interest rate rule that is too narrowly focused on inflation destabilizes asset markets and the broader economy. Adjustments to the interest rate rule can remove this source of welfare-reducing instability. For example, allowing an

independent role for credit growth (beyond its role in constructing the inflation forecast) would reduce the volatility of output and asset prices."

The term on the growth of velocity can also be interpreted as a growth of credit in the following way:

$$
\begin{aligned}
V_t &= \frac{1}{1-\left(1-\frac{m_t}{c_t}\right)} = \frac{1}{\frac{m_t}{c_t}}, \\
V\widehat{V}_t &= \left(1-\frac{m}{c}\right)\left(\widehat{1-\frac{m_t}{c_t}}\right)
\end{aligned}
$$

so that

$$
\overline{g}_{V,t} = \frac{m}{c}\left(1-\frac{m}{c}\right)\overline{g}_{\left(1-\frac{m}{c}\right),t}
$$

where $\overline{g}_{\left(1-\frac{m}{c}\right),t}$ is the growth rate of credit (per real money). The modified Taylor condition in the linear deviation form (with consumption growth) is

$$
\begin{aligned}
\overline{R}_t - \overline{R} &= \Omega\theta E_t\left(\overline{g}_{c,t+1}-\overline{g}\right) + \Omega\psi\left(1-\theta\right)\frac{l}{1-l}E_t\overline{g}_{l,t+1} + \Omega E_t\left(\overline{\pi}_{t+1}-\overline{\pi}\right) \\
&\quad -\Omega_R E_t\left(\overline{R}_{t+1}-\overline{R}\right) - \Omega_{\left(1-\frac{m}{c}\right)}E_t\overline{g}_{\left(1-\frac{m}{c}\right),t+1}. \qquad (25)
\end{aligned}
$$

Corollary 4 *The Taylor condition in equation (25) is characterized by* $\Omega_{\left(1-\frac{m}{c}\right)} \equiv \frac{(R-1)(1-\gamma-\omega)^2}{(1-\gamma)^2}\frac{\omega}{R(1-\omega)}$*; given* w*, then* $\frac{\partial\Omega_{\left(1-\frac{m}{c}\right)}}{\partial R} > 0$, $\frac{\partial\Omega_{\left(1-\frac{m}{c}\right)}}{\partial\gamma} > 0$.

A positive expected credit growth causes a negative effect on the current net nominal interest rate $\overline{R}_t$, which rises in magnitude as the BGP R rises and as the credit technology parameter γ rises.

4 Calibration

Here we follow Benk et al. (2010) in using postwar US data for the calibration:

Preferences		
θ	1	Relative risk aversion parameter
ψ	1.84	Leisure weight
β	0.96	Discount factor
Goods Production		
α	0.64	Labor share in goods production
δ_K	0.031	Depreciation rate of goods sector
A_G	1	Goods productivity parameter
Human Capital Production		
ε	0.83	Labor share in human capital production
δ_H	0.025	Depreciation rate of human capital sector
A_H	0.21	Human capital productivity parameter
Banking Sector		
γ	0.11	Labor share in credit production
A_F	1.1	Banking productivity parameter
Government		
σ	0.05	Money growth rate

Table 1: Parameters

g	0.024	Avg. annual output growth rate
π	0.026	Avg. annual inflation rate
R	0.0944	Nominal interest rate
l_G	0.248	Labor used in goods sector
l_H	0.20	Labor used in human capital sector
l_F	0.0018	Labor used in banking sector
i/y	0.238	Investment-output ratio in goods sector
m/c	0.38	Share of money transactions
x	0.55	Leisure time
$l \equiv 1 - x$	0.45	Productive time

Table 2: Target Values

For the calibration, we can now derive values of the coefficients expected in the estimation of the primary Taylor equation that will be used for estimation, that of equation (22):

$$\begin{aligned}\overline{R}_t - \overline{R} &= \Omega E_t\left(\overline{\pi}_{t+1} - \overline{\pi}\right) + \Omega\theta E_t\left(\overline{g}_{c,t+1} - \overline{g}\right) + \Omega\psi\left(1-\theta\right)\frac{l}{1-l}E_t\overline{g}_{l,t+1} \\ &\quad - \left(\Omega - 1\right)E_t\left(\overline{R}_{t+1} - \overline{R}\right) - \Omega_V E_t\overline{g}_{V,t+1}.\end{aligned}$$

For the inflation coefficient Ω, the calibrated value is

$$\begin{aligned}\Omega &= 1 + \frac{\omega}{R\left(1-\omega\right)} = 1 + \frac{\left(1-\gamma\right)\left(1-\frac{m}{c}\right)}{R\left[1-\left(1-\gamma\right)\left(1-\frac{m}{c}\right)\right]} \\ &= 1 + \frac{\left(1-0.11\right)\left(1-0.38\right)}{1.0944\left(1-\left(1-0.11\right)\left(1-0.38\right)\right)} \\ &= 2.125.\end{aligned}$$

And for $R = 1.0$, only cash is used, so that $\frac{m}{c} = 1$, and this coefficient goes to one; $\Omega = 1$. This similarly happens with zero credit productivity when $A_F = 0$ and only cash is used in exchange.

The other coefficients except for velocity are simple functions of the inflation coefficient. The consumption growth coefficient is $\Omega\theta$ which with $\theta = 1$ for log-utility, this coefficient is zero. With leisure preference at 1.84, and the productive time $1 - x \equiv l$ given at 0.55, the coefficient on the growth in productive time is

$$\Omega\psi\left(1-\theta\right)\frac{l}{1-l} = \left(2.125\right)\left(1.84\right)\left(1-1\right)\frac{0.55}{0.45} = 0.$$

Of course this still equals zero because of log utility, but any underestimation in the model of θ can then be seen to be factored by $\left(2.125\right)\left(1.84\right)\frac{0.55}{0.45} = 4.78$. The coefficient on the forward interest term is 1.125 given the inflation coefficient of 2.125, and the velocity coefficient Ω_V is

$$\begin{aligned}
\Omega_V &\equiv \frac{(R-1)}{R}\left(\frac{1-\gamma-\omega}{1-\omega}\right) \\
&= \frac{(R-1)}{R}\left(\frac{1-\gamma-(1-\gamma)\left(1-\frac{m}{c}\right)}{\left[1-(1-\gamma)\left(1-\frac{m}{c}\right)\right]}\right) = \frac{(R-1)}{R}\left(\frac{(1-\gamma)\frac{m}{c}}{\left[1-(1-\gamma)\left(1-\frac{m}{c}\right)\right]}\right) \\
&= \frac{(1.0944-1)}{1.0944}\left(\frac{(1-0.11)\,0.38}{(1-(1-0.11)\,(1-0.38))}\right) \\
&= 0.065088.
\end{aligned}$$

