An Adjustment Cost Model of Distributional Dynamics

Parantap Basu[1] Yoseph Getachew[2]

Durham University Durham University

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[1]Economics Department, Durham University, 23/26 Old Elvet, Durham DH1 3HY, UK.e-mail: parantap.basu@durham.ac.uk.

[2]Durham Business School, Durham University, DH1 3LB, Durham, UK; e-mail: yoseph.getachew@durham.ac.uk

Abstract

We analyze the distributional effect of adjustment cost, in an environment with incomplete capital market. We find that a higher adjustment cost for human capital slows down the intergenerational mobility and results in persistent inequality across generations. This result is robust in alternative environments. In addition to adjustment cost, other productivity parameters such as the output elasticity of human capital, labour supply elasticity of home production and the productivity of child care determine the intergenerational mobility. A public redistributive policy favouring poor helps the intergnerational mobility. A pro-rich redistributive policy can also help this mobility as long the rich bias is kept in a limit. The adjustment cost generally slows down the convergence process even in an environment where the government is actively following such a redistributive policy.

1 Introduction

There are two distinct dimensions of inequality. First is the cross sectional inequality at a given moment of time. Second is the variation of income or status of a given family across generations. The latter notion of inequality is inherently dynamic as it is determined by intergenerational mobility. These two dimensions of inequality may or may not be connected. If they do, the implication is that the initial cross sectional inequality could play a role in determining the intergenerational mobility. It is an open question whether the son of a poor farmer with meagre resources to invest in education will become a software engineer. The evidence during the last two decades, however, point to the direction that such intergenerational mobility is not taking place (Machin, 2004).

The seminal paper of Becker and Tomes (1979) analyze these two notions of inequalities in an equilibrium framework and draw the conclusion that a stable distribution of income can emerge which could be explained by individual and market lucks alone. Their crucial assumption is that the credit market is perfect which means that individuals with low wealth and high marginal product of capital could borrow from individuals with the opposite trait and could tend to equalize the differences in saving. Thus the residual inequality could be attributed to luck. Since then a considerable literature (Loury, 1981, Mulligan, 1997, Banerjee and Newman, 1993, Galore and Zeira, 1993, Bandyopadhyay and Basu, 2005), evolved emphasizing the role of credit market imperfection in perpetuating the inequality.

The aim of this paper is to explore how some factors which are important to the investment climate (such as adjustment cost) could affect distributional dynamics in a heterogenous society where market is incomplete and diminishing marginal returns to factors prevail. We analyze the effects of human capital adjustment cost on distribution dynamics in a neoclassical setting with missing credit markets. As in Loury (1981), and Benabou (2000, 2002), we continue with a scenario with missing credit and insurance markets. Individuals thus differ in terms of initial distribution of human capital and innate abilities. The differences in abilities are due to idiosyncratic shocks to productivity which cannot be hedged using an insurance market. The new feature in our setting is the introduction of an investment technology which has been hitherto ignored in the inequality and intergenerational mobility literature. This special feature is a convex capital adjustment cost technology. Such an adjustment cost technology basically means that the marginal cost of investment in reproducible capital (measured in terms of foregone consumption) is rising in the level of investment in human capital. If all societies face common such adjustment cost technology, the degree of persistence in economic inequality and the intergenerational mobility could depend on the curvature of the adjustment cost function in a nontrivial way. A society facing a steep adjustment cost function would experience perpetual inequality and low intergenerational mobility measured by the serial correlation between the wealth of current and future generations.

Why does a higher adjustment cost slow down intergenerational mobility? When the credit market is missing, individuals investment opportunities (which is investment in human capital in our model) are limited to the resource they have in hand. Because of diminishing marginal return to investment, the return to investment is higher for poor than the rich due to poor's naturally low investment rate. Therefore, technological factors that impede individual investment opportunities (such as a higher adjustment cost) could disproportionately harm the poor households. This in turn affects negatively the intergenerational mobility of households. Thus adjustment cost can aggravate the persistence of economic inequality that is driven by missing credit markets.

