SOCIAL STRUCTURE AND HUMAN CAPITAL DYNAMICS*

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First version, February 15, 2010
This version, June 9, 2010

Abstract. This paper studies how social structures affect the dynamics of growth and inequality. We investigate how societies that are identical in terms of economic primitives, such as preferences, technology and endowment, can have different equilibrium dynamics. We do this by explicitly embedding networks that resemble social structures into an otherwise standard framework with overlapping generations, in which parents invest in the education of their offspring. We show that even if the population is initially heterogeneous, there exists a balanced growth path with no inequality for all networks, which is independent of the social structure. However, its local stability and transition dynamics depend on the network at hand, summarized by a measure of network cohesion. We find that as cohesion increases, the parameter region for which the balanced growth path is stable becomes larger; i.e. it becomes more likely that society will converge to a path of equality. Unlike the typical approach in the literature, which concentrates on segregated versus integrated societies, we also quantify the transition of a range of networks that represent stylised versions of society with varying degrees of social cohesion. Of those, the star network (representing for example a city with high human capital, which is linked to the periphery) provides relatively low inequality and the highest level of growth during transition.

Keywords: Human capital, growth, social structure, local externality, networks

JEL Classification: O40, E24, D62, D85

*We are grateful to Aqib Aslam, Paul Beaudry, Edouard Challe, Dean Corbae, Tom Crossley, Tatiana Damjianovic, Partha Dasgupta, Sanjeev Goyal, Jonathan Heathcote, Migel Leon-Ledesma, Marco van der Leij, Hamish Low, José Tavares, Flavio Toxvaerd, and Klaus Waelde for interesting conversations and useful comments on this project. We also thank participants at the 2nd Cambridge-Oxford-London Macro Meeting, the 2009 SAET meeting, 18th CEPR ESSIM, and seminar participants at University of Manchester.

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1. Introduction

Empirical evidence suggests that peer group effects on human capital accumulation are significant, despite the difficulty of disentangling neighborhood effects from other omitted and endogenous variables.[1] Examples of how social interactions affect individual behavior or choices are provided by Akerlof (1997) and Glaeser and Scheinkman (2002), among others. Such findings suggest that residential location, friends and neighbors greatly affect youth behavior and human capital through a variety of characteristics correlated with neighborhood wealth, such as school quality and safety from crime. The willingness of families to pay substantially higher house prices and rents to move to locations with better social environments and schools means that typical households believe that neighborhood and school social composition affect their children's life prospects. As Lucas (1988) states, " [H]uman capital accumulation is a *social* activity, involving groups of people in a way that has no counterpart in the accumulation of physical capital."

Motivated by these observations, we ask, *what is the importance of social structures for human capital accumulation and thus for economic growth and inequality*? We investigate how societies that are populated by agents of different initial human capital levels, but are otherwise identical in terms of economic primitives such as preferences, technology and endowment, can have different equilibrium dynamics and long run behavior. Our model is a stylized economy with overlapping generations, in which parents invest in the education of their offspring. A key ingredient for our analysis is the evolution of human capital: we assume that future human capital depends on four distinct factors, namely investment in education, intergenerational human capital transmission (inherited capital), a global externality (education system or common school curriculum in the economy) and finally on a local externality, which represents peer effects. Our novelty is in the way that the local externality is modeled: We describe the social structure by a network and assume that the local externality is a summary statistic of the human capital of a household's neighbors in the network, e.g. the average human capital of the neighbors. In this way, we have an operational framework where differences in social structures may affect individual outcomes through local externalities, since the return on investment in education depends on family connections (e.g., *peer effects*), or its social capital.[2]

The main findings of the paper are the following. First, if the population is homogenous in all respects, i.e. everyone has the same initial human capital, then the social structure is irrelevant for human capital dynamics and the economy is always on a stable balanced growth path with equality, since there is no uncertainty in the model, irrespective of the network at hand. Second, we show that if the population is heterogeneous with respect to initial human capital, but identical in all other dimensions, then the long run behavior of the economy depends on the social structure in a non-trivial way. We demonstrate that if the two externalities are important enough for the accumulation of human capital, there is a unique balanced growth path

[1] For example, Hanushek, Kain, Markman, and Rivkin (2003) estimate that a one standard deviation increase in peer test scores increases individuals test scores by 35 percent of a standard deviation. Neighborhood effects are also important in high school drop outs and other educational outcomes (e.g., Coleman (1988), Cutler and Glaeser (1997)), youth crime and delinquency (e.g., Kling, Ludwig, and Katz (2005)), and in the physical and mental health of children and adults (e.g., Katz, Kling, and Liebman (2001)).

[2] In this respect, our paper borrows the idea from Coleman (1988) that social capital or networks might be an important input in the formation of human capital.

with long run equality. Moreover, as sociery becomes more cohesive, long run equality prevails for a larger range of externality parameters. We prove this by carefully defining an appropriate measure of *network cohesion* and then show how it uniquely determines the parameter regions for which the balanced growth path with equality is locally stable.[3] We also show numerically that when the two externalities are weak and the network cohesion is low, there exist balanced growth paths with long run inequality. We calibrate the model to the U.S. economy and find that although long run inequality is a theoretical possibility, it occurs for a very small range of externality parameters; the calibrated parameters indicate that almost all networks and externality parameters imply income equality in the (possibly very) long run.

Third, our model predicts that high long run growth rates are generally associated with high inequality, but this is not necessarily true during the transition. To illustrate this, we analyze the transition dynamics for various special cases of networks that represent stylized versions of society, ranging from a society of total isolation, where there are no links between any households, to total network cohesion, where all households are linked with each other. During transition to long run equality, the more cohesive the society is, the less inequality there is; however there is no corresponding monotonic relationship between short run growth rates and cohesion. Growth rates during the transition can vary from very high to very low, irrespective of the degree of inequality. We find that the social structure that is most likely to produce both low inequality during the transition and high long run growth and welfare levels for all households is the star network, which can be interpreted as a city (high initial level of human capital) linked to all the households/neighborhoods of the periphery (low initial levels of human capital).

Our paper is not the first to address the relevance of peer effects for economic growth and inequality.[4] First, Lucas (1988) emphasized the role of human capital external effects on productivity. Following this, there is a large number of articles that also assume spillover effects on human capital formation, e.g. de la Croix and Doepke (2003). This literature, however, does not study how social structures affect the dynamics of the economy. It focuses on different questions. For instance, de la Croix and Doepke (2003) investigate how inequality affects economic growth in a model with fertility differentials. There is also an important strant of literature that examines the importance of capital market imperfections for human capital formation and the dynamics of inequality and growth (e.g. Galor and Zeira (1993) and Banerjee and Newman (1993)), which however abstracts from explicit social structures. More closely related to our paper is the work of Bénabou (1996) who studies a similar question to ours and investigates how social community affects growth and inequality.[5] In his model,

[3] We deliberately avoid the terms *integration* and *segregation* as these are often associated to races and minority groups. We simply want to capture the variability of the intensity with which households interact with each other and our measure depends on the network structure rather than on the partition of households into different groups.

[4] Our work also belongs to the vast literature that studies the relationship and interactions of inequality and growth. A comprehensive summary of this literature can be found in Garcia-Penalosa and Turnovsky (2006).

[5] In a related article, Bénabou (1993) shows how local externalities affect the incentive of agents to segregate with effects on productivity. This model, however, is static and focuses on location choice. Durlauf (1996) also demonstrates how endogenous community formation can generate segregated societies resulting on inequality persistence and poverty traps. Our analysis is different, since we focus on the impact of different social structures on growth and inequality. We investigate whether such structures can be a primary cause of inequality in the short and long run.

the acquisition of human capital also reflects the influence of family, local (community), and economy wide factors. Our article differs in that social structures are represented by explicit networks. This makes our framework more tractable and flexible for analyzing a variety of networks representing how different communities are interlinked and not only focusing on two types of societies (e.g., segregated versus integrated).[6] Due to the tractability of our framework, we are able to map a simple measure of cohesion, which depends on the network structure, onto the dynamics of the economy. To our knowledge, we are the first to integrate explicit networks into a standard model of economic growth.

There is also a growing literature relating networks to a variety of other economic issues. Goyal (2007), Jackson (2008) and Jackson (2009) provide an overview of the recent research and models on social networks in economics and techniques for analyzing different economic issues. Our article is closer to papers that investigate the effects of social networks in equilibrium allocations. In this respect, our paper is related to Ghiglino and Goyal (2010), Calvó-Armengol and Jackson (2004), Chantarat and Barrett (2008), and van der Leij and Buhai (2008). Ghiglino and Goyal (2010), for instance, show how social networks shape equilibrium allocations and prices in a standard exchange economy in which the utility of an individual is negatively affected by the consumption of their neighbors. Calvó-Armengol and Jackson (2004) embed networks in a model of the labor market and study their implications on employment status and unemployment duration.[7] Chantarat and Barrett (2008) build a two period model with two technologies (low and high) to investigate the effects of networks on poverty traps. Networks in their model are a form of social capital and reduce the cost of using the advanced technology. They show how the cost of network formation might be a barrier for poor agents to use the better technology, with persistent effects on inequality and poverty. van der Leij and Buhai (2008) study how social interactions map into path dependence on occupational segregation. As in Bénabou (1993), they show that segregation is an equilibrium outcome and can be welfare enhancing.

The paper proceeds as follows. Section 2 presents the environment in terms of economic primitives: preferences, endowments and technologies. We describe how we introduce social structures through networks, it also defines the equilibrium and presents our measure of social cohesion. It shows how we can map different networks onto our simple measure, which ranges from zero to one, and the larger the measure, the more network cohesion there is. Section 3 derives our main analytical result, which shows how the convergence to a no inequality long run equilibrium depends on the network cohesion measure. Section 4 calibrates the model and simulates the economy for different networks and parameters. Section 5 concludes.