5 Artificial Data Estimation Results

The model is simulated to generate all of the variables of the economy, for a data sample of 100 years, with each period one year. Then the model is estimated over this 100 year annual simulated set of time series variables, for which the expected next period variables are computed exactly within the model. This simulation of 100 data points for all variables and the estimation of the Taylor condition is repeated 1000 times, and then the average of the 1000 estimation equation coefficients are presented. In particular, we draw 100 random sequences for the shock vector innovations and then use control functions of the log-linearized model to compute 100 sequences for the variables, including the one-period ahead forecasts, entering the Taylor condition. The BGP equilibrium values are used for the target values of the Taylor condition. Then we apply an OLS estimation to each model simulation, as in Orphanides AER (2001), while in addition using instrumental variable (IV) estimation to account for possible simultaneity, with both sets of results presented.

We report these results for for sets of data: 1) raw data, 2) HP filtered data, 3) a Christiano and Fitzgerald (2003) band pass filter that uses a 3 by 8 window for business cycles, and 4) a Christiano and Fitzgerald (2003) band pass filter with a 2 by 15 window that leaves in some of the longer run effects that shocks can have in an endogenous growth economy. For experimentation, a 2 by 100 Christiano and Fitzgerald (2003) band pass filter in effects takes out the mean and was found to give almost the same results as the raw data, which is stationary (results not reported).

Most Taylor rule estimations would use data filtered in ways similar to the second and third filters, and so these might provide the best analogue for a typical Taylor rule estimation. However we consider the 2 by 15 band pass results to be the "best" results as they also include the long run effects from changes in endogenous growth that happen over considerable periods for the shock persistence assumed here, and as is usual. In this model, the endogenous growth leads to effects over frequencies lower than the business cycle, according to findings in our previous related work (Benk et al, 2010). The shocks can cause the growth rate and the permanent income levels to shift over considerable periods of time.

The results are reported in turn for three different models: one as in equation (22), a second model in which the consumption growth rate is arbitrarily replaced by the output growth rate, and third an equation with only the standard Taylor rule of the inflation term, the output growth term and the interest smoothing term. But it is important to emphasize that for all three models we use the artifiicially simulated data from the full model, albeit using the four different filters. Therefore only the first model is correctly specified in accordance with the data. The other two models are counterfactual experiments designed to show the results with certain model misspecification.

Note that for each of the three models estimated from the simulated data, we also arbitrarily eliminate the forward looking interest term as an alternative in each estimation.

The main equation for estimation it that derived from equation

(22); it can be written as

$$R_t = \beta_0 + \beta_1 E_t \pi_{t+1} + \beta_2 E_t g_{c,t+1} + \beta_3 E_t g_{l,t+1} + \beta_4 E_t g_{V,t+1} + \beta_5 E_t R_{t+1} + \varepsilon_t.$$

For the instrument variables estimation, the following are used as instruments:

$$R_{t-1}, R_{t-2}, R_{t-3}, R_{t-4};$$
$$\pi_{t-1}, g_{c,t-1}, g_{l,t-1}, g_{v}, {}_{t-1}.$$

Results are presented in 12 tables, four for each of the three estimated equations, using each of the four data filters. Tables 3, 4, 5 and 6 present the results for the raw, HP filtered, 3x8 band-pass filtered data, and the 2x15 band-pass filtered data respectively for the main Taylor condition of the model (equation (22)), as given in the estimation form of the equation above. Generally the results move more towards the theoretical model as the reader procedes from the first to the fourth of the four tables of results.

In particular, in Table 6 all coefficients are in a sense as predicted by the theory except for one main issues. First, all variables are statistically significant for the full model; this feature is lost when the forward interest term is arbitrarily dropped. Second, the inflation coefficient is 2.1759 with OLS and 2.248 with IVs as compared to the theoretical value of 2.125, so this is clearly a good fit.

However the consumption growth term and productive time growth terms both have coefficients as if the relative risk aversion coefficient is less than one, rather than equaling one as in log-utility. Consider that for the consumption term the theoretical value is $\Omega\theta$. With $\Omega = 2.248$, then with an IV coefficient in column two of 0.3171, this simplies for the IV estimation that $\Omega\theta = (2.248)\,\theta = 0.3171$, so that $\theta = \frac{2.248}{0.3171} = 7.089$ which is too high.

However from the growth in productive time term, the IV estimate is -0.3764. Theoretically, this term is given by $\Omega\psi\,(1-\theta)\,\frac{l}{1-l} =$

$(2.248)(1.84)(1-\theta)\frac{0.55}{0.45} = 0.3764$, implying that

$$\begin{aligned} 0.3764 &= (2.248)(1.84)(1-\theta)\frac{0.55}{0.45}, \\ \theta &= 1 - \frac{0.3764}{(2.248)(1.84)\frac{0.55}{0.45}} \\ &= 0.9256. \end{aligned}$$

A coefficient of $\theta = 0.9256$ is quite close to the calibration of $\theta = 1$.

For velocity, the theoretical coefficient is -0.06508, as compared to the Table 6 statistically significant value of -0.1956 for OLS and -0.3125 for IVs, in the first two columns, which are higher. Finally, consider the forward looking interest rate coefficient in Table 6. These values in the first two columns are -1.758 and -1.592, and compare to the theoretical value of the negative of the inflation coefficient minus one. As estimated with IVs, this value is -1.248, so again these estimated values seem to be pretty much in the ballpark.

The upward bias in the consumption growth term may be an econometric feature that arises due to the estimation procedure that we have not identified, although the instrument sets for IV estimations do pass the Sargan test. On the other hand, the estimation results of Table 6 are very much what a researcher estimating a Taylor rule might call a successful estimation of the Taylor rule. And this is true even if it was estimated without any theoretical model to guide judgement of the results! Such Taylor rule estimations can be found in various forms throughout the literature.

The "Taylor principle" of the inflation coefficient exceeding one is robust, and robust in an interesting way. In Table 3, with raw data, the inflation coefficient is significant and less than one in all four cases. In Table 4, with HP data, the inflation coefficient now rises significantly above one for the model including the forward interest rate term, but below or bordering on one for the third and fourth columns without the forward interest term. Further the HP data finds the inflation coefficient between 1.5 and 1.7 when including the forward interest term, well below the theoretical value of 2.125. Once the 3x8 band pass filter is used in Table 5, the values are 2.175, 2.770 when including the forward interest term, but 0.634 and 2.4146 without the forward term. This variation settles down in the 2x15 Table 6 results, with coefficients near the theoretical value when including the forward interest term, but at 0.6136 and 0.9587 without the forward term. In other words, getting inflation coefficients less than one may be a result of model misspecification as is clear here.