What could possibly give rise to an adjustment cost in human capital production? Our explanation for adjustment cost in the human capital technology rests on family ties. Alesina and Giuliano(2010) document that countries with greater family ties experience less youth geographical mobility, more specialization in home production and less education of women and more fertility. We model this greater family tie as a human capital technology exhibiting a convex adjustment cost. Such an adjustment cost technology means that the elasticity of future income to current positive shock is less than unity. In other words, agents respond less to good luck or positive shock to income.

We set up a model in which households are heterogenous in terms of initial human capital and innate ability. They receive warm-glow utility from investing in child's education. As in Loury (1981) human capital is the only form of reproducible capital in the economy. Idiosyncratic productivity shocks together with initial difference in wealth could give rise to current cross-sectional inequality. The absence of credit and insurance markets prevent agents from mitigating these idiosyncratic shocks. An unlucky agent suffering a bad productivity shock invests less resources and time to her child's education which means her child inherits less human capital. How quickly her offspring gets over this disadvantage depends on structural factors which can be broadly subdivided in two categories: (i) technological factors which relate to the productivity of human capital, (ii) public redistributive policy factors. Regarding (i), we find that the process of convergence can be aggravated if (a) adjustment cost is higher, (b) parents have lower labour supply elasticity and (c) the output elasticity of human capital is higher. Regarding (ii), we find that a public service programme that disproportionately benefits poor could help intergenerational mobility. On the other hand, a pro-rich programme can also help this convergence process as long as the pro-rich bias is kept at a minimum. In all these alternative model environments, a higher adjustment cost of human capital generally slows the process of convergence.

The paper is organized as follows. Section 2 presents the model. Section 3 characterizes the dynamics and equilibrium of individual wealth accumulation. Section 4 examines intergenerational mobility and inequality dynamics. Section 4 studies the distributional dynamics in an environment with public funding. Section 6 concludes.

2 The model

2.1 Preference and technology

Suppose a continuum heterogeneous households $i \in [0,1]$ in overlapping generations. Each household i consists of an adult of generation t attached to a child of generation $t+1$. When young, agents who are born at $t-1$, do not make any decisions; they inherit human capital from their parents while their consumption is already included in that of their parents. . When adult, at t, they use their acquired human capital (h_t^i) for the production of final goods and services to earn income (y_t^i) using the following Cobb-Douglas production function.

$$y_t^i = \epsilon_t^i \left(h_t^i\right)^\alpha \tag{2.1}$$

Individuals are subject to an i.i.d. idiosyncratic productivity shocks ϵ_t^i which drive their total marginal productivity.[1] Further assume

$$\epsilon_t^i = \epsilon_t \varphi_t^i \tag{2.2}$$

where the idiosyncratic shock φ_t^i follows the process: $\ln \varphi_t^i \sim N(-\upsilon^2/2, \upsilon^2)$ and ϵ_t is an aggregate shock which obeys $\ln \epsilon_t \sim N(0,1)$. By construction $\mathrm{E}(\epsilon_t^i) = \epsilon_t$.

[1] Eq. (2.1) could be interpreted as income net of the cost of physical capital (e.g., Benabou 2002).

Technology of human capital production

Our important innovation here is the specification of the investment technology. The production of human capital at date t (h_{t+1}^i) takes place with the aid of three factors: (i) past human capital (h_t^i), (ii) investment in schooling (s_t^i), (iii) home production such as child care which is a function of raw labour (l_t^i). Assume the following Cobb-Douglas form as follows:

$$h_{t+1}^i = Q_t^i \left(h_t^i\right)^{1-\theta} \left((1-\delta)h_t^i + s_t^i\right)^{\theta} \tag{2.3}$$

where $0 < \theta < 1$, $\omega > 0$, $0 < \delta < 1$. Note that there are increasing returns to scale in the production of human capital.[2] The parameter δ is the rate of depreciation of human capital. Alternatively, $1-\delta$ can be thought of as the degree of intergenerational spillover knowledge as in Mankiw, Romer and Weil (1992). For example, a musician's child may inherit $(1-\delta)$ fraction of the human capital (h_t^i) of her father due to the musical environment where she is raised.

The variable Q_t^i is the productivity of human capital which can be thought of as cognitive skills as in Hanushek and Weissman (2008).[3] This cognitive skill is endogenous as it depends on the parental time in child care. We posit the following cognitive skills technology:[4]

$$Q_t^i = \left(l_t^i\right)^{\omega}$$

where $\omega > 0$. The parameter ω is the elasticity of cognitive skills with respect to time spent on child raising or schooling of kids.