2. The Model

2.1. Environment. The economy is inhabited by overlapping generations of individuals who live two periods, childhood and adulthood. In each generation, there are n households

[6]See also de la Croix and Doepke (2004) and Fernandez and Rogerson (1996). In a recent article, Mookherjee, Napel, and Ray (2009) emphasize how geographical locations affect parents' aspiration. In their model, which also assumes an exogenous location, a household will have higher aspiration if it lives in a neighborhood with a large fraction of educated neighbors.

[7]See also Calvó-Armengol and Jackson (2007). They show that there is a higher return from investing in education if one expects to have more friends investing in education, since unemployed workers hear about job positions via their friend and educated workers are more likely to hear jobs requiring education than uneducated workers.

indexed by $i = 1, 2, ..., n$. Each household consists of one adult and one child, such that the population is constant and equal to n. Adults are generally assumed heterogeneous in their initial endowment of human capital. Let adult i at time t has human capital h_{it}. At time t, adult i cares about her own consumption c_{it} and the future human capital of her offspring h_{it+1}.[8] She works and her productivity is proportional to her skill. The decisions of household i at time t are taken by the parent who chooses consumption and investment in the human capital of her offspring, e_{it}. At the end of period t the parent of household i dies, and the offspring becomes the adult/parent in the new period $t+1$, who then has a new child, and so on.

Firm. There is one firm in this economy, with one production input, human capital. The aggregate production technology is linear in the efficiency unit of labor, H_t:

$$Y_t = H_t. \tag{1}$$

Since the labor productivity of workers is proportional to their human capital, the income for household i at time t is equal to the human capital of the parent, h_{it}.

Households. The utility function for household i is given by

$$\ln c_{it} + \psi \ln h_{it+1},$$

where $\psi > 0$ is the altruism factor. The human capital h_{it+1} for the child of household i depends multiplicatively on four factors. First, it depends on investment in schooling provided by her parent, e_{it}; second, on the human capital of her parent h_{it}; third, on the average human capital of her parent's neighbors $\bar{h}_{it}$ (local externality), and fourth on the average human capital $\bar{h}_t$ for the whole economy, i.e. the human capital of the teachers (global externality) or the national school curriculum. We then express the evolution of human capital as

$$h_{it+1} = (\theta + e_{it})^{\eta} h_{it}^{1-\beta_1-\beta_2} \bar{h}_{it}^{\beta_2} \bar{h}_t^{\beta_1}, \tag{2}$$

where $\theta > 0$, $\beta_1 \geq 0$, $\beta_2 \geq 0$, $\eta > 0$, and $0 < \beta_1 + \beta_2 \leq 1$. The assumption that $\theta > 0$ allows for the possibility that the parent may opt not to educate her child, since it implies that $h_{it+1} > 0$ even if it may be that $e_{it} = 0$. The parameters β_1 and β_2 determine the importance of own human capital and the two externalities for determining future human capital. We allow for the possibility of $\beta_1 = 0$ or $\beta_2 = 0$ in order to study the extreme cases where we switch off the externalities.

The budget constraint for household i is given by

$$c_{it} + e_{it} p_t = h_{it},$$

where p_t is the price of investment in education in terms of the consumption good at period t.

[8]The introduction of physical capital does not change any of the qualitative implications of the model in a significant way. Therefore, in order to focus on the effects of different social structures on growth and inequality we abstract from physical capital accumulation.

The problem of an arbitrary household i is then summarized by the following:

$$\max_{c_{it},e_{it}} [\ln c_{it} + \psi \ln h_{it+1}] \tag{3}$$

$$s.t. \;\; c_{it} + e_{it}p_t = h_{it}, \tag{4}$$

$$h_{it+1} = (\theta + e_{it})^{\eta} h_{it}^{1-\beta_1-\beta_2} \bar{h}_{it}^{\beta_2} \bar{h}_t^{\beta_1}, \tag{5}$$

$$e_{it}, c_{it} \geq 0. \tag{6}$$

The solution to the households problem is:

$$e_{it} = \begin{cases} 0, & \text{if } \frac{h_{it}}{p_t} \leq \frac{\theta}{\psi\eta} \\ \frac{\psi\eta \frac{h_{it}}{p_t} - \theta}{1+\psi\eta}, & \text{if } \frac{h_{it}}{p_t} > \frac{\theta}{\psi\eta} \end{cases} \quad \text{and} \quad c_{it} = \begin{cases} h_{it}, & \text{if } \frac{h_{it}}{p_t} \leq \frac{\theta}{\psi\eta} \\ \frac{p_t}{1+\psi\eta}\left(\frac{h_{it}}{p_t} + \theta\right), & \text{if } \frac{h_{it}}{p_t} > \frac{\theta}{\psi\eta} \end{cases}.$$

This means that if the price of education is too high for some households, they will optimally choose not educate their children and they will therefore consume all their income.

Social Structures. We now describe how we introduce social structures in this economy. We consider a static network in which agents inherit the network structure from their parents. In effect, this means that at $t = 0$ an arbitrary household i is linked to some other households and these links remain unchanged thereafter.[9] A network is a set of nodes (here households), $N = \{1, 2, ..., n\}$. Let $a_{ij} \in \{0, 1\}$ be a relationship between two agents i and j. It is assumed that $a_{ij} = a_{ji}$ (i.e. that the network is undirected) and that $a_{ij} = 1$ if there is a link between households i and j and $a_{ij} = 0$ otherwise. This notation allows us to represent the network with the adjacency matrix A. Given that we consider an undirected network, this is a symmetric matrix of zeros and ones, of which the $ij-$th entry is a_{ij}. Last, let $G = A + I_n$, i.e. G is the adjacency matrix with ones on the diagonal and typical element g_{ij}. We define the local externality variable $\bar{h}_{it}$ as the average human capital of the neighbors of household i, that is

$$\bar{h}_{it} = \frac{\sum_{j=1}^{n} g_{ij} h_{jt}}{\sum_{j=1}^{n} g_{ij}}.$$

where g_{ij} is the $ij - th$ element of matrix G. It is now clear that the assumption that $g_{ii} = 1$ ensures that a household's human capital is non-zero, even if it is not linked to any other households.[10] Table 1 shows some examples of standard networks that we consider throughout the paper.

2.2. Equilibrium.

We are now ready to define the competitive equilibrium in this economy.

Definition 1: Competitive Equilibrium. *Given a social structure described my the matrix G, a competitive equilibrium at time t is a collection of households' allocation $\{c_{it}, e_{it}\}_{i=1}^{n}$,*

[9] A clearly interesting extension is to assume a dynamic network structure, whereby households decide optimally their *location*, i.e. whether to make or break links with other households. We defer this to future work. Here, we essentially assume large mobility costs, as in Mookherjee, Napel, and Ray (2009).

[10] This is only one of many possible ways of defining the local externality, which we consider as a reasonable starting point. Other formulations for $\bar{h}_{it}$, include an aggregate measure of the human capital of a household's neighborhood, or a measure of human capital that calculates the average human capital of a household's neighbors plus the capital of the neighbors' neighbors, etc.

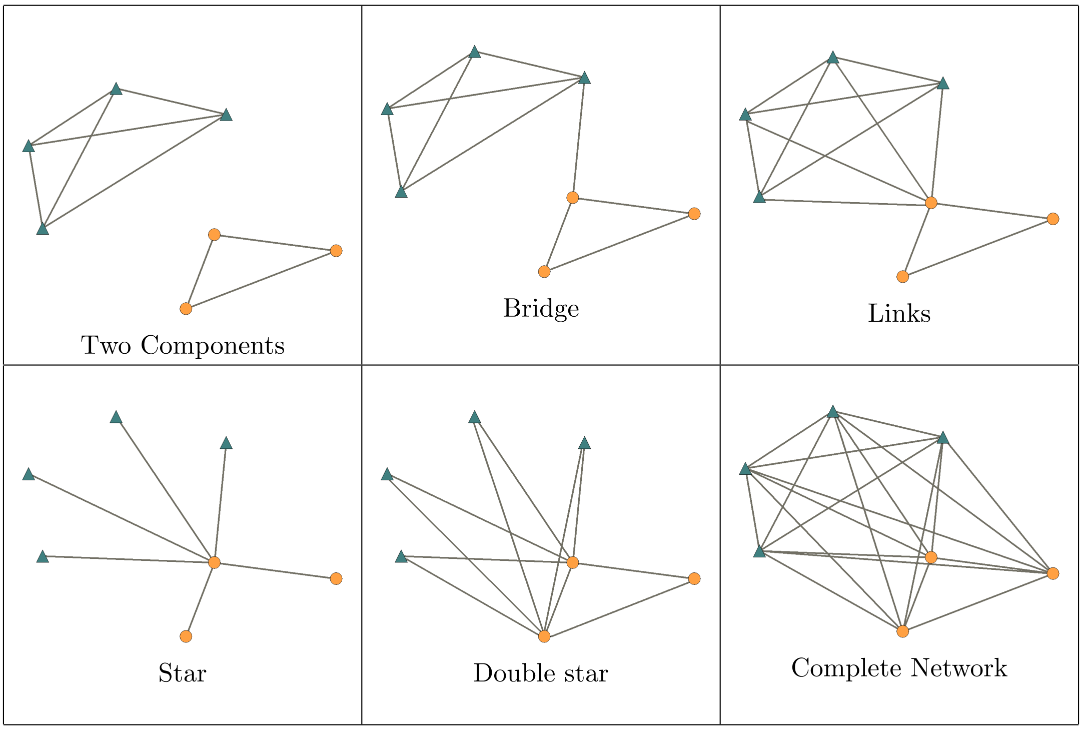

Table 1: Social Structures

firm output H_t, *and price* p_t, *such that:*

(a) *Given price* p_t, *the parent of household* i *chooses* $\{c_{it}, e_{it}\}$ *to solve problem (3), subject to constraints (4), (5) and (6);*

(b) *Output,* H_t, *maximizes the firm's profits and*

(c) *The goods market clears, i.e.*

$$\sum_{i=1}^{n} c_{it} + \sum_{i=1}^{n} e_{it} p_t = \sum_{i=1}^{n} h_{it} = H_t. \tag{7}$$

In this economy any positive and finite price p_t clears the goods market. There are different approaches in the literature for determining p_t. Firstly, we could assume that investment in education is in terms of parent's time (home schooling), such that the budget constraint could be written as $c_{it} = h_{it}(1 - e_{it})$. This is a standard approach when agents are homogenous (e.g. Galor and Weil (2000)), since in this case it does not matter who provides education for children (i.e. parents or teachers). However, when agents are heterogenous this would imply that parents would face different prices for investment in education and prices would be lower for relatively poor households. As a result, in equilibrium parents would invest the same proportion of the their income in education regardless of their income level.[11] In this case, we can easily show that there is a unique global stable balanced growth with no inequality. Network structures will

[11] Glomm and Ravikumar (1992) and Bénabou (1996) use this assumption, but they also consider another variable in the offspring's human capital formation, which is education quality. Parents face the same relative price for the quality of education. In this case, although parents' time devoted to help their offspring's learning would be the same, investment in the quality of education would vary with income.

not have any effect in the long run, but would affect the transition dynamics of inequality and growth.