The other interesting point of the Taylor principle is that here it results only when velocity is greater than one. Then the theory predicts a coefficient greater than one. This takes away the aura of mystery involved with a central bank that is acting by aggressive action to offset inflation increases by raising the interest rate by more than the inflation increase. Rather, it is a simple consequent of setting the money supply growth rate.

The next estimation equation and two tables with results takes a counter-factual view of our model. Instead of using the growth in consumption term we incorrectly substitute in a growth in output term, and do an otherwise similar set of estimations. The estimation equation is

$$R_t = \beta_0 + \beta_1 E_t \pi_{t+1} + \beta_2 E_t y_{g,t+1} + \beta_3 E_t l_{g,t+1} + \beta_4 E_t v_{g,t+1} + \beta_5 E_t R_{t+1} + \varepsilon_t,$$

and the instruments now are

Raw data			Assumed $\beta_5 = 0$	
	OLS	IV	OLS	IV
β_0	8.15E-06	3.19E-06	-2.33E-06	4.43E-06
	(0.0004) [0.0186]	(0.0004) [0.0081]	(0.0003) [-0.0069]	(0.0004) [0.0121]
$E_t\pi_{t+1}$	0.8367*	0.6051	0.7017*	0.8076*
	(0.2623) [3.1892]	(0.4755) [1.4563]	(0.0692) [10.1454]	(0.1113) [7.2544]
$E_tg_{c,t+1}$	0.177*	0.2021*	0.1636*	0.2516*
	(0.0282) [6.2724]	(0.1024) [1.9741]	(0.0154) [10.6043]	(0.0523) [4.8054]
$E_tg_{l,t+1}$	-0.0311	0.0848	-0.0174	0.0969
	(0.0818) [-0.3799]	(0.1437) [0.59]	(0.0738) [-0.2363]	(0.1335) [0.7257]
$E_tg_{V,t+1}$	-0.1693*	-0.2154*	-0.1671*	-0.2584*
	(0.0264) [-6.4048]	(0.0952) [-2.2628]	(0.0248) [-6.7515]	(0.0558) [-4.6312]
E_tR_{t+1}	-0.1698	0.3402	N/A	N/A
	(0.3076) [-0.5521]	(0.6309) [0.5392]		
R-square	0.7859	0.527	0.7755	0.6094
Adjust R-sqaure	0.7742	0.5004	0.7660	0.5921
F statistic	75.5656*	47.1891*	89.308*	67.1322*
Sargan's statistic	N/A	2.4628* {3}	N/A	4.361* {4}
Notes: () is standard error; [] is t-statistic; * represents significant at 95% confidence interval; {} degree of freedom				

Table 3: Taylor Condition Estimation, Raw Data, 100 Years Simulated, 1000 Estimations Average

HP data			Assumed $\beta_5 = 0$	
	OLS	IV	OLS	IV
β_0	-3.89E-06	-1.78E-06	-3.28E-06	-1.38E-06
	(6.49E-05) [-0.06]	(3.92E-05) [-0.0454]	(4.95E-05) [-0.0663]	(3.51E-05) [-0.0393]
$E_t\pi_{t+1}$	1.6811*	1.5211*	0.7159*	1.0028*
	(0.2345) [7.1673]	(0.5902) [2.5773]	(0.0667) [10.7357]	(0.1639) [6.1183]
$E_tg_{c,t+1}$	0.243*	0.2754*	0.159*	0.2261*
	(0.0215) [11.3137]	(0.0742) [3.71]	(0.0152) [10.4481]	(0.0516) [4.3809]
$E_tg_{l,t+1}$	-0.1894*	-0.1506	-0.1065	-0.0742
	(0.0723) [-2.6193]	(0.1426) [-1.056]	(0.0702) [-1.518]	(0.1144) [-0.6488]
$E_tg_{V,t+1}$	-0.1942*	-0.2855*	-0.1718*	-0.2496*
	(0.0244) [-7.9655]	(0.0613) [-4.6556]	(0.0242) [-7.0966]	(0.0498) [-5.0115]
E_tR_{t+1}	-1.1375*	-0.7118	N/A	N/A
	(0.2668) [-4.264]	(0.7986) [-0.8913]		
R-square	0.7999	0.5604	0.7085	0.5227
Adjust R-square	0.7891	0.5357	0.696	0.5015
F statistic	80.2562*	51.3676*	60.9254*	52.2328*
Sargan's statistic	N/A	3.1274* {3}	N/A	4.3747* {4}
Notes: () is standard error; [] is t-statistic; * represents significant at 95% confidence interval; {} degree of freedom				

Table 4: Taylor Condition Estimation, HP Filtered Data, 100 Years Simulated, 1000 Estimations Average

BP Filter:			Assumed $\beta_5 = 0$	
3x8 Window	OLS	IV	OLS	IV
β_0	-8.09E-08	-4.56E-07	-7.32E-07	-3.06E-06
	(1.77E-05) [-0.0456]	(1.28E-05) [-0.0356]	(1.48E-05) [-0.0496]	(4.41E-05) [-0.0694]
$E_t\pi_{t+1}$	2.1746*	2.7704*	0.634*	2.4156*
	(0.3826) [5.6835]	(0.3731) [7.4262]	(0.1956) [3.2417]	(1.1099) [2.1764]
$E_t g_{c,t+1}$	0.2833*	0.3297*	0.1549*	0.1744*
	(0.0427) [6.642]	(0.0369) [8.9336]	(0.0288) [5.3773]	(0.0737) [2.3655]
$E_t g_{l,t+1}$	-0.238	-0.4413*	-0.2218	-0.5969
	(0.1299) [-1.8325]	(0.1205) [-3.6638]	(0.1331) [-1.6667]	(0.3599) [-1.6584]
$E_t g_{V,t+1}$	-0.1522*	-0.3042*	-0.1743*	-0.6034*
	(0.0428) [-3.5603]	(0.0571) [-5.3271]	(0.0519) [-3.357]	(0.2492) [-2.4212]
$E_t R_{t+1}$	-2.0338*	-2.2182*	N/A	N/A
	(0.4254) [-4.7809]	(0.4535) [-4.8913]		
R-square	0.7907	0.7368	0.5762	N/A
Adjust R-square	0.7794	0.722	0.5582	N/A
F statistic	86.8317*	80.0176*	35.4542*	18.7186*
Sargan's statistic	N/A	7.4585* {3}	N/A	2.0652* {1}
Notes: () is standard error; [] is t-statistic; * represents significant at 95% confidence interval; {} degree of freedom The instruments when $\beta_5 = 0$ are $R_{t-1}, \pi_{t-1}, g_{c,t-1}, g_{l,t-1}, g_{v,t-1}$				