The parameter θ is the degree of adjustment cost in the production of human capital in the same spirit as in Lucas and Prescott (1971), Hercowitz and Sampson (1911) and Basu (1987). This technology parameter is the central interest in this paper. If θ is a fraction, greater investment in schooling (s_t^i) alone gives rise to diminishing returns (because $\frac{\partial h_{t+1}^i}{\partial s_t^i}$ is decreasing in s_t^i) unless it is accompanied by an increase in raw labour (l_t^i) such as parental care and past human capital (h_t^i). In other words, unless knowledgeable parents (with high h_t^i) spend more time (l_t^i) on child care, schooling

[2] This gives rise to non-convexity in the investment technology. This is not a problem for stability of the solution as long as ω is bounded.

[3] Hanushek and Weissman (2008) measure this cognitive skill in terms of international test scores in science and mathematics.

[4] Similar cognitive skill technology is used in Basu and Bhattrai (2011).

(s_t^i) alone would give rise to diminishing returns. A larger θ will slow down the pace of decline in the marginal return to schooling ($\frac{\partial h_{t+1}^i}{\partial s_t^i}$). Thus, the parameter θ could proxy the quality of schools. If $\theta = 1$ (zero adjustment cost), and $\omega = 0$, the investment technology reduces to a standard linear depreciation rule.

Utility function and budget constraint

Agents care about their own consumption (c_t^i) and the human capital stock of their children (h_{t+1}^i), which can be justified by "joy of giving". Since h_{t+1}^i has a home production component , such a utility function can be thought of as a reduced form as in Greenwood et. al. (1995). In other words, the utility of the adult at date t is given by:[5]

$$u\left(c_t^i, l_t^i, h_{t+1}^i\right) = \ln\left(c_t^i - b\left(l_t^i\right)^\eta\right) + \beta \ln h_{t+1}^i \tag{2.4}$$

The parameters $b > 0$ and $\eta > 1$ determine the time parents allocate to care for their children; $1/(\eta - 1)$ is the intertemporal elasticity of substitution in labor supply that will be used for child care.

At the end of the period, parents allocate after tax income between current consumption c_t^i and saving s_t^i. The latter is used for investment in human capital accumulation of the offspring as shown in (2.3). The budget constraint is thus given by:

$$c_t^i + s_t^i = y_t^i\left(1 - \tau\right) \tag{2.5}$$

where h_{t+1}^i and τ represent the human capital of the offspring of agent i and the tax rate facing the agent respectively.

2.2 Initial distribution of human capital

At the beginning, each adult of the initial generation is endowed with human capital h_0^i. She supplies it inelastically to a privately-held firm, which is subject to idiosyncratic shock that brings individual-

[5] Note that models with unitary inter-temporal elasticity of substitution utility function and altruistic agents with a "joy of giving" motive are ubiquitous in growth literature (see, for instance, Glomm and Ravikumar, 1992, Galor and Zeira, 1993, Saint-Paul and Verdier 1993(Saint-Paul and Verdier 1993), and Benabou, 2000). The results are similar in a dynastic altruism model as in Barro (1985), which we show in the appendix.

specific risk to income and investment.[6] Households are then both consumers and entrepreneurs. The distribution of h_0^i takes a known probability distribution, $\ln h_0^i \sim N(\mu_0, \sigma_0^2)$. and it evolves over time along an equilibrium trajectory.

2.3 Equilibrium

An individual household of cohort t solves the following problem, obtained by substituting (2.5) and (2.3) into (2.4),

$$\max_{s_t^i, l_t^i} \ln\left(y_t^i(1-\tau) - s_t^i - b\left(l_t^i\right)^\eta\right) + \beta \ln\left(l_t^i\right)^\omega h_t^i \left(1 - \delta + \frac{s_t^i}{h_t^i}\right)^\theta \tag{2.6}$$

taking as given τ and h_t^i. The optimization yields,

$$s_t^i \quad : \quad s_t^i + \frac{1-\delta}{1+\theta\beta} h_t^i = \frac{\theta\beta}{1+\theta\beta}\left(y_t^i(1-\tau) - b\left(l_t^i\right)^\eta\right) \tag{2.7}$$

$$l_t^i \quad : \quad \left(l_t^i\right)^\eta = \frac{\beta\omega}{b(\eta+\beta\omega)}\left(y_t^i(1-\tau) - s_t^i\right) \tag{2.8}$$