We could also assume that the relative price of education in terms of the consumption good is constant over time, for example that $p_t = 1$, such as in Galor and Moav (2004). This would imply that education is relatively more expensive for poor than for rich parents, and consequently the latter will invest more in education than the former, however this does not allow for time variation in the price of education.

Alternatively, we consider the case, as in de la Croix and Doepke (2003) and de la Croix and Doepke (2004), that children are educated by teachers under a common school curriculum and we assume that there is a notional continuum of teachers with unit measure, whose human capital is equal to the average human capital $\bar{h}_t = \left(\sum_{i=1}^{n} h_{it}\right)/n$. In this case, $p_t = \bar{h}_t$ corresponds to the price of education relative to the consumption good and not only all parents face the same price, but as the average human capital grows the price of education relative to the consumption good increases. This last fact is also consistent to the empirical evidence provided by Theil and Chen (1995) who show that there is a positive correlation between income and the relative price of education in a cross section of countries.[12]

For $p_t = \bar{h}_t$, we have that households' decisions depend only on their relative human capital $x_{it} = h_{it}/\bar{h}_t$. As long as $x_{it} > \frac{\theta}{\psi\eta}$, rich households will invest more in education than relatively poor parents. In order to ensure that investment in education is positive when all households are identical, we must assume that

$$\psi\eta > \theta. \tag{8}$$

This is generally true when the society is highly altruistic (i.e. ψ is large) or for example when the threshold θ for having non-zero human capital is relatively small.

Note that optimal consumption and investment in human capital depends directly only on the global externality, i.e. the human capital of the teachers. The local externality only plays an indirect role, via the expression for the evolution of human capital (2). Therefore, from the perspective of a household i, the equilibrium in a given period is does not depend on neighbors' decisions. This is true because of the way we have defined human capital formation technology and due to the assumption of the "warm glow" utility function.[13] Consequently, at any given period t, the decision of household i is the same irrespective of the strategies of all other households, since this is based on *state* variables only and not on concurrent decisions by other households. In other words, there is a unique equilibrium given the state variables in each period.

[12] The Commonfund Institute in the United States calculates the Higher Education Price Index (HEPI), which corresponds to an index of the cost in high education in the United States. According to the HEPI (2009) report, from 1983 to 2009 the HEPI rose (it went from 1 to 2.79) by roughly 28 percent more than the Consumer Price Index (it increased from 1 to 2.18).

[13] The "warm glow" utility function, whereby the offspring's human capital enters directly in the utility function, has been widely used in economic growth models. See, for instance, Galor and Weil (2000) and de la Croix and Doepke (2003). In addition, it appears to be more consistent with empirical evidence (Andreoni (1989)).

3. LONG RUN DYNAMICS

We first provide some definitions that we will use in the remainder of paper. Let the growth rate of the average human capital be

$$\gamma_t = \frac{\bar{h}_{t+1}}{\bar{h}_t},$$

and the growth rates for individual households be

$$\gamma_{it} = \frac{h_{it+1}}{h_{it}}.$$

We also define the auxiliary parameters

$$\delta = \frac{\eta}{1+\theta} \text{ and } \phi = \left(\frac{\psi\eta}{1+\psi\eta}\right)^{\eta}.$$

It is then straightforward to derive that in equilibrium the dynamic system is described by the following system of difference equations (see Appendix A):

$$x_{it+1} = \frac{\gamma_{it}}{\gamma_t} x_{it} = \frac{x_{it}\gamma_{it}}{\frac{1}{n}\sum_{k=1}^{n} x_{kt}\gamma_{kt}}, \text{ for } i = 1, ..., n \quad (9)$$

$$\gamma_{it} = \left(\theta + \max\left\{0, \frac{\psi\eta x_{it} - \theta}{1+\psi\eta}\right\}\right)^{\eta} x_{it}^{-\beta_1-\beta_2} \left(\frac{\sum_{j=1}^{n} g_{ij}x_{jt}}{\sum_{j=1}^{n} g_{ij}}\right)^{\beta_2}, \text{ for } i = 1, ..., n \quad (10)$$

A balanced growth path is defined by having the same growth rate for all households, i.e. $\gamma_{it} = \gamma_t = \gamma$ so that the relative human capitals denoted by x_i remain constant along the balanced growth path. We are essentially looking for x_i and γ such that

$$\frac{1}{n}\sum_i x_i = 1 \text{ and}$$

$$\left(\theta + \max\left\{0, \frac{\psi\eta x_i - \theta}{1+\psi\eta}\right\}\right)^{\eta} x_i^{-\beta_1-\beta_2} \left(\frac{\sum_{j=1}^{n} g_{ij}x_j}{\sum_{j=1}^{n} g_{ij}}\right)^{\beta_2} = \gamma \text{ for all } i = 1, ..., n.$$

Solving this system analytically, is in general impossible. However we can characterize some special cases, but also the case of long run equality.

We first define a measure of network cohesion, which is important for determining the long run behavior of the system. For a network defined by the matrix $G = A + I_n$, where A is the adjacency matrix, let

$$r_{il}^{g} = \frac{g_{il}}{\sum_j g_{ij}}.$$

From the perspective of household i, this parameter gives a measure of intensity of the social interaction of household i with household l. [14] Moreover, we define the relative intensity of the social interaction of household i with household l as

$$f_{il}^{g} = r_{il}^{g} - \bar{r}_{l}^{g},$$

[14] It is implicit in our definition that every household has equally intense social interactions with all its neighbours. Conceivably, we could include a weight that multiplies this expression, that would allow for unequally weighted social relations.

where $\bar{r}_l^g = \left(\sum_i r_{il}^g\right)/n$. In other words, the quantity f_{il}^g represents the deviation of the intensity of the social interaction of i with l from the average intensity of social interactions of l, so that the larger these deviations f_{il}^g are, the more variability there is in the importance of social interactions in this economy. Let F_g be an $n \times n$ matrix with typical il-element $f_{il}^g(g)$. We are now ready to define the measure of network cohesion κ_g.

Definition 2: Network Cohesion. *For a network defined by the matrix G, a measure of network cohesion is*

$$\kappa_g = 1 - \rho(F_g),$$

where $\rho(F_g)$ is the spectral radius of the matrix F_g and $\kappa_g \in [0,1]$.

It can be shown that the larger κ_g is, the more network cohesion there is. In general, we can also show that $\kappa_g \in [0,1]$ (see Cavalcanti and Giannitsarou (2010)). The elements of the matrix F become larger in absolute value as the intensity of households' social interactions becomes more uneven, i.e. as there is less cohesion in the society. Consider for example two extreme cases of a society where everyone is linked to each other (complete network), depicted in the lower right panel of Table 1 and the case of a society which consists of two unconnected groups, as shown in the upper left panel of Table 1. In the former case, the social interaction with l is of importance $1/n$ for household i, and that is true for all households i, so that the average $\bar{r}_l$ is also $1/n$. Therefore, all the elements of F are zero, and $\kappa_g = 1$, i.e. there is the highest possible network cohesion. In the latter case, the society is divided into two groups. The relative importance of an interaction of two households within a group is very large compared to the relative importance of a (non-existent) interaction between two households from the two groups. Therefore r_{il}^g will be quite far from the average. This high variability in the intensity of social interactions means that there is no network cohesion, since there are two groups of households that never interact with each other and $\kappa_g = 0$. In some sense, the network cohesion measure is an index of the variability of the network and, as in Echenique and Fryer Jr (2007), it may be disaggregated at the individual level. However, it is independent of how we label households (e.g., black and white). The main difference between our measure and the Social Segregation Index (SSI) of Echenique and Fryer Jr (2007) is that our measure represents a aggregate statistic for the whole society, while the SSI is a statistic calculated for particular subgroups within the society.[15]

With this definition in place, we can now prove the following main results of the paper (derivations and proofs in Appendix).