Table 5: Taylor Condition Estimation, Band Pass Filtered Data (8 years), 100 Years Simulated, 1000 Estimations Average

BP Filter			Assumed $\beta_5 = 0$	
2x15 Window	OLS	IV	OLS	IV
β_0	-2.48E-06	-2.06E-06	-1.78E-06	-1.98E-06
	(4.35E-05) [-0.057]	(3.07E-05) [-0.0672]	(3.35E-05) [-0.0532]	(3.08E-05) [-0.0642]
$E_t\pi_{t+1}$	2.1759*	2.2484*	0.6136*	0.9587*
	(0.1945) [11.1867]	(0.4885) [4.6022]	(0.107) [5.7356]	(0.2451) [3.9112]
$E_t g_{c,t+1}$	0.2769*	0.3171*	0.1701*	0.2707*
	(0.016) [17.2929]	(0.0347) [9.139]	(0.0174) [9.7592]	(0.0429) [6.3073]
$E_t g_{l,t+1}$	-0.294*	-0.3764*	-0.2103*	-0.2586*
	(0.0676) [-4.3476]	(0.1012) [-3.7192]	(0.0882) [-2.3847]	(0.1248) [-2.072]
$E_t g_{V,t+1}$	-0.1956*	-0.3125*	-0.1583*	-0.2998*
	(0.0245) [-79976]	(0.0378) [-8.2683]	(0.0315) [-5.0211]	(0.0514) [-5.8295]
$E_t R_{t+1}$	-1.7576*	-1.5923*	N/A	N/A
	(0.1989) [-8.8347]	(0.5732) [-2.7779]		
R-square	0.8288	0.6648	0.6247	0.3197
Adjust R-square	0.8196	0.646	0.6087	0.2895
F statistic	95.5996*	61.6562*	41.5342*	35.3063*
Sargan's statistic	N/A	4.897* {3}	N/A	8.5164* {4}
Notes: () is standard error; [] is t-statistic; * represents significant at 95% confidence interval {} degree of freedom				

Table 6: Taylor Condition Estimation, Band Pass Filtered Data (15 years), 100 Years Simulated, 1000 Estimations Average

$$R_{t-1}, R_{t-2}, R_{t-3}, R_{t-4};$$

$$\pi_{t-1}, y_{g,t-1}, l_{g,t-1}, v_{g,t-1}.$$

Tables 7-10 are directly comparable to the Tables 3-6 above except for the arbitrary, counter-theoretical change in one variable, now including output growth, as in the "speed limit" definition of the output gap, in place of the consumption growth term. The results for the inflation coefficient are markedly different from the correct model results in Tables 3-6. The HP results show inflation coefficient above one in several cases, similar to results seen in the literature. However using the band pass filters the results become much more erratic, and do not come close to the theoretical value for the coefficient as in Tables 5 and 6. Rather they jump to 3.5 and 19 for the 3x8 band pass results with the forward interest term, and to 3.9 without the forward term and with IVs, while being below one for the case without the forward interest term and with OLS. The 2x15 band pass results are similar with coefficients of 4 and 11 with the forward interest term, and of 0.5 and 2.3 without the forward term.

Interesting the velocity growth term remains signficant, of the right sign and of a similar magnitude in most of the results. This implies that excluding this variable may lead to misspecification such as in almost all Taylor rule estimations found in the literature.

The forward interest term itself becomes insignificant in five of six of the filtered results. The output growth, growth in productive time, and velocity terms also have much more variable results across the estimations. This makes it difficult to choose the "best" model as their is little confidence that one such model is best. Basically it is surprising how this one deviation from the true model that generated the artificial data could give such more volatile results.

Raw data			Assumed $\beta_5 = 0$	
	OLS	IV	OLS	IV
β_0	-4.68E-06	-3.5E-07	-8.21E-07	-2.6E-05
	(0.0002) [-0.022]	(0.0006) [0.0005]	(0.0004) [-0.002]	(0.0006) [-0.0427]
$E_t\pi_{t+1}$	0.2582	0.357	0.7412*	1.1486*
	(0.213) [1.2126]	(1.0202) [0.3499]	(0.0944) [7.8495]	(0.1684) [6.8202]
$E_t g_{y,t+1}$	0.0094	0.0672	0.0566*	0.2031*
	(0.0244) [0.3853]	(0.1998) [0.3366]	(0.0155) [3.6516]	(0.095) [2.1377]
$E_t g_{l,t+1}$	-0.0881	-0.2823	-0.3366*	-0.8679*
	(0.1308) [-0.6736]	(0.8729) [-0.3234]	(0.1041) [-3.2346]	(0.3255) [-2.6659]
$E_t g_{V,t+1}$	-0.1447*	-0.0587	-0.1329*	-0.0978
	(0.0252) [-5.7478]	(0.0936) [-0.6275]	(0.0282) [-4.7107]	(0.0717) [-1.3646]
$E_t R_{t+1}$	0.5832*	0.995	N/A	N/A
	(0.2586) [2.2552]	(1.2053) [0.8255]		
R-square	0.6224	N/A	0.5852	N/A
Adjust R-square	0.6021	N/A	0.5676	N/A
F statistic	34.3579*	18.2004*	37.1515*	23.976*
Sargan statistic	N/A	2.5382* {3}	N/A	5.2205* {4}
Notes: () is standard error; [] is t-statistic; * represents significant at 95% confidence interval {} degree of freedom				

Table 7: Output Growth instead of Consumption Growth, Raw Data, 100 Years Simulated, 1000 Estimations Average

It illustrates that if our model was the true model of the economy, and this second estimation model was used, then the results would become much more difficult to have confidence in. The results here echo back to the lack of robustness that has been found for example for interest smoothing terms, and for whether the Taylor principle holds or not in different periods. It should hold in ALL periods, once other factors are included within the model.