Combining (2.7) and (2.8), we obtain

$$s_t^i \quad = \quad y_t^i \frac{(1-\tau)\eta\theta\beta}{\eta+\beta\omega+\theta\beta\eta} - \frac{(1-\delta)(\eta+\beta\omega)}{\eta+\beta\omega+\theta\beta\eta} h_t^i \tag{2.9}$$

$$l_t^i \quad = \quad \left(\frac{\omega\beta(1-\tau)}{b(\eta+\beta\omega+\theta\beta\eta)} y_t^i\right)^{\frac{1}{\eta}} \tag{2.10}$$

Most of the results are standard. The first term on the right hand side of eq. (2.9) shows an agent's optimal saving as a function of her income, when human capital is fully depreciated. The presence of the second term in the right hand side of (2.9) shows that individual's optimal saving decision constitutes of both new investment plus a replacement of depreciated capital. It captures how agents compensates the depreciation of capital through shifting resources between saving and consumption. According to (2.10), agents choose effort proportional to the level of their income. The parameter $\frac{1}{\eta}$ is the usual elasticity of income to labor supply.

[6] Such type of undiversified entrepreneurial and investment risks are a common feature of the developing word, according to Angeletos and Calvet, (2006). Even in the United States, they argue, the majority of firms constitutes of privately owned companies.

3 Individual capital dynamics

We begin characterizing the dynamics and equilibrium of the economy at the individual level. We suppose a complete depreciation of human capital, $\delta = 1$.[7] Then, the ith individual optimal human capital accumulation is given by, from (2.1), (2.2), (2.3), (2.9) and (2.10),

$$h_{t+1}^{i} = \phi_1 \left(h_t^i\right)^{\varrho_2} \left(\epsilon_t \varphi_t^i\right)^{\varrho_1} \tag{3.11}$$

where

$$\varrho_1 \equiv \frac{\omega}{\eta} + \theta \tag{3.12}$$

$$\varrho_2 \equiv \frac{\alpha\omega}{\eta} + \theta(\alpha - 1) + 1 = \alpha\left(\frac{\omega}{\eta} + \theta\right) + 1 - \theta \tag{3.13}$$

$$\phi_1 \equiv (\eta\theta)^{\theta} \left(\frac{\omega}{b}\right)^{\frac{\omega}{\eta}} \left(\frac{\beta\epsilon(1-\tau)}{(\eta + \beta\omega + \theta\beta\eta)}\right)^{\varrho_1} \tag{3.14}$$

We see immediately from (3.11) that the difference in luck and parental wealth among individuals determine the distribution of the next period human capital.

4 Intergenerational Mobility and the Dynamics of Inequality

In the extant literature a precise measure of intergenerational mobility is the correlation between the child's income and the parent's income (see, for example, Machin, 2004). If this correlation is low, the mobility is high and vice versa. In the context of our model, this correlation is summarized by ϱ_2 appearing in the dynamics of individual human capital (3.11). This key equation also determines the evolution of inequality which can be measured by the cross sectional variance of the log of the level of human capital.[8] Since the preference and technology parameters are the same for all individuals, the cross sectional variance of $\ln\phi$ is zero. Based on (3.11) the cross sectional variance of capital stock thus evolves as:

$$\sigma_{t+1}^2 = (\varrho_2)^2 \sigma_t^2 + (\varrho_1)^2 \upsilon^2 \tag{4.15}$$

[7] We provide results for the case $\delta \neq 1$ for the basic model in the appendix, using a loglinear approximation method of Campbell, (1994).

[8] There are numerous measures of inequality (see for example, Sala-i-Martin, 2002). The variance of the log level of human capital is a reasonable measure compared to the variance of the level of human capital itself. Sala-i-Martin points out that the variance of the level is a poor measure because it depends on the scale of measurement.

Note that the root of the difference equations (3.11) and (4.15) are stable as long as $0 < \varrho_2 < 1$, which implies $\omega < \alpha^{-1}\theta(1-\alpha)\eta$.[9] Thus even though the human capital production technology (2.3) is subject to increasing returns to scale, we have a stable steady state configuration as long as ω is bounded.