Proposition 1: *When all households are identical and have equal initial capital h_0 then the local externality is irrelevant and the economy is always at a balanced growth path where $x_{it} = x^* = 1$ for all $i \in N$ and $t = 1, 2, ...$ and $\gamma^* = \phi(1+\theta)^\eta$.*

Proposition 2: Long Run Equality. *Suppose that the population is initially heterogeneous, i.e. that households may have different initial human capital. Then, there exists a locally*

[15] A more detailed account of a generalized cohesion measure is presented in Cavalcanti and Giannitsarou (2010).

stable balanced growth path where $x_{it} = x^* = 1$ *for all* $i \in N$ *(equality) and* $\gamma^* = \phi(1+\theta)^\eta$ *if and only if*

$$\beta_1 + \kappa_g \beta_2 > \delta \tag{11}$$

Proposition 2 states that there exists a balanced growth path with no inequality and the long run growth rate γ^* does not depend on the social network structure; however the local dynamics around the balanced growth path depend on the particular network that generates the local externality. The network structure affects not only the speed of convergence towards this long run equality balanced growth path equilibrium (we will show this numerically), but also whether the economy converges or not to this equilibrium. The stability condition can be nicely interpreted as an inequality that gives a lower bound of the importance of the two externalities relative to the importance of a household's own investment in education (summarized in δ) in order to achieve equality in the long run. It shows that the global externality (captured by β_1) and the local externality (captured by $\kappa_g\beta_2$) have to be stronger than own investment in education in order to eliminate inequality in the long run. However, one needs to keep in mind that the parameters β_1 and β_2 also summarize the importance of externalities *relative* to importance of inherited human capital. Assuming that the externalities become stronger (i.e. that β_1 and β_2 increase) implies that at the same time, inherited human capital becomes less important, since $1-\beta_1-\beta_2$ decreases.

Whenever the externality parameters are such that the inequality (11) is not satisfied, it is not possible to find closed form solutions to the system or characterize the dynamics analytically. Therefore, in order to establish some properties in such cases, we perform a series or numerical simulations of the system, by keeping certain parameters fixed and varying externality parameters, networks and initial conditions. The general picture that emerges from this exercise is that there may exist many balanced growth paths with inequality, i.e. such that in general $x_i \neq x_j$. As long as the local externality parameter values are such that $\beta_1+\kappa_g\beta_2 >> \delta$, the unique stable balanced growth path is that of equality, as given in Proposition 2. As the externality parameters take values that imply $\beta_1+\kappa_g\beta_2 \approx \delta$, other solutions may also be locally stable; which one is reached very much depends on the initial conditions, as well as *on the network structure*. For parameter values such that $\beta_1+\kappa_g\beta_2 << \delta$ the solution $x_i = 1$ (long run equality) becomes unstable, and the system converges to balanced growth paths that imply long run inequality. As an illustration only, figure 1 shows the long run values of relative capital and growth rate of average human capital, for the star network.[16]

We observe the following features when there is long run inequality: First, the growth rate of the economy with a balanced growth with inequality is in general larger than the growth rate of the balanced growth path with equality, i.e. $\gamma > \gamma^*$, as shown for example in the lower panel of figure 1. Second, convergence to any of these balanced growth paths is local, and therefore the long run outcomes are very sensitive to the initial conditions of human capital. Third, in

[16] The long run is approximated by simulating the system for $T = 600$ and selecting $x_{i,600}$ and γ_{600}. Figure 1 presents the result for a star network with seven nodes and the following parameter values $\beta_1 = 0.25$, $\theta = 1/2$, $\psi = 10$, $\eta = 3/4$. We vary the local externality parameter β_2 between 0 and $1-\beta_1$. Similar outcomes emerge when we vary initial conditions, networks, the number of nodes of the network and other parameters.

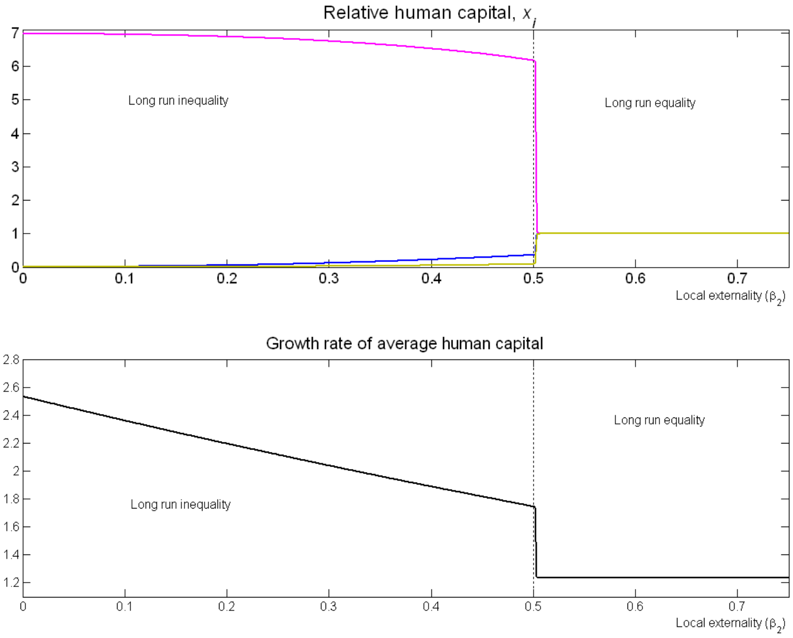

Figure 1: Long run inequality for the star network.

contrast to the case of long run equality, where γ^* is independent of the social structure and the externality parameters, the long run growth rate depends both on the network at hand and on the externality parameters. Elaborating a bit more on this last point, it is straightforward to establish numerically that long run growth rates and Gini coefficients decrease in both the externality parameters β_1 and β_2. This is very intuitive. Since in our model there is a trade-off between growth and long run equality, as externalities become more important for human capital accumulation than one's inherited capital and own investment, societies have more equality, at the expense of lower growth rates.

Perhaps more surprising is what we find when we vary the networks and examine what happens to long run growth and inequality. The long run growth rates with inequality are *not* monotonic in the measure of network cohesion. For most parameterizations we consider, the empty network generates the highest long run growth rate and the complete network generates the lowest growth rate, however there is no way to order the long run outcomes for all other networks that are between these two extreme cases (these are the benchmark structures commonly used in the literature). Even more surprisingly, in a few particular cases, the empty network actually yields growth rates that are lower than other networks, or the complete network yields growth rates that can be higher than those of other networks. This is possibly counter intuitive: one would expect that the empty and complete networks give long run growth rates that represent upper and lower bounds of possible long run outcomes.

Figure 2 summarizes all our findings for the long run properties of our economies. It shows the region of parameter values of β_1 and β_2 for which the balanced growth path with equality is stable. We fix deep parameters ψ, η and θ, and present the stability conditions in the β_1 and β_2 space, since these two parameters represent the importance of the two types of externalities.

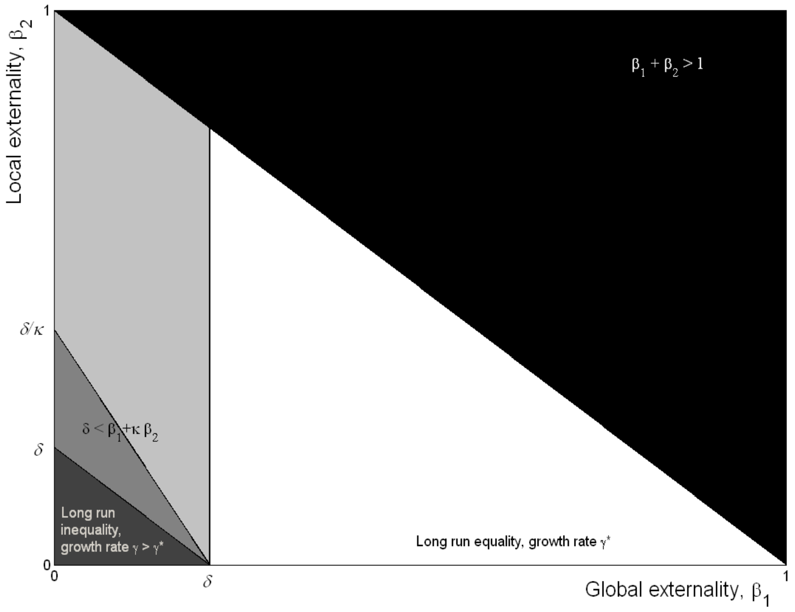

Figure 2: Summary of long run stability properties.

Combinations in the black area violate the assumption $\beta_1 + \beta_2 < 1$ and thus not considered. In the white area of the figure, all network structures imply stable balanced growth path with long run equality and growth rate γ^*. In the dark grey area, all network structures imply long run inequality (γ^* is unstable) and a balanced growth path where $\gamma > \gamma^*$. The line defined by $\delta = \beta_1 + \kappa_g \beta_2$ determines the regions of long run equality or inequality for a given network with cohesion κ_g. For all combinations of β_1 and β_2 below that line there is long run inequality, while above this line there is long run equality and γ^* is stable. As cohesion κ_g increases, the parameter region of long run equality increases as well.

Additionally, we have also analyzed the case of home schooling in which $p_{it} = h_{it}$, and we can easily show that all households invest the same quantity of education, $e_{it} = \frac{\psi\eta - \theta}{1+\psi\eta}$, regardless of their income level. Moreover, there is a unique globally stable balanced growth path with $\gamma^* = \phi\,(1+\theta)^{\eta}$, with a degenerate income distribution; this is because the corresponding stability condition is now $0 < \beta_1 + \kappa_g \beta_2$ which is always satisfied for the parameter restrictions at hand. As in our case, the global externality is a force that brings households together, but more importantly, in the case of home schooling all parents invest the same proportion of their income in education; this reduces the wedge between rich and poor households more than in the case of a common schooling.

4. TRANSITION

Next, we do numerical experiments in order to address questions of the following nature: What is the relation of network cohesion, growth and inequality during the transition to a balanced growth path of equality? How do social structures affect growth, long run levels of output and welfare? What changes could we impose on the social structure in order to bring long run equality in an economy that is at a balanced growth path with inequality? In general, what kind of policy lessons can we draw from our analysis?