Finally consider a way to take one step further using the same simulated data from the general equilibrium economy, while now estimating a bog-standard Taylor rule equation. The estimated equation for such a model is given here as

$$R_t = \alpha + \beta_1 \pi_t + \beta_2 g_{y,t} + \beta_3 E_t R_{t+1} + \varepsilon_t,$$

HP data			Assumed $\beta_5 = 0$	
	OLS	IV	OLS	IV
β_0	-3.17E-06	-4.31E-06	-2.37E-06	-8.84E-07
	(4.12E-05) [-0.077]	(7.36E-05) [-0.0586]	(3.6E-05) [-0.0659]	(5.47E-05) [-0.0161]
$E_t\pi_{t+1}$	1.0886*	2.1411*	0.7243*	1.4884*
	(0.5873) [1.8537]	(2.849) [0.7515]	(0.1017) [7.1206]	(0.2455) [6.0625]
$E_t g_{y,t+1}$	0.0736	0.2234	0.0458*	0.1545
	(0.0474) [1.5539]	(0.3396) [0.6579]	(0.0147) [3.1214]	(0.1166) [1.3247]
$E_t g_{l,t+1}$	-0.5084	-1.3189	-0.3266*	-0.9081
	(0.3119) [-1.6299]	(1.8816) [-0.7009]	(0.1106) [-2.9534]	(0.5026) [-1.807]
$E_t g_{V,t+1}$	-0.1308*	-0.1413	-0.1318*	-0.1273
	(0.0314) [-4.1656]	(0.1071) [-1.3197]	(0.0305) [-4.3232]	(0.0696) [-1.8279]
$E_t R_{t+1}$	-0.4029	-0.7159	N/A	N/A
	(0.6526) [-0.6174]	(3.189) [-0.2245]		
R-square	0.4556	N/A	0.4461	N/A
Adjust R-square	0.4264	N/A	0.4225	N/A
F statistic	16.9992*	10.5689*	20.7177*	15.5229*
Sargan statistic	N/A	3.7252* {3}	N/A	5.5669* {4}
Notes: () is standard error; [] is t-statistic; * represents significant at 95% confidence interval; {} degree of freedom				

Table 8: Output Growth instead of Consumption Growth, HP Filtered, 100 Years Simulated, 1000 Estimations Average

with instruments of :

$$R_{t-1}, R_{t-2}, R_{t-3}, R_{t-4};$$
$$\pi_{t-1}, g_{y,t-1}.$$

Tables 11-14 show first that the inflation coefficient is above one, in a reasonable range, and statistically significant for only the IV estimation for the HP filters, when the forward interest rate term is dropped. For the 2x15 filter the IV coefficient again becomes too high. For all other filtered cases, the inflation coefficient is only significant in the OLS estimation without the forward interest term, but with coefficients between 0.25 and 0.5. Thus the Taylor principle fails in all but one of the 12 filtered estimation results.

Similarly the output growth term becomes insignificant in all but one of the filtered estimation results. For the HP filter, with OLS

BP Filter			Assumed $\beta_5 = 0$	
3x8 Window	OLS	IV	OLS	IV
β_0	-1.34E-06	-3.59E-06	-6.38E-07	-7.97E-06
	(1.88E-05) [-0.0711]	(0.0001) [-0.0308]	(1.28E-05) [-0.0499]	(8.21E-05) [-0.0971]
$E_t\pi_{t+1}$	3.4709*	19.4083	0.7351*	3.8595
	(3.2984) [1.0523]	(10.3917) [1.8677]	(0.2582) [2.8464]	(1.9897) [1.9397]
$E_t g_{y,t+1}$	0.2325	1.4064	0.0357	0.1509
	(0.2426) [0.9582]	(0.8027) [1.7521]	(0.0261) [1.3675]	(0.1067) [1.414]
$E_t g_{l,t+1}$	-1.6512	-9.9752	-0.306	-1.7217
	(1.7057) [-0.9681]	(5.5867) [-1.7855]	(0.2591) [-1.1809]	(1.1505) [-1.4965]
$E_t g_{V,t+1}$	-0.1465*	-0.406*	-0.1464*	-0.8419
	(0.0679) [-2.1566]	(0.1724) [-2.3555]	(0.0659) [-2.2231]	(0.465) [-1.8106]
$E_t R_{t+1}$	-2.974	-19.3281	N/A	N/A
	(3.5332) [-0.8417]	(11.4718) [-1.6848]		
R-square	0.3407	N/A	0.2631	N/A
Adjust R-square	0.3053	N/A	0.2318	N/A
F statistic	11.234*	10.9357*	9.3531*	7.4618*
Sargan statistic	N/A	5.4738* {2}	N/A	2.8875 {1}

Notes: () is standard error; [] is t-statistic; * represents significant at 95% confidence interval; {} degree of freedom

The instruments when $\beta_5 = 0$ are $R_{t-1}, \pi_{t-1}, g_{y,t-1}, g_{l,t-1}, g_{v,t-1}$

The instruments when $\beta_5 \neq 0$ are $R_{t-1}, R_{t-2}, R_{t-3}, \pi_{t-1}, g_{y,t-1}, g_{l,t-1}, g_{v,t-1}$

Table 9: Output Growth instead of Consumption Growth, Band Pass Filtered data (3x8 years), 100 Years Simulated, 1000 Estimations Average

BP Filter			Assumed $\beta_5 = 0$	
2x15 Window	OLS	IV	OLS	IV
β_0	-3.69E-06	-7.87E-06	-8.79E-07	-4.94E-06
	(6.46E-05) [-0.0571]	(0.0001) [-0.0631]	(2.49E-05) [-0.0353]	(9.61E-05) [-0.0514]
$E_t\pi_{t+1}$	4.1282*	10.8617*	0.5416*	2.3462*
	(1.6764) [2.4625]	(5.2505) [2.0687]	(0.1696) [3.1927]	(1.1765) [1.9943]
$E_t g_{y,t+1}$	0.2958*	0.9448	0.0384*	0.2096
	(0.1223) [2.418]	(0.4948) [1.9093]	(0.0196) [1.9632]	(0.3167) [0.6616]
$E_t g_{l,t+1}$	-2.0516*	-6.3939	-0.2833	-1.4587
	(0.8772) [-2.3388]	(3.3234) [-1.9239]	(0.1894) [-1.4958]	(1.7819) [-0.8187]
$E_t g_{V,t+1}$	-0.1175*	-0.3193*	-0.0952*	-0.2468
	(0.0422) [-2.7816]	(0.1471) [-2.1711]	(0.0428) [-2.2249]	(0.1643) [-1.502]
$E_t R_{t+1}$	-3.7842*	-10.2081	N/A	N/A
	(1.7718) [-2.1358]	(5.7383) [-1.7789]		
R-square	0.3583	N/A	0.2459	N/A
Adjust R-square	0.3238	N/A	0.2138	N/A
F statistic	11.3053*	7.4463*	8.389*	6.084*
Sargan statistic	N/A	6.9086* {3}	N/A	3.6385* {1}
Notes: () is standard error; [] is t-statistic; * represents significant at 95% confidence interval; {} degree of freedom				
The instruments when $\beta_5 = 0$ are $R_{t-1}, \pi_{t-1}, g_{y,t-1}, g_{l,t-1}, g_{v,t-1}$				

Table 10: Output Growth instead of Consumption Growth, Band Pass Filtered data (2x15 years), 100 Years Simulated, 1000 Estimations Average

and without the forward interest term, the output growth term is significant, of the right sign, but close to zero at 0.028. The last variable, the forward interest term, is insignificant in all 6 filtered estimation cases.