The intergenerational mobility and the dynamics of (the cross sectional) wealth inequality are, therefore, characterized jointly by ϱ_2. Four crucial parameters, namely θ, η, ω and α, as seen from (3.13) determine the intergenerational mobility and the dynamics of inequality. The following proposition is self evident from (3.11) and (4.15).

Proposition 1 *A higher degree of adjustment cost (lower θ) or a higher elasticity (lower η), a higher labour productivity ω and a higher capital share α make the intergenerational mobility slower and the inequality process more persistent.*

The intuition that a higher adjustment cost (lower θ) lead to inequality persistence is straightforward. When the credit market is missing, individuals investment opportunities are limited to the resource they have in hand. Because of diminishing marginal return to investment, i.e. $\alpha < 1$ in (2.1), the poor have higher marginal productivity than the rich due to their naturally low investment rate. Therefore, technological factors that impede individual investment opportunities (such as a higher adjustment cost) must disproportionately harm the poor households.[10]

The intuition with respect to the effects of η, α and ω in inequality dynamics are also simple. Intergenerational mobility and inequality dynamics decrease in α due to its effect in relative marginal productivity of individuals. The relative marginal productivity of the rich increases in α. The difference in saving between the rich and the poor is proportional to α. This can be easily demonstrated by rewriting (5.32) for two households i and j: $\ln s_t^i - \ln s_t^j = \alpha\left(\ln h_t^i - \ln h_t^j\right) + \left(\ln \varphi_t^i - \ln \varphi_t^j\right)$. If $h_t^i > h_t^j$, then, the difference in saving between the ith and the jth households increases in α.

Applying similar reasoning we can show that the rich spend relatively more time on child education

[9] Because $\varrho_2 < 1$ iff $\alpha\left(\frac{\omega}{\eta} + \theta\right) < \theta$.

[10] Note that, however, if the production technology were increasing returns to scale ($\alpha > 1$), – which is infeasible due to stability condition (4.15)–, then a higher θ could have led to inequality persistence. The intuition is similar as before: Increasing returns to investment implies rich households disproportionately benefit from additional investment. In this case, a favorable investment environment such as a lower adjustment cost (higher θ) may lead to inequality persistence as it disproportionately benefits the rich households.

as $1/\eta$ – the labour supply elasticity with respect to y_t^i – becomes larger.[11] Therefore, a higher human capital inequality will perpetuate if η is small or the labour supply elasticity of income is large. On the other hand, effort is rewarded according to ω – the productivity of labor in the human capital production sector (2.3). Thus, if the rich devote more efforts in child education than the poor, – which is the case in this paper,[12] – then a higher ω could slow down intergenerational mobility.

4.1 Inequality decomposition

How does inequality transmit through generations? Are today's children poor because their parents are resource poor? Or is it due to bad luck suffered by their parents and grandparents? Although this is a much researched question, in the context of our model, we find that the relative importance of these two factors depends on the adjustment cost.

When capital market is incomplete, both luck and difference in initial human capital of the first generation play a central role in transmitting initial inequality through generations. We see this easily when rewriting (3.11) as the difference in human capital of two adjacent children i and j,

$$\ln h_{t+1}^i - \ln h_{t+1}^j = \varrho_1 \left(\ln \varphi_t^i - \ln \varphi_t^j \right) + \varrho_2 \left(\ln h_t^i - \ln h_t^j \right) \tag{4.16}$$

The first term shows the effect of luck while the second term is the effect of difference in human capital of parents. However, further decomposing (**??**) shows that the latter difference could be also due to luck of grandparents or difference in human capital of grandparents. Thus, further iterating (**??**) one more generation**:**

$$\begin{aligned} \ln h_{t+1}^i - \ln h_{t+1}^j \quad = \quad & \varrho_1 \left(\ln \varphi_t^i - \ln \varphi_t^j \right) + \varrho_2 \varrho_1 \left(\ln \varphi_{t-1}^i - \ln \varphi_{t-1}^j \right) \\ & + (\varrho_2)^2 \left(\ln h_{t-1}^i - \ln h_{t-1}^j \right) \end{aligned} \tag{4.17}$$

The second interaction term picks up the effect of past luck or shock experienced by grandparents on today's generation. The past shock impacts the current child's human capital in proportion to

[11] From (2.10), if two households differ in their income, their labour supply response will differ depending on the magnitude of η. For example, for two households i and j, we can also rewrite: $\ln l_t^i - \ln l_t^j = \frac{1}{\eta}(\ln y_t^i - \ln y_t^j)$. Thus, within the tth generation if the ith household has a higher income than the jth household ($y_t^i > y_t^j$), the ith household will spend considerably more time on child education compared to the jth household if $\frac{1}{\eta}$ is large.