4.1. Transition to long run equality. We first look at the transition and long run levels for settings that imply long run equality for all social structures, i.e. cases for which $\beta_1 > \delta$. We consider $n = 7$ and compare outcomes for the six networks presented in Table 1, against the benchmark cases of an empty and a complete network. We consider the following simple structure. Of the $n = 7$ households, $m = 3$ belong to group A and $n - m = 4$ belong to group B. In table 1, group A is represented by the orange circles and group B is represented by the green triangles. Initial conditions for human capital are clearly important for the transition to the balanced growth path. We assume that the initial conditions for all households within a group are identical, but are different for the two groups. We assume that group A has high initial human capital, set to $h_{A0} = 5$ and group B has low initial human capital, $h_{B0} = 1$.[17]

Next, we fix the values of the deep parameters of the model that determine δ such that in the balanced growth path equilibrium the model matches some empirical features of the United States economy. We start by calibrating three parameters: ψ, η and θ. We set the model period to be 35 years and we choose these three parameters such that three conditions hold. First, we use the fact that the long run annual growth rate of output per capita is 2%. This is consistent to the post World War II annual growth rate of the United States economy (see Heston, Summers, and Aten (2006)). Given that the model period is 35 years, we have that the long run growth rate is $\gamma^* = 1.02^{35} = 1.9999 \approx 2$. Therefore:

$$\left(\frac{\psi\eta}{1+\psi\eta} (1+\theta) \right)^{\eta} = 2. \tag{12}$$

Second, we use the fact that the returns to investment in education is 10%. This number is in the range of estimates of the return to schooling for the United States and OECD economies (see Psacharopoulos (1999), Psacharopoulos and Patrinos (2004) and Krueger and Lindahl (2001)). The returns to investment in education in our model is:

$$\frac{\partial h_{t+1}}{\partial e_t} \frac{1}{h_{t+1}} = \eta(\theta + e_t)^{-1}.$$

In the balanced growth path

$$\frac{\partial h}{\partial e} \frac{1}{h} = \frac{\eta}{\theta + e} = \frac{\eta}{\theta + \frac{\psi\eta - \theta}{1+\psi\eta}} = \frac{1+\psi\eta}{\psi(1+\theta)} = 0.1. \tag{13}$$

Last, observe that in our model the national income identity implies that income is divided between consumption and investment in education only. In the data, the ratio of investment in education to final consumption expenditures in the United States is roughly 10.64%.[18] In our

[17] Note that the levels of initial human capital do not matter very much for the qualitative questions we are asking, this is why we do not provide extensive justification for these initial conditions. What matters a lot is which group has high or low human capital, because for certain networks the initial human capital for a particular node (e.g. the centre of a star) is critical for the dynamics for all the economy.

[18] According to the OECD (2009) expenditure on education in the United States as a percentage of GDP was 7.4% in 2006. The National Income and Product Accounts show that personal consumption expenditures as a share of GDP was 69.58% in 2006. Therefore, the ratio of education expenditures to consumption expenditures is about 10.64%.

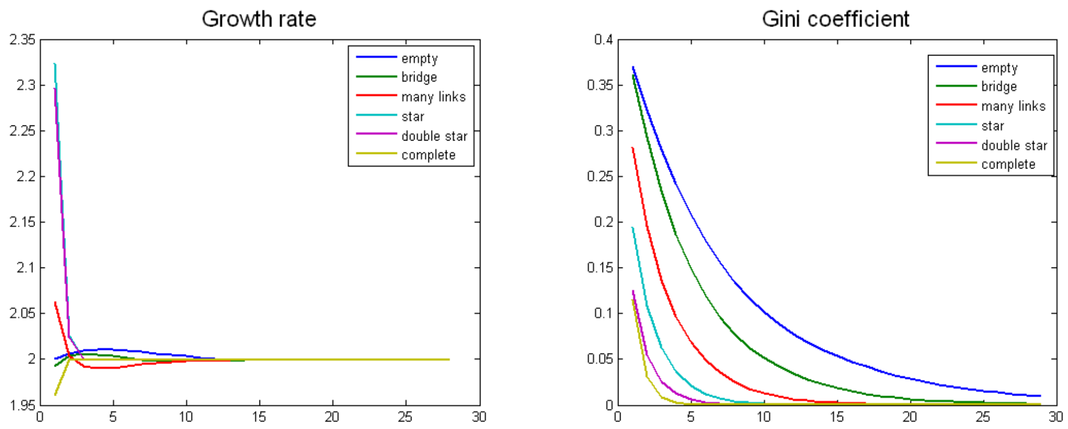

Figure 3: **Long run equality.** For $h_{A0} = 5$, $h_{B0} = 1$, $\beta_1 = 0.2$ and $\beta_2 = 0.6$. All social structures imply balanced growth with equality. *Note*: The dynamics for the empty and two components networks are identical.

model, investment in education as a fraction of total consumption is:

$$\frac{\sum_{i=1}^{n} e_{it}\bar{h}_t}{\sum_{i=1}^{n} c_{it}}.$$

In the balanced growth path:

$$\frac{\sum_{i=1}^{n} e_i\bar{h}}{\sum_{i=1}^{n} c_i} = \frac{\frac{\psi\eta-\theta}{1+\psi\eta}}{\frac{1+\theta}{1+\psi\eta}} = \frac{\psi\eta-\theta}{1+\theta} = 0.1064. \tag{14}$$

Using equations (12), (13), and (14) we find that

$$\theta = 4.4688, \eta = 0.45649 \text{ and } \psi = 11.0640.$$

In this case $\delta = \frac{\eta}{1+\theta} = 0.0835$. Note that these parameters satisfy the condition that in an economy with no inequality investment in human capital is positive, $\psi\eta > \theta$. Also, note that the a small δ implies the balanced growth path with equality is stable for almost all combinations of externality parameters.

With this set of parameters in place, we restrict attention to $\beta_1 > \delta = 0.0835$, so that all social structures we consider generate a balanced growth path with long run equality. Parameters β_1 and β_2 (externalities), do not affect the household decision and the growth rate γ^* at the balanced growth path, but they determine the equilibrium dynamics. Empirical studies (e.g., Rosenzweig and Wolpin (1994)) show that although family background is an important determinant of human capital, the effect is quantitatively small. Therefore, as in de la Croix and Doepke (2003) and de la Croix and Doepke (2004), we assume that $1-\beta_1-\beta_2 = 0.2$. Since peer effects on human capital accumulation are estimated to be large (e.g., Hanushek, Kain, Markman, and Rivkin (2003)), we let $\beta_2 = 0.6$. The implied elasticity for the global externality is therefore $\beta_1 = 0.2$.

In Figure 3, we show the following for each of the networks considered: the left graph shows the growth rates of average human capital for each of the networks we consider; and the right graph shows the Gini index of income over time for six networks. We first compare the two extreme cases of zero and full network cohesion (empty and complete networks, depicted by the blue and light green lines respectively). Although the two economies converge to the long run equilibrium, the speed of convergence depends on the social network structure. Convergence is much faster under full cohesion than zero cohesion. Nevertheless, during the transition, the average growth rate of human capital is larger when $\kappa_g = 0$ (blue line) than when $\kappa_g = 1$ (light green line). During the transition, inequality is always higher when $\kappa_g = 0$. The initial Gini index under zero cohesion is roughly three times larger than the case of full cohesion.

In general, group A (high income group) is worse off under full cohesion than under no cohesion. In the short run, group B (low income group) is better off under full cohesion than under no cohesion, however, in the long run group B households are also worse off under full cohesion than under no cohesion. The intuition for this result is straightforward: For poor households, the local effect of being linked to rich households dominates the global externality of a low average growth rate. In the long run, however, as the human capital of both groups converges, the global externality dominates and since the long run average level of human capital is lower under full cohesion, group B households are overall worse off. Therefore, if intergenerational transfers are possible (e.g., social planner, government), it is possible to design a policy, such that all agents are better off under segregation than integration.

What happens for social structures with cohesion between zero and one? First, there seems to be a monotonic ranking of inequality with network cohesion. As κ increases, the Gini index becomes smaller at any given point in time during the transition. This is shown very sharply in the right graph of Figure 3 and is intuitive: as societies become less fragmented, they converge to equality faster. Once again, what is surprising is the non-monotonic relationship between short-run growth rates and network cohesion. We observe the following: the highest levels of average human capital are achieved for the star network. This is true provided that the centre of the star network is a node that belongs to group A, i.e. has high initial human capital.[19] The double star network also generates high levels of human capital, provided that both centres have high initial human capital. If the initial levels of human capital are high enough, then high growth rates in the transition imply that on average everyone is better off than if there was zero network cohesion.

Having tried a variety of experiments with different initial conditions and networks, we summarize our findings through a simple numerical exercise. We consider 50 different networks with seven nodes. In these, we include many of the standard named networks (e.g. empty, complete, bridge, star, ring, wheel, friendship, tree, mesh, periphery, bus, etc.). The networks are ordered by the measure of network cohesion, κ. We then draw initial conditions for the human capital of each node from a uniform distribution on the interval $[1, 10]$ and we calculate the average growth rate and Gini index for each network over the first $T = 5$ periods of the transition (which corresponds to 175 years, given our calibration). Because initial conditions are randomly drawn and outcomes are sensitive to those, we repeat this experiment 100,000 times and report the averages, minimum and maximum realizations. The results are shown in

[19] In fact, if it belongs to group B, then the star network generates the lowest human capital levels.

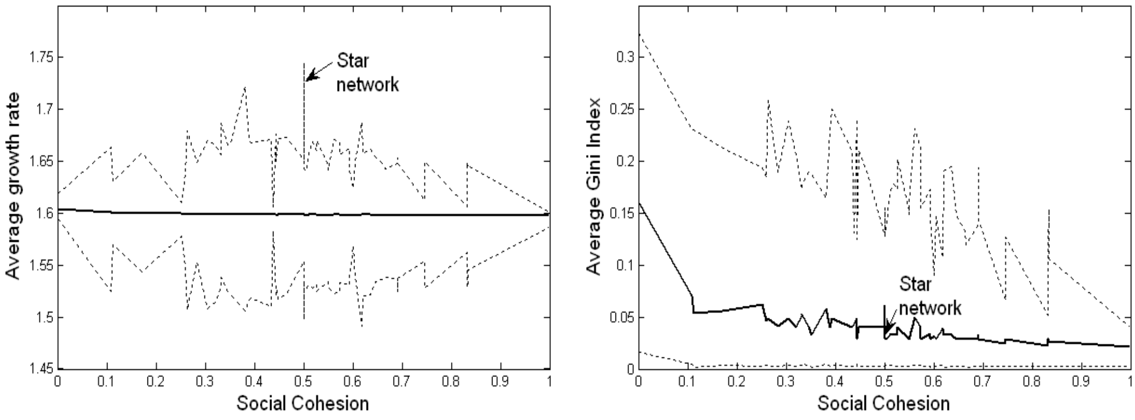

Figure 4: Average growth rates and average Gini indeces over $T = 5$ periods (175 years), ordered by social cohesion. The solid lines are the sample means over 100,000 replications of the experiment. The dotted lines show minimum and maximum realizations.