The results of Tables 11-14 show a breaking down of the Taylor principle, a lack of interest smoothing, and little or no effect of the output growth deviations. Such results in the literature have been simply interpreted as implying that the central bank was strong on inflation, with little feedback from output deviation, and that there was no interest smoothing during the estimation period. And maybe the one result with the Taylor principle holding would be presented, or maybe the other results would also be presented. It might be concluded, again simply, that the Taylor principle was not holding, or it was, while the central bank was had a strong inflation focus,

Raw data			Assumed $\beta_3 = 0$	
	OLS	IV	OLS	IV
β_0	-8.43E-06	1.02E-06	-6.38E-06	-1.81E-06
	(0.0002) [-0.035]	(0.0005) [0.002]	(0.0004) [-0.0165]	(0.0009) [-0.0021]
π_t	0.1405	0.0468	0.579*	0.9708*
	(0.1822) [0.771]	(0.4676) [0.0979]	(0.0842) [6.8801]	(0.2983) [3.2546]
$g_{y,t}$	0.0154	0.0069	0.0384*	0.032
	(0.0147) [1.048]	(0.1185) [0.0582]	(0.0129) [2.9691]	(0.3055) [0.1046]
$E_t R_{t+1}$	0.5438*	1.301*	N/A	N/A
	(0.2132) [2.5505]	(0.5954) [2.185]		
R-square	0.5084	N/A	0.4676	N/A
Adjust R-square	0.5235	N/A	0.4565	N/A
F statistic	37.8863*	25.3571*	46.2738*	28.9338*
Sargan statistic	N/A	2.6444* {3}	N/A	5.8272* {2}
Notes: () is standard error; [] is t-statistic; * represents significant at 95% confidence interval; {} degree of freedom				
The instruments when $\beta_3 = 0$ are $R_{t-1}, R_{t-2}, \pi_{t-1}, g_{y,t-1}$				

Table 11: Output Growth in Standard Taylor Rule, Raw Data, 100 Years Simulated, 1000 Estimations Average

although maybe not an "aggressive" one as in the Taylor principle.

Of course all such interpretations of the results would be spurrious as they result simply from model misspecification.

6 Taylor Condition under Special Cases

Below we show how the form of the GE Taylor rules changes when the modelling setup changes. First we explore how the Taylor principle is lost in a cash only economy, when the only change is to exclude the exchange credit as an alternative to cash. Second, in an economy without physical capital, the output growth term replaces the consumption growth term, in that theoretical the two are the same. If we also make exchange credit prohibitive and eliminate leisure preference, then the traditional Taylor (1993) rule with the additional restriction that the coefficient on inflation is one. Finally the exogenous growth version of the model is presented, whereby

HP data			Assumed $\beta_3 = 0$	
	OLS	IV	OLS	IV
β_0	-1.74E-06	1.61E-07	-1.64E-06	2.05E-06
	(2.48E-05) [-0.0701]	(6.18E-05) [0.026]	(2.47E-05) [-0.0662]	(6.11E-05) [0.0336]
π_t	0.2089	0.1679	0.4913*	1.2463*
	(0.2836) [0.7366]	(1.0468) [0.1604]	(0.0845) [5.8126]	(0.4415) [2.8232]
$g_{y,t}$	0.0172	0.0155	0.0281*	0.0155
	(0.0152) [1.1307]	(0.1305) [0.1189]	(0.012) [2.3509]	(0.2085) [0.0743]
$E_t R_{t+1}$	0.3201	1.3556	N/A	N/A
	(0.3084) [1.0381]	(1.11) [1.2213]		
R-square	0.3354	N/A	0.3148	N/A
Adjust R-square	0.3144	N/A	0.3005	N/A
F statistic	16.9139*	12.3363*	23.4052*	20.2213*
Sargan statistic	N/A	4.5173* {3}	N/A	7.6602* {3}
Notes: () is standard error; [] is t-statistic; * represents significant at 95% confidence interval; {} degree of freedom				
The instruments when $\beta_3 = 0$ are $R_{t-1}, R_{t-2}, R_{t-3}, \pi_{t-1}, g_{y,t-1}$				

Table 12: Output Growth in Standard Taylor Rule, HP Filtered Data, 100 Years Simulated, 1000 Estimations Average

BP Filter			Assumed $\beta_3 = 0$	
3x8 Window	OLS	IV	OLS	IV
β_0	-4.75E-07	-2.12E-05	-4.06E-07	-5.65E-06
	(1.33E-05) [-0.0358]	(0.0003) [-0.061]	(1.33E-05) [-0.0306]	(0.0001) [-0.0512]
π_t	0.315	2.2666	0.2541*	1.1365
	(0.5936) [0.5308]	(9.395) [0.2413]	(0.0832) [3.0558]	(3.4309) [0.3313]
$g_{y,t}$	0.0184	0.5236	0.0173	0.3275
	(0.0246) [0.7474]	(2.3498) [0.2228]	(0.0176) [0.9837]	(0.3067) [1.068]
$E_t R_{t+1}$	-0.0737	-2.9147	N/A	N/A
	(0.6201) [-0.1189]	(11.221) [-0.2598]		
R-square	0.1349	N/A	0.1117	N/A
Adjust R-square	0.1076	N/A	0.0932	N/A
F statistic	5.2231*	3.0513*	6.3547*	7.1777*
Notes: () is standard error; [] is t-statistic; * represents significant at 95% confidence interval; {} degree of freedom The instruments when $\beta_3 = 0$ are $\pi_{t-1}, g_{y,t-1}$ The instruments when $\beta_3 \neq 0$ are $R_{t-1}, \pi_{t-1}, g_{y,t-1}$				