[12] To see this rewrite (2.10): $\ln l_t^i - \ln l_t^j = \frac{\alpha}{\eta}\left(\ln h_t^i - \ln h_t^j\right) + \frac{1}{\eta}\left(\ln \varphi_t^i - \ln \varphi_t^j\right)$. And, suppose $h_t^i > h_t^j$. Then, $\frac{\alpha}{\eta} > 0$ implies that the rich spend relatively more time on children education than the poor.

how it affected her parents given that there was difference in human capital between the ith and jth grandparents. This explains the interaction term $\varrho_2\varrho_1$. Thus a combination of past luck and initial inequality determine both the dynamics and the steady state inequality. As time advances initial inequality (the third term in the right hand side of (4.17)) has a decaying effect on inequality (if $\varrho_2 < 1$) while the first and second term determine the long run inequality. If the adjustment cost is high (θ is low), this initial disadvantage persists implying that higher adjustment cost unambiguously leads to inequality persistence. On the hand, from the first (second) term, the effect of past luck on inequality is weakened (ambiguous) if adjustment cost is high. Therefore, there is no a clear cut effect of adjustment cost in steady state inequality.

Iterating (4.16) backward until date zero, one gets the following decomposition of the cross sectional variance due to ancestral difference in wealth and the difference due to luck.

$$\Delta_{t+1}^{h} = \varrho_2^{t+1}\Delta_0^{h} + \varrho_1 \sum_{k=0}^{k=t} \varrho_2^{k}\Delta_{t-k}^{\varphi} \tag{4.18}$$

where $\Delta_0^h \equiv \ln h_0^i - \ln h_0^j$ and $\Delta_{t-k}^{\varphi} \equiv \ln \varphi_{t-k}^i - \ln \varphi_{t-k}^j$. The appendix provides the derivation of (4.18). Note also that the decomposition is done independent of the lognormal distribution assumption. The first term of (4.18) represents the difference due to initial inequality in wealth. The second term picks up the effect of current and past lucks on the current inequality. The initial wealth difference tends to have a more persistent effect on the current inequality in the presence of higher adjustment cost which explains why the first term is decreasing in θ. On the other hand, the adjustment cost has ambiguous effects on the second term in the right hand side of (4.18). While the adjustment cost dampens the luck effect of parents on the current inequality (considering ϱ_1 in the second term), it also intensifies the past effects of shocks due to the interaction between luck and the ancestral difference in wealth ($\varrho_2^k\Delta_{t-k}^{\varphi}$). This tension between current luck effect and past luck effects explains why the second term behaves ambiguously with respect to θ.

4.2 Income inequality

The derivation of income variance is straightforward. Based on (2.1), the cross sectional variance of income at date t is given by

$$\sigma_{y,t}^2 \equiv \mathrm{var}(\ln y_t^i) = \alpha^2\sigma_t^2 + \upsilon^2 \tag{4.19}$$

Plugging (4.15) into (4.19) and using (4.19) repeatedly one gets the following equilibrium law of motion for income inequality:

$$\sigma_{y,t}^2 = (\varrho_2)^2 \sigma_{y,t-1}^2 + \upsilon^2 \left(1 - (\varrho_2)^2 + \alpha^2 (\varrho_1)^2\right) \tag{4.20}$$

The steady state income inequality is thus given by:

$$\sigma_{y,t}^2 = \upsilon^2 + \frac{\alpha^2 (\varrho_1)^2 \upsilon^2}{1 - (\varrho_2)^2} \tag{4.21}$$

Upward bias in the income based estimate of intergenerational mobility

In the extant literature, intergenerational mobility is measured by a regression of child's income on the parent's income. We will demonstrate now that such a measure is likely to overestimate the mobility if adjustment cost is present.