Figure 4.

The results from these experiments can be summarized as follows. First, the sample mean of the average γ over the first five periods is almost constant, irrespective of the social structure. We detect a mild decrease as network cohesion increases. The sample mean of the average Gini index shows an overall pattern of decreasing with network cohesion. This is in line with the simple intuition that on average, more fragmented societies experience large growth but higher inequality. However, there is an interesting variation for these averages. The dotted lines give the ranges from the minimum to the maximum, selected from the 100,000 realizations. If cohesion is small or large, the ranges become smaller, indicating smaller variation of the average growth rates for the extreme cases. Note that these are the cases that correspond to the standard growth models with global externalities only (empty network or complete network). At intermediate levels of network cohesion, the variations from network to network become larger and more extreme. The largest range of average growth rates is observed for the star network, which varies approximately between 1.5 and 1.75. Translated to annualized growth, this means that a household at the highest end would have approximately 35% more income at the end of their life than a household at the lowest end of this range. For the Gini index, variations in inequality are larger for networks with low cohesion.

These experiments demonstrate an important insight. In our economy, it is possible to have high short and medium run growth associated with both *high or low* inequality, depending on the social structure and the initial distribution of human capital. It is in this sense that our framework may be able to reconcile conflicting empirical evidence regarding the relationship between growth and inequality (a brief but insightful review of this empirical literature can be found in Garcia-Penalosa and Turnovsky (2006)).

The case of the star network is particularly interesting. It always implies the largest range of average growth rates and one of the smallest range of average Gini index. In fact, we generally observe that the highest growth rates and lowest Gini coefficients for the star network are achieved for high realizations of initial human capital for the central hub. Therefore, if a policy

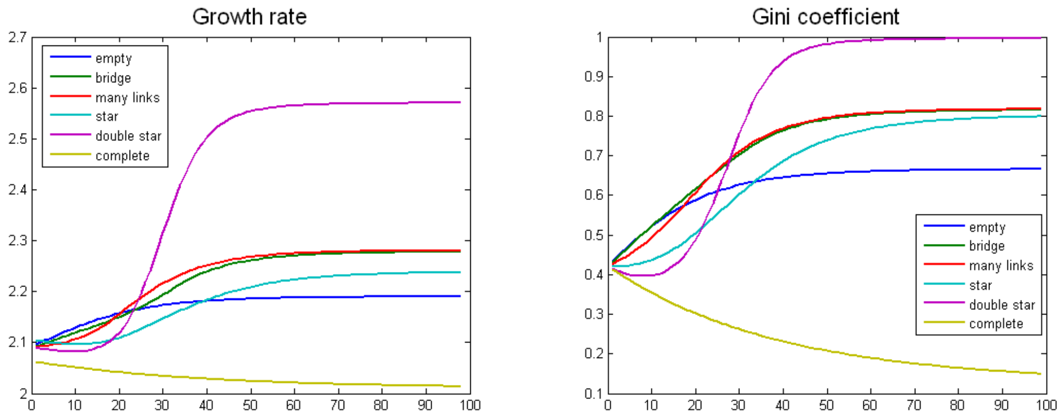

Figure 5: **Long run inequality.** For $h_{A0} = 5$, $h_{B0} = 1$, $1 - \beta_1 - \beta_2 = 0.93$ and $\beta_2 = 0.06$. All social structures imply balanced growth with inequality.

maker's objective were just to minimize inequality in the transition to the balanced growth path and at the same time maximize growth, the optimal social structure would be one of the type of the star network, through public policies that would link rich (high human capital) cities to the periphery, e.g. by bridges, roads and generally infrastructure, schools, etc. Through intergenerational transfers it is possible to make all households better off under the star network with a high human capital in the central node, than any other social structure.

4.2. Transition to long run inequality. Here we compare the transition to balanced growth paths with inequality for various networks. We use the same number for the calibrated parameters (θ, η and ψ), the same number of nodes and initial conditions of the previous subsection.[20] In order to ensure that all networks imply long run inequality, we need to assume that the two externalities are relatively weak, i.e. that $\beta_1 + \beta_2 < 0.0835$. Therefore, in order to analyze all networks with long run inequality, we set $\beta_1 = 0.01$, and $\beta_2 = 0.06$. A large $1-\beta_1-\beta_2$ implies that besides formal schooling the main determinant of human capital formation is the family background. In this case, peer effects and global externalities have a small impact on human capital formation.[21]

Figure 5 shows an example of the transition of growth and inequality to the balanced growth path for six standard networks. The left graph of figure 5 shows the growth rates of average human capital and the right graph shows the Gini coefficients along the transition. From this figure we can see that the double star network generates the highest long run growth and Gini index, which is equal to one in the long run.[22] The long run growth rate of the double star

[20]Strictly speaking, the calibration in the previous section was based on the assumption of growth rate γ^* which corresponds to the case long run equality. We use the same parameterization for more meaningful comparisons of the dynamics.

[21]Although earlier estimates show that family income and education have a positive but small effect on child's income (e.g., Leibowitz (1974) and Rosenzweig and Wolpin (1994)), a recent literature (e.g., Cunha and Heckman (2007)) based on psychological studies has emphasized the importance of shocks that happen in early childhood and even at birth (e.g., being born in disadvantaged family environment) on child's future income and skill acquisition.

[22]In the previous section we noted the possibility that a network other than the empty may generate higher long run growth. Our experiment here is a manifestation of this possibility.

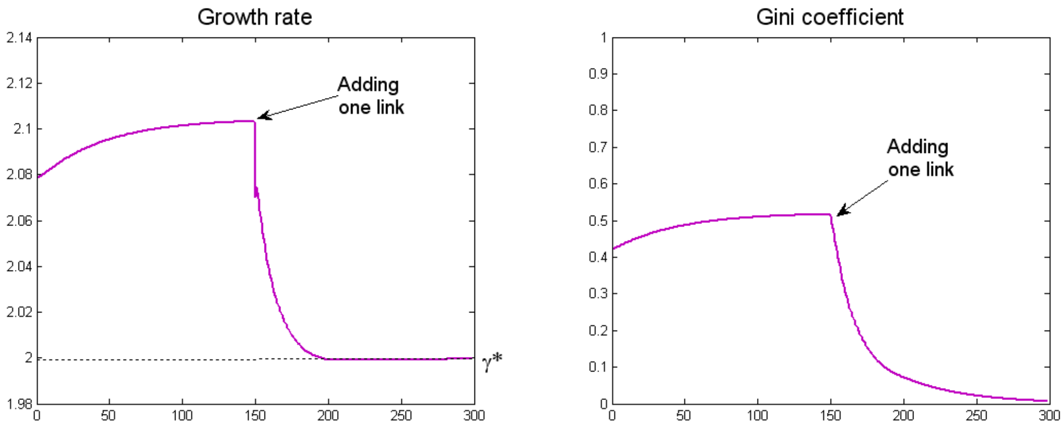

Figure 6: **Switching network structures.**

network is about 30 percent larger than the growth rate of the economy under no long run inequality. The complete network displays the lowest long run growth and it also converges to the lowest long run Gini index, which is less than one. In fact, there is a clear positive correlation of growth and inequality among all networks: more unequal societies are associated with high long run growth. Therefore, in this case, there is a clear trade-off between growth and inequality both in the short and in the long run.

Note however, that as discussed earlier, there is no monotonic relationship between our measure of network cohesion and growth and inequality. This case can also generate social mobility over time. Some households that are initially rich, might become relatively poorer if they were connected to households with relatively low human capital. This depends on the particular network and on the wealth/capital of one's neighbors (peer effects).

4.3. Transition when network structure changes. A last numerical experiment we perform is to show how inequality and the growth rate evolves when the network structure changes at some arbitrary period from one that implies long run inequality to one that implies long run equality. This is not essentially different from what we do in section 4.1, but we find it useful to show a more complete picture of how the dynamics may change and how it is possible to generate an economy that gives rise to the inverted U-shape for inequality, i.e. the Kuznets curve.

The experiment we perform is the following. We begin with the same parametrization of the deep parameters ($\theta = 4.4688$, $\eta = 0.45649$ and $\psi = 11.0640$), number of nodes, and initial conditions as in the previous two subsections. We then let $1 - \beta_1 - \beta_2 = 0.36$, and $\beta_2 = 0.6$. Compared to the parameter values of subsection 4.1, we kept the same value for the local externality parameter but increased the value of the the parameter that governs the impact of family background on human capital formation. Then for the first T' periods, the social structure is given by a disconnected network of seven nodes with two complete components, as in the top-middle panel of Table 1. Given our parameters, this structure implies a balanced growth path with inequality. At period $T' + 1$ the network switches to being the middle-left panel of Table 1 (i.e. the bridge or the network that links one node of a component with one of the other component). For our parameters, this network implies long run equality.

Figure 6 plots the evolution of inequality (Gini index) and growth rate over time when the switch occurs at period $T' = 150$. The right panel can be interpreted loosely as a Kuznets curve: over time inequality first increases and then decreases when the two components are linked until it is eliminated in the long run. Thus our framework provides a novel mechanism that can generate this well known empirical regularity. Clearly, a more satisfactory framework would generate an endogenous switch of networks at some point in time, perhaps via some optimization process; we defer this exercise to future work.