Table 13: Output Growth in Standard Taylor Rule, Band Pass filter Data (8 years), 100 Years Simulated, 1000 Estimations Average

BP Filter			Assumed $\beta_3 = 0$	
2x15 Window	OLS	IV	OLS	IV
β_0	-6.03E-07	-8.11E-05	-5.32E-07	-0.0003
	(2.6E-05) [-0.0232]	(0.0078) [-0.0104]	(2.59E-05) [-0.0206]	(0.0119) [-0.0244]
π_t	0.3114	12.9747	0.3265*	10.336
	(0.4455) [0.6989]	(480.1135) [0.027]	(0.0993) [3.2887]	(246.9293) [0.0419]
$g_{y,t}$	0.0198	-0.6661	0.0207	-1.056
	(0.0173) [1.1461]	(33.8783) [-0.0197]	(0.012) [1.7299]	(32.9489) [-0.0321]
$E_t R_{t+1}$	0.0071	-5.7142	N/A	N/A
	(0.4727) [0.015]	(387.8748) [-0.0147]		
R-square	0.1684	N/A	0.1533	N/A
Adjust R square	0.1422	N/A	0.1356	N/A
F statistic	6.7896*	1.3647	9.2247*	2.6632
Notes: () is standard error; [] is t-statistic; * represents significant at 95% confidence interval; {} degree of freedom The instruments when $\beta_3 = 0$ are $\pi_{t-1}, g_{y,t-1}$				
The instruments when $\beta_3 \neq 0$ are $R_{t-1}, \pi_{t-1}, g_{y,t-1}$				

Table 14: Output Growth in Standard Taylor Rule, Band Pass filter Data (15 years), 100 Years Simulated, 1000 Estimations Average

only investment in human capital is eliminated. The difference in the derived Taylor condition is that the targets are now strictly exogenous as well. In contrast these are the BGP solutions with endogenous growth.

Note that for all of these special cases of our economy, we could run the same experiment of generating artificial data and then estimating the Taylor condition to show that these estimations reveal only how the central bank is conducting its money supply policy. Again it would be true that these are not the result of some type of aggressive or passive reaction to inflation in terms of setting the interest rate. But in truth, each is still equivalent to the other, and either interpretation is reasonable. The point remains to derive the precise form of the Taylor condition within the best general equilibrium structure that we can, so that when we estimate such conditions from the actual economy, we can understand our results and where such misspecification bias might lay. This is a huge murky grey area at present, which motivates looking at these other cases of the model.

6.1 GE Taylor for CIA Endogeneous Growth Economy

One special case of the economy is when there is no banking, no credit production and a cash-only economy occurs. This holds if $A_F = 0$, with the result of the endogenous growth model with a CIA constraint as in Gomme (1993) or relatedly Jones, Manuelli, and Siu (2007). In this case, the Taylor condition can be expressed as

$$\overline{R}_t - \overline{R} = \theta E_t \left(\overline{g}_{c,t+1} - \overline{g}\right) + \psi \left(1 - \theta\right) E_t \overline{g}_{l,t+1} + E_t \left(\overline{\pi}_{t+1} - \overline{\pi}\right).$$

The most striking differences here are that the Taylor condition does not contain the interest-rate-smoothing terms and the Taylor principle is only marginally met in that now $\Omega = 1$, rather than $\Omega > 1$. Therefore the endogenization of velocity through credit production is key to establishing in theory the Taylor principle and forward-looking interest-smoothing terms.

6.2 GE Taylor with No Physical Capital

Keeping in endogenous velocity, with $A_F > 0$, but having an economy without physical capital give the special case of a human capital only economy, as in the framework of Gillman and Kejak (2005) or relatedly Hromcova (2008). Now the growth of consumption coincides with the growth of output, but there are still employment and and interest smoothing terms, along with the Taylor principle being satisfied.

$$\begin{aligned}\overline{R}_t - \overline{R} &= \Omega\theta E_t \left(\overline{g}_{y,t+1} - \overline{g}\right) + \Omega\psi \left(1 - \theta\right) \frac{l}{1-l} E_t \overline{g}_{l,t+1} + \Omega E_t \left(\overline{\pi}_{t+1} - \overline{\pi}\right) \\ &\quad - \Omega_R E_t \left(\overline{R}_{t+1} - \overline{R}\right) - \Omega_V E_t \overline{g}_{V,t+1}.\end{aligned}$$

However if further, there is no leisure choice, in that $\psi = 0$, or alternatively if there is log-utility ($\theta = 1$), then the Taylor condition no longer includes the term for the expected growth in productive time :

$$\begin{aligned}\overline{R}_t - \overline{R} &= \Omega\theta E_t\left(\overline{g}_{y,t+1} - \overline{g}\right) + \Omega E_t\left(\overline{\pi}_{t+1} - \overline{\pi}\right) \\ &\quad -\Omega_R E_t\left(\overline{R}_{t+1} - \overline{R}\right) - \Omega_V E_t \overline{g}_{V,t+1}.\end{aligned}$$

And if there is no banking, in that $A_F = 0$, then GE Taylor Rule collapses to the commonly used Taylor rule without interest rate smoothing as in Taylor (1993), except for a marginal Taylor principle of $\Omega = 1$:

$$\overline{R}_t - \overline{R} = \theta E_t\left(\overline{g}_{y,t+1} - \overline{g}\right) + E_t\left(\overline{\pi}_{t+1} - \overline{\pi}\right).$$

6.3 GE Taylor Rule for Exogeneous Growth Economy

Interestingly, the GE Taylor rule has the same form even for the exogenous growth economy

$$\begin{aligned}\overline{R}_t - \overline{R} &= \Omega\theta E_t\left(\overline{g}_{c,t+1} - \overline{g}\right) + \Omega\psi\left(1-\theta\right)\frac{l}{1-l}E_t\overline{g}_{l,t+1} + \Omega E_t\left(\overline{\pi}_{t+1} - \overline{\pi}\right) \\ &\quad -\Omega_R E_t\left(\overline{R}_{t+1} - \overline{R}\right) - \Omega_V E_t \overline{g}_{V,t+1}\end{aligned}$$

in which case $\overline{g}$ stands for the exogenous growth rate of labor-augmenting productivity. And if there is no physical capital, then again $y = c$ and

$$\begin{aligned}\overline{R}_t - \overline{R} &= \Omega\theta E_t\left(\overline{g}_{y,t+1} - \overline{g}\right) + \Omega\psi\left(1-\theta\right)\frac{l}{1-l}E_t\overline{g}_{l,t+1} + \Omega E_t\left(\overline{\pi}_{t+1} - \overline{\pi}\right) \\ &\quad -\Omega_R E_t\left(\overline{R}_{t+1} - \overline{R}\right) - \Omega_V E_t \overline{g}_{V,t+1}.\end{aligned}$$