To see this, ignore aggregate risk for the moment. Note first that based on the production function (2.1), the log of income of the ith family in the tth cohort is given by:

$$\ln y_t^i = \alpha \ln h_t^i + \ln \varphi_t^i \tag{4.22}$$

Using (3.11), we get the ARMA(1,1) representation for income:

$$\ln y_t^i = \alpha \ln \phi_1 + \varrho_2 \ln y_{t-1}^i - (1-\theta) \ln \varphi_{t-1}^i + \ln \varphi_t^i \tag{4.23}$$

which represents the evolution of the income of the ith individual. Note that the current idiosyncratic shock φ_t^i impacts the income but the capital wealth of the current generation.

Thus an ordinary least square regression of the child's income on the parent's income in the ith family will involve an error term which has a negative autocorrelation. It is easy to verify that the OLS estimate of the coefficient $\widehat{\varrho}$ is given by:

$$\widehat{\varrho} = \varrho_2 - (1-\theta) \frac{\operatorname{cov}(\ln \varphi_{t-1}^i, \ln y_{t-1}^i)}{\operatorname{var}(\ln y_{t-1}^i)} \tag{4.24}$$

Since the covariance between contemporaneous income and luck is positive (see 4.23), the immediate implication is that $\widehat{\varrho} < \varrho_2$. Thus the standard income based estimate of intergenerational mobility is likely to produce an upward bias. The bias will be greater in an economy with higher adjustment cost (low θ).

4.3 Steady state Inequality

Based on (4.15) the steady state variance of capital stock is given by:

$$\sigma^2 = \frac{(\varrho_1)^2 \upsilon^2}{1-(\varrho_2)^2} = \frac{\left(\frac{\omega}{\eta}+\theta\right)^2 \upsilon^2}{1-\left(\alpha\left(\frac{\omega}{\eta}+\theta\right)+1-\theta\right)^2} \tag{4.25}$$

Inequality in the long run is mainly the result of individuals' differences in labor and capital investment decision as a response to differences in luck. From (4.25), we have the following proposition:

Proposition 2 *The long run distribution of wealth (σ^2) is a function of initial distribution in luck (υ^2) and independent of the initial distribution of σ_0^2. σ^2 increases in ω,$\frac{1}{\eta}$,and α whereas θ has an ambiguous effect on the steady state inequality, which depends on the values of other parameters.*

Proof. Large values of ω,$\frac{1}{\eta}$,and α increase the nominator and/or decrease the denominator of (4.25) whereas large θ increases both. See also Fig. 1 and 2. ■

The equilibrium and the time path of inequality thus depend on the parameters of the production and accumulation technologies including the adjustment cost. In economies with low adjustment cost, initial wealth difference tends to be less persistent. However, they may not have a smaller steady state distribution. To illustrate this numerically, suppose a low ($\theta = 0.8$) and a high ($\theta = 0.1$) adjustment cost economies. The other parameters are set at $\alpha = 0.36$, $\eta = 1.5$, and $\upsilon^2 = 0.25$[13] while we experiment between two values of ω, 0.12 and 0.22.

Figures 1 and 2 plot the inequality for two economies that differ in adjustment cost under two specification of ω. For high adjustment cost economy ($\theta = 0.1$), initial wealth difference tends to have a persistent effect in all cases. Inequality starts from the initial variance of wealth and slowly declines over time and settles down to the steady state level. For a low adjustment cost economy, initial inequality effect washes out very quickly and the current luck effect dominates. The economy starts off from the initial variance and the variance jumps to a higher level in response to luck. But, the steady state inequalities are 0.32 and 0.69 for a high ($\theta = 0.1$) and low ($\theta = 0.8$) adjustment cost respectively when $\omega = .22$ whereas they are 0.27 and 0.12 when $\omega = 0.12$.

[13] The simulation is mainly for illustrative purpose that no calibration exercise is performed.