5. Closing Comments

In this paper we studied the effects of social structures on growth and inequality. We did this by constructing a simple model of endogenous growth, where human capital formation depends on a local externality determined by an underlying network structure. We show that for some calibrations such local externalities can introduce interesting dynamics and generate low wealth inequality without compromising high economic growth and welfare. Some of our numerical simulations indicate, for instance, that the star network is the optimal social structure in the sense of maximizing growth and welfare, and minimizing inequality. Most other studies (e.g. Bénabou (1996)) on growth and inequality, emphasized a trade-off between inequality and growth, but they all focused on segregated versus integrated societies. We study a variety of social structures and contrary to the traditional *efficiency-equity* trade-off, the star network might increase efficiency relatively to more integrated societies and might income distribution relatively to more segregated societies. In some sense this is unsurprising: the star network is known to be *efficient*, in that it maximizes connectivity and minimizes number of links among the network nodes.

Our model is very stylized and thus mostly serves as a framework for qualitative analysis. It is only a first step towards a deeper understanding of how social structures affect the macreoeconomy but further work should study richer frameworks that can be used for more precise policy analysis. Our work offers a simple toolbox for embedding networks in dynamic macroeconomic models that can possibly be extended to many other interesting frameworks.

APPENDIX

A. Dynamic system and BGP

We have that

$$h_{it+1} = (\theta + e_{it})^{\eta} h_{it}^{1-\beta_1-\beta_2} \bar{h}_{it}^{\beta_2} \bar{h}_t^{\beta_1} = \left(\theta + \max\left\{\frac{\psi\eta x_{it} - \theta}{1+\psi\eta}\right\}\right)^{\eta} h_{it}^{1-\beta_1-\beta_2} \bar{h}_{it}^{\beta_2} \bar{h}_t^{\beta_1}$$

Therefore

$$\begin{aligned} x_{it+1} &= \frac{h_{it+1}}{\bar{h}_{t+1}} = \frac{1}{\gamma_t}\left(\theta + \max\left\{\frac{\psi\eta x_{it} - \theta}{1+\psi\eta}\right\}\right)^{\eta} x_{it}^{1-\beta_1-\beta_2} \left(\frac{\bar{h}_{it}}{\bar{h}_t}\right)^{\beta_2} \\ &= \frac{1}{\gamma_t}\left(\theta + \max\left\{\frac{\psi\eta x_{it} - \theta}{1+\psi\eta}\right\}\right)^{\eta} x_{it}^{1-\beta_1-\beta_2} \left(\frac{\sum_j g_{ij} x_{jt}}{\sum_j g_{ij}}\right)^{\beta_2} \end{aligned}$$

Also,

$$\begin{aligned} \gamma_t &= \frac{\bar{h}_{t+1}}{\bar{h}_t} = \frac{1}{n}\frac{\sum_k h_{kt+1}}{\bar{h}_t} = \frac{1}{n}\frac{\sum_k \left(\theta + \max\left\{\frac{\psi\eta x_{kt}-\theta}{1+\psi\eta}\right\}\right)^{\eta} h_{kt}^{1-\beta_1-\beta_2} \bar{h}_{kt}^{\beta_2} \bar{h}_t^{\beta_1}}{\bar{h}_t} \\ &= \frac{1}{n}\sum_k \left(\theta + \max\left\{\frac{\psi\eta x_{kt} - \theta}{1+\psi\eta}\right\}\right)^{\eta} h_{kt}^{1-\beta_1-\beta_2} \bar{h}_{kt}^{\beta_2} \bar{h}_t^{\beta_1-1} \\ &= \frac{1}{n}\sum_k \left(\theta + \max\left\{\frac{\psi\eta x_{kt} - \theta}{1+\psi\eta}\right\}\right)^{\eta} x_{kt}^{1-\beta_1-\beta_2} \left(\frac{\sum_j g_{kj} x_{jt}}{\sum_j g_{kj}}\right)^{\beta_2} \\ &= \frac{1}{n}\sum_k \left[\left(\theta + \max\left\{\frac{\psi\eta x_{kt} - \theta}{1+\psi\eta}\right\}\right)^{\eta} x_{kt}^{1-\beta_1-\beta_2-1} \left(\frac{\sum_j g_{kj} x_{jt}}{\sum_j g_{kj}}\right)^{\beta_2}\right] x_{kt} \\ &= \frac{1}{n}\sum_k \gamma_{kt} x_{kt} \end{aligned}$$

In other words the system is described by

$$\begin{aligned} x_{it+1} &= \frac{\gamma_{it}}{\gamma_t} x_{it} = \frac{x_{it}\gamma_{it}}{\frac{1}{n}\sum_{k=1}^{n} x_{kt}\gamma_{kt}} \\ \gamma_{it} &= \left(\theta + \max\left\{\frac{\psi\eta x_{it} - \theta}{1+\psi\eta}\right\}\right)^{\eta} x_{it}^{1-\beta_1-\beta_2-1} \left(\frac{\sum_j g_{ij} x_{jt}}{\sum_j g_{ij}}\right)^{\beta_2} \end{aligned}$$

i.e. the growth rate of the average human capital is the weighted average of the growth rates of all households, weighted by their relative human capital.

A balanced growth path is defined by having the same growth rate for all households, i.e. $\gamma_{it} = \gamma_t = \gamma^*$. In that case, it has to be that the x_is remain constant in the balanced growth path, say x_i. From (9), we also have that

$$x_i = \frac{x_i\gamma^*}{\frac{1}{n}\gamma^*\sum_{k=1}^{n} x_i} \Longrightarrow \frac{1}{n}\sum_{k=1}^{n} x_i = 1 \tag{15}$$

B. STABILITY PROPERTIES OF DYNAMIC SYSTEM

B.1. Case $x_{it} > \theta/\psi\eta$. We first examine the stability properties of solutions that satisfy $x_{it} > \frac{\theta}{\psi\eta}$ as $t \longrightarrow \infty$. First let

$$z_{it} = (\theta + x_{it})^{\eta} x_{it}^{1-\beta_1-\beta_2}$$

Also, we have that if $x_{it} > \frac{\theta}{\psi\eta}$ then

$$(\theta + e_{it})^{\eta} = \left(\theta + \max\left\{\frac{\psi\eta x_{it} - \theta}{1+\psi\eta}\right\}\right)^{\eta} = \left(\frac{\psi\eta}{1+\psi\eta}\right)^{\eta} (\theta + x_{it})^{\eta} = \phi\,(\theta + x_{it})^{\eta}$$

so that

$$\gamma_{it} = \left(\frac{\theta + x_{it}}{1+\psi\eta}\psi\eta\right)^{\eta} \left(\frac{1}{x_{it}}\right)^{1-1-\beta_1-\beta_2} \left(\frac{\sum_{j=1}^{n} g_{ij} x_{jt}}{\sum_{j=1}^{n} g_{ij}}\right)^{\beta_2} = \phi \frac{z_{it}}{x_{it}} \left(\frac{\sum_{j=1}^{n} g_{ij} x_{jt}}{\sum_{j=1}^{n} g_{ij}}\right)^{\beta_2}. \tag{16}$$

We can then write

$$x_{it}\gamma_{it} = \phi z_{it} \left(\frac{\sum_{j=1}^{n} g_{ij} x_{jt}}{\sum_{j=1}^{n} g_{ij}}\right)^{\beta_2} = \phi\,(\theta + x_{it})^{\eta} x_{it}^{1-\beta_1-\beta_2} \left(\frac{\sum_{j=1}^{n} g_{ij} x_{jt}}{\sum_{j=1}^{n} g_{ij}}\right)^{\beta_2}. \tag{17}$$

Dynamics. Suppose that there a solution of the dynamic system (9) and (10) such that (15) holds and $\gamma_{it} = \gamma_t = \gamma^*$. Then it is a BGP, if it is locally stable. To check this we rewrite the system in vector form

$$\mathbf{x}_{t+1} = \mathbf{W}(\mathbf{x}_t)$$

where W is a (non-homogeneous) and highly non-linear function of x_t. We then examine whether the Jacobian of the system at a solution has all its eigenvalues inside the unit circle. If yes, the solution is a stable BGP. The Jacobian is defined by

$$J = \frac{d\mathbf{W}(\mathbf{x})}{d\mathbf{x}} = \begin{pmatrix} \frac{\partial W_1}{\partial x_1} & & \frac{\partial W_1}{\partial x_n} \\ & \ddots & \\ \frac{\partial W_n}{\partial x_1} & & \frac{\partial W_n}{\partial x_n} \end{pmatrix}$$

For a household i we have that

$$x_{it+1} = W_i(\mathbf{x}_t) = \frac{\gamma_{it}}{\gamma_t} x_{it}$$

We need the following auxiliary derivatives.