With no leisure preference, and no banking ($A_F = 0$), again the simple standard from results.

$$\overline{R}_t - \overline{R} = \theta E_t\left(\overline{g}_{y,t+1} - \overline{g}\right) + E_t\left(\overline{\pi}_{t+1} - \overline{\pi}\right).$$

Despite the fact that the above Taylor conditions looks the same for the models with exogenous and endogenous growth, there is a fundamental difference between these two. First, and most importantly, under the exogenous growth setup the targeted inflation rate and growth rate of the economy are completely unrelated (and so considered independently). In contrast, under the endogenous growth setup the targets for inflation, the growth rate, and the nominal interest rate are all endogenously determined. Only in the log-utility case is the nominal interest R independent of the growth rate, and rather a simple additive function approximately of the stationary money supply growth rate and the rate of time preference. However, in the log-utility case, the BGP inflation rate and growth rate are still simultaneously determined as endogenous variables.

7 Discussion

While there is an extensive and a growing body of literature on the issues of indeterminacy in economies starting with Sargent and Wallace (1975) and McCallum (1981) and recently by Cochrane (2007), Atkeson, Chari and Kehoe (2009) and Adao, Correia and Teles (2011), here we consider only the equilibrium within the general equilibrium economy. The price path is quite determinate. And the policy associated with the Taylor condition is fully implementable, through money supply policy in the sense of Alvarez, Lucas and Weber (2001). "Targets" now have a precise economic equilibrium determination rather than being arbitrarily set exogenous parts of a Taylor rule. The targets are the economy's BGP equilibrium values.

The structure of a general equilibrium Taylor Condition results in a specific form from the economy. Therefore, if the economy contains an equilibrium condition for the nominal interest rate, in the form here that we call the Taylor Condition, while derived from a flexible price DSGE model with exogenous money supply, then the economy behaves as the one under an exogenous growth money supply process, even while exhibiting a type of Taylor rule, "central bank interest targeting", behavior. This is the key point of the paper. Further, estimation of the Taylor condition using simulated data verifies that such a Taylor rule estimation would spuriously be called central bank interest targeting behavior; rather such estimations in the literature may simply be estimating the equilibrium condition of the economy with a money supply policy.

Note that regarding inflation persistence, Dittmar, Gavin and Kydland (2005) focus on how assumptions about money supply within a Taylor rule context can lead to any degree of inflation persistence. This would also be expected within our model here. Exploring this would be an interesting extension.

There are some real results in the above tables. First the "Taylor principle" of an above unity inflation coefficient happens in the theory only when money velocity is greater than one, and indeed it is found to hold very near to its theoretical value in the estimation of the correct model in the first set of tables. At the same time, velocity plays a key role as being both a significant variable in itself, and being the key to how big the coefficient of the inflation coefficient will be. The significance of velocity comes from variation in velocity growth over the sample data period, while the magnitude of the inflation coefficient comes from the BGP level of velocity.

Replacing consumption growth by output growth is seen to have its perils when the correct model is with consumption growth. And a forward interest term is indeed robust in the correct "well-formulated" model, in McCallum (2008) terms, but not robust when using misspecified models, something of focus in Taylor and Wieland (2010).

8 Conclusion

The paper has derived a general equilibrium condition in dynamic and balanced growth path cases for a constant relative risk aversion economy with leisure, endogenous growth through a separate human capital investment sector as in Lucas (1988), and with endogenous velocity through production of exchange credit in a financial intermediary as based in that literature. The importance of velocity in producing the Taylor principle is consistent with velocity importance found in Reynerd (2004). The paper also looked at special cases with log-utility, and with a simple cash-in-advance constraint in which velocity is one. While providing a theoretical means to overview the Taylor empirical literature, such as reviewed by Siklos and Wohar (2005), here the immediate focus was first to show that estimation of a Taylor rule may result in a spurious inference that the central bank is engaged in Taylor principle behaviour, rather than simply supplying money. We go some distance in proving this point in the sense of generating artificial data and estimating rather successfully our theoretical Taylor Condition that is simply an equilibrium condition in the economy in which the central bank makes changes in the money supply growth rate. For example, such money supply changes tend to occur whenever the "fiscal inflation tax" needs to be resorted to in order to finance deficits such as during the current banking and recession crisis, or during war in general, which some construe to be the source of the deficit problem of the last decade.

Money velocity growth itself enters as a variable and ends up playing a potentially significant role; in particular when velocity is changing significantly such as during the recent bank crisis and the 1930s when velocity cycled downwards, as identified in Benk et al. (2010). This velocity theoretically is made endogenous in the model following the banking financial intermediation microeconomic literature of producing financial services with a Cobb-Douglas production function including deposited funds as a factor.

The paper exhibits thereby how the banking production of exchange credit is surprising crucial to the derivation of a Taylor principle whereby the coefficient on the inflation term is in fact greater than one. This results only through the endogenization of velocity; a simple cia cash-only constraint with a velocity of one gives a unity coefficient on inflation. Through the endogenization of growth, we can derive an output gap measure not inconsistent with Taylor and Wieland's (2010) emphasis on changes in output as a measure for the output gap. In our model, the output growth term does enter if we also include an investment growth term; otherwise the consumption growth is the "output gap" term of the model's Taylor Condition.

Estimation results are also given for certain misspecified models, relative to the generated data, and indeed the effects on the results are substantial. The results may give a way to better understand to how the estimate Taylor rules so as to better account for changes in underlying conditions, including times when changes in velocity are significant because of bank productivity collapses, such as has apparently occurred during the recent/current recession and bank crisis. Omitting this term as in almost all standard Taylor rules may induce significant omitted variable bias. Misspecification bias does seem apparent in our estimation of a simple Taylor rule when using the original data from the general equilibrium model. Therefore the results hold promise for explaining comparisons of estimated rules across different periods and countries, as well as during bank crises, sudden financial deregulations, or times of other significant shifts in money velocity, a task which would help organize this disparate literature.

By simulating data of the model and estimating successfully a

Taylor Rule from the data, the paper implies that identification of a Taylor rule econometrically may simply be identification of part of the economy's asset price behaviour when a central bank prints money. It may be spurious to claim that such Taylor estimations show how the central bank actually conducts policy through interest rate targeting rather than through a more simple fiscal satisfaction of its spending needs through direct and indirect taxes including inflation.

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