First,

$$\begin{aligned} \frac{\partial z_{it}}{\partial x_{it}} &= \frac{\partial}{\partial x_{it}} \left[(\theta + x_{it})^{\eta} x_{it}^{1-\beta_1-\beta_2}\right] = \eta\,(\theta + x_{it})^{\eta-1} x_{it}^{1-\beta_1-\beta_2} + (1-\beta_1-\beta_2)\,(\theta + x_{it})^{\eta} x_{it}^{\alpha_1-1} \\ &= (\theta + x_{it})^{\eta} x_{it}^{1-\beta_1-\beta_2} \left(\frac{\eta}{\theta + x_{it}} + \frac{1-\beta_1-\beta_2}{x_{it}}\right) = z_{it} \left(\frac{\eta}{\theta + x_{it}} + \frac{1-\beta_1-\beta_2}{x_{it}}\right) \end{aligned}$$

At BGP this is

$$\left.\frac{\partial z_{it}}{\partial x_{it}}\right|_{BGP} = (\theta + x_i)^{\eta} x_i^{1-\beta_1-\beta_2} \left(\frac{\eta}{\theta + x_i} + \frac{1-\beta_1-\beta_2}{x_i}\right)$$

We also need

$$\frac{\partial (\gamma_{it} x_{it})}{\partial x_{it}} = \frac{\partial}{\partial x_{it}} \left[\phi (\theta + x_{it})^{\eta} x_{it}^{1-\beta_1-\beta_2} \left(\frac{\sum_j g_{ij} x_{jt}}{\sum_j g_{ij}}\right)^{\beta_2}\right] = \gamma_{it} \left(\frac{\eta}{\theta + x_{it}} + \frac{1-\beta_1-\beta_2}{x_{it}} + \frac{\beta_2}{\sum_j g_{ij} x_{jt}}\right)$$

At the BGP this is

$$\left.\frac{\partial (\gamma_{it} x_{it})}{\partial x_{it}}\right|_{BGP} = \gamma^* \left(\frac{\eta}{\theta + x_i} + \frac{1-\beta_1-\beta_2}{x_i} + \frac{\beta_2 g_{ii}}{\sum_j g_{ij} x_j}\right).$$

Next we need

$$\frac{\partial (\gamma_{kt} x_{kt})}{\partial x_{it}} = \beta_2 \left(\frac{\sum_j g_{kj} x_{jt}}{\sum_j g_{kj}}\right)^{-1} \frac{g_{ki}}{\sum_k g_{kj}} \gamma_{kt} = \gamma_{kt} \frac{\beta_2 g_{ki}}{\sum_j g_{kj} x_{jt}}$$

At the BGP this becomes

$$\left.\frac{\partial (\gamma_{kt} x_{kt})}{\partial x_{it}}\right|_{BGP} = \gamma^* \frac{\beta_2 g_{ki}}{\sum_j g_{kj} x_j}$$

Finally we need

$$\frac{\partial \gamma_t}{\partial x_{it}} = \frac{\phi}{n} \left\{\beta_2 \sum_k \left[\left(\frac{\sum_j g_{kj} x_{jt}}{\sum_j g_{kj}}\right)^{\beta_2 - 1} \frac{g_{ki}}{\sum_k g_{kj}} z_{kt}\right] + \left(\frac{\sum_j g_{ij} x_{jt}}{\sum_j g_{ij}}\right)^{\beta_2} \frac{\partial z_{it}}{\partial x_{it}}\right\}.$$

Then at the BGP we have

$$\left.\frac{\partial \gamma_t}{\partial x_{it}}\right|_{BGP} = \frac{\gamma^*}{n} \left(\frac{\eta}{\theta + x_i} + \frac{1-\beta_1-\beta_2}{x_i} + \beta_2 \sum_k \frac{g_{ki}}{\sum_j g_{kj} x_j}\right).$$

Now we are ready to calculate the Jacobian.

1. We need the diagonal elements

$$\frac{\partial W_i (\mathbf{x}_t)}{\partial x_{it}} = \frac{1}{\gamma_t} \left(\frac{\partial (\gamma_{it} x_{it})}{\partial x_{it}} - \frac{\gamma_{it} x_{it}}{\gamma_t} \frac{\partial \gamma_t}{\partial x_{it}}\right) = \frac{1}{\gamma_t} \left(\frac{\partial (\gamma_{it} x_{it})}{\partial x_{it}} - x_{it+1} \frac{\partial \gamma_t}{\partial x_{it}}\right),$$

   at BGP this becomes

$$\left.\frac{\partial W_i (\mathbf{x}_t)}{\partial x_{it}}\right|_{BGP} = \frac{\eta}{\theta + x_i} + \frac{1-\beta_1-\beta_2}{x_i} + \beta_2 \frac{g_{ii}}{\sum_j g_{ij} x_j} - \frac{x_i}{n} \left(\frac{\eta}{\theta + x_i} + \frac{1-\beta_1-\beta_2}{x_i} + \beta_2 \sum_k \frac{g_{ki}}{\sum_j g_{kj} x_j}\right).$$

2. The off-diagonal elements are given by

$$\frac{\partial W_i (\mathbf{x}_t)}{\partial x_{lt}} = \frac{1}{\gamma_t} \left(\frac{\partial (\gamma_{it} x_{it})}{\partial x_{lt}} - \frac{\gamma_{it} x_{it}}{\gamma_t} \frac{\partial \gamma_t}{\partial x_{lt}}\right) = \frac{1}{\gamma_t} \left(\frac{\partial (\gamma_{it} x_{it})}{\partial x_{lt}} - x_{it+1} \frac{\partial \gamma_t}{\partial x_{lt}}\right),$$

at BGP this becomes

$$\left.\frac{\partial W_i(\mathbf{x}_t)}{\partial x_{lt}}\right|_{BGP} = \beta_2 \frac{g_{il}}{\sum_j g_{ij} x_j} - \frac{x_i}{n}\left(\frac{\eta}{\theta + x_l} + \frac{1-\beta_1-\beta_2}{x_l} + \beta_2 \sum_k \frac{g_{kl}}{\sum_j g_{kj} x_j}\right).$$

In summary,

$$\begin{aligned}
\left.\frac{\partial x_{it+1}}{\partial x_{it}}\right|_{BGP} &= \left(1 - \frac{x_i}{n}\right)\left(\frac{\eta}{\theta + x_i} + \frac{1-\beta_1-\beta_2}{x_i}\right) + \beta_2\left(\frac{g_{ii}}{\sum_j g_{ij} x_j} - \frac{x_i}{n}\sum_k \frac{g_{ki}}{\sum_j g_{kj} x_j}\right) \\
\left.\frac{\partial x_{it+1}}{\partial x_{lt}}\right|_{BGP} &= -\frac{x_i}{n}\left(\frac{\eta}{\theta + x_l} + \frac{1-\beta_1-\beta_2}{x_l}\right) + \beta_2\left(\frac{g_{il}}{\sum_j g_{ij} x_j} - \frac{x_i}{n}\sum_k \frac{g_{kl}}{\sum_j g_{kj} x_j}\right)
\end{aligned}$$

Special Case: BGP with Equality. Note that the BGP with equality, i.e. $x_i = 1$ trivially satisfies (15) and yields $\gamma^* = \phi(1+\theta)^\eta$. In that case the matrix $J(\mathbf{x})$ reduces to

$$J(\mathbf{x}) = (\delta + 1 - \beta_1 - \beta_2)\left(\mathbf{I}_n - \frac{1}{n}\mathbf{1}_{n\times n}\right) + \beta_2\left(\mathbf{I}_n - \frac{1}{n}\mathbf{1}_{n\times n}\right)R$$

where $\delta = \eta/(\theta+1)$. Therefore, given a network g, the balanced growth path where $x^* = 1$ and $\gamma^* = \phi(1+\theta)^\eta$ is stable if the eigenvalues of J are inside the unit circle.

There is no general result that gives the eigenvalues of the sum of two matrices as a function of the eigenvalues of the summands. However, in the special case of two matrices that commute, an extension of the theorem by Frobenius on commuting matrices ensures that the spectral radius of the sum of two commuting square matrices is the sum of the two spectral radii of the two matrices, i.e. that $\rho(A+B) = \rho(A) + \rho(B)$. We next show that the matrices $(\delta+1-\beta_1-\beta_2)\left(\mathbf{I}_n - \frac{1}{n}\mathbf{1}_{n\times n}\right)$ and $\beta_2 F_g$ indeed commute. First, note that

$$\sum_i f_{il} = \sum_i (r_{il} - \bar{r}_l) = \sum_i r_{il} - \sum_i \bar{r}_l = n\bar{r}_l - n\bar{r}_l = 0,$$

and

$$\begin{aligned}
\sum_l f_{il} &= \sum_l (r_{il} - \bar{r}_l) = \sum_l r_{il} - \sum_l \bar{r}_l = \sum_l \frac{g_{il}}{\sum_j g_{ij}} - 1 \\
&= \frac{\sum_l g_{il}}{\sum_j g_{ij}} - 1 = 0.
\end{aligned}$$

Therefore

$$\mathbf{1}_{n\times n} F_g = \mathbf{0}_n, \text{ and } F_g \mathbf{1}_{n\times n} = \mathbf{0}_n.$$

We then have that

$$\begin{aligned}
&\left[(\delta+1-\beta_1-\beta_2)\left(\mathbf{I}_n - \frac{1}{n}\mathbf{1}_{n\times n}\right)\right][\beta_2 F_g] = \beta_2(\delta+1-\beta_1-\beta_2)\left(\mathbf{I}_n F_g - \frac{1}{n}\mathbf{1}_{n\times n} F_g\right) = \beta_2(\delta+1-\beta_1-\beta_2) F \\
&\quad = \beta_2(\delta+1-\beta_1-\beta_2)\left(F_g \mathbf{I}_n - \frac{1}{n} F_g \mathbf{1}_{n\times n}\right) = [\beta_2 F_g]\left[(\delta+1-\beta_1-\beta_2)\left(\mathbf{I}_n - \frac{1}{n}\mathbf{1}_{n\times n}\right)\right]
\end{aligned}$$

and the two matrices commute. From the aforementioned theorem, the spectral radius (i.e. the maximum eigenvalue in absolute value) of the Jacobian is equal to the sum of the spectral radii of the matrices $\beta_2 F_g$ and $(\delta + 1 - \beta_1 - \beta_2)\left(\mathbf{I}_n - \frac{1}{n}\mathbf{1}_{n\times n}\right)$. We have that

$$\begin{aligned} \rho\left((\delta + 1 - \beta_1 - \beta_2)\left(\mathbf{I}_n - \frac{1}{n}\mathbf{1}_{n\times n}\right)\right) &= \delta + 1 - \beta_1 - \beta_2, \\ \rho\left(\beta_2 F_g\right) &= \beta_2 \rho\left(F_g\right). \end{aligned}$$

Then a necessary and sufficient condition for stability of the balanced growth path is that

$$\beta_1 + \beta_2 \kappa_g > \delta.$$

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