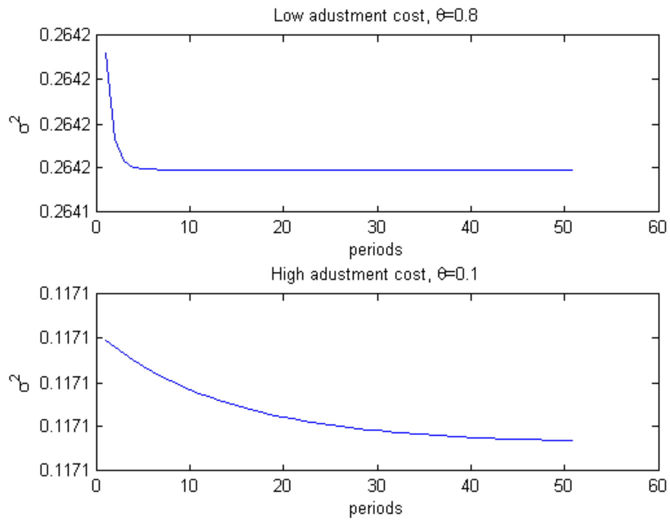

Figure 1: Time path of inequality ($\omega = 0.12$)

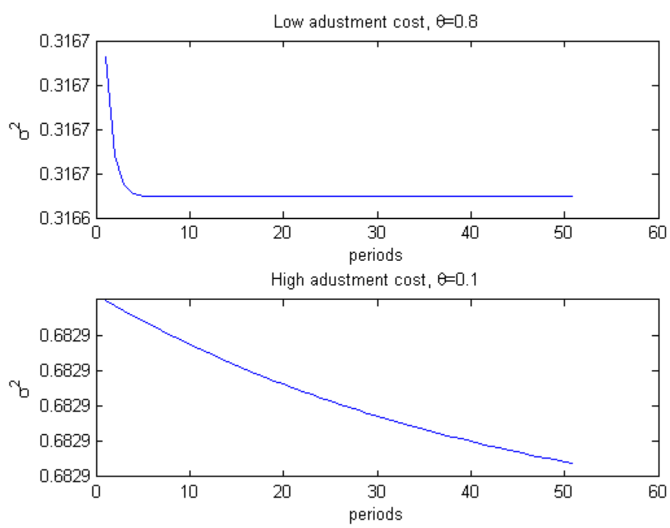

Figure 2: Time path of inequality ($\omega = 0.22$)

4.4 Inequality and aggregate efficiency

Aggregating (3.11), we get the difference equation that characterizes the dynamics of aggregate (average) capital that determines growth[14]

$$\begin{aligned} \ln h_{t+1} &= \ln \phi_1 + \varrho_2 \ln h_t + \frac{1}{2}\sigma_t^2 \varrho_2 (\varrho_2 - 1) \\ &\quad + \frac{1}{2}\upsilon^2 (\varrho_1)(\varrho_1 - 1) \end{aligned} \tag{4.26}$$

Therefore, the dynamics of the economy is determined by (4.15) and (4.26). Both are stable and monotonically converge to their respective steady state when $0 < \varrho_2 < 1$. From (4.26), we have the following proposition:

Proposition 3 *Inequality at date t reduces the average capital stock and hence growth at date $t+1$.*
Proof. *Given, the stability condition $0 < \varrho_2 < 1$, $\partial \ln h_{t+1}/\partial \sigma_t^2 < 0$.* ■

The intuition is simple. The poor have a relatively high marginal productivity due to diminishing returns to investment. But, since they cannot borrow from those who have a lower marginal product and invest due to missing credit market, Pareto efficiency cannot be achieved in this economy, which is often implicitly assumed in representative agent models with complete market. Therefore, a higher inequality leads to a greater inefficiency as it correlates to missing productive opportunities.

5 Public fund provision, and inequality dynamics

5.1 Human capital accumulation

Let us now look at a situation where the government provides part of the funds that are required for human capital production. This is in fact more consistent with empirical evidence that in most industrialized countries, education funding comprises public and private resources. For instance, in the UK, until recently, about 35% of universities' total funding comes from the government (BBC, June 27, 2011).

Thus, let now the government levy a proportional tax τ on the total output and provides g_t bundle of public services to each households. But, suppose that the use and efficiency of the public resource

[14] During aggregation, we use the fact that $\ln E\left[x_t^i\right] = E\left[\ln x_t^i\right] + \frac{1}{2}\operatorname{var}\left[\ln x_t^i\right]$. Note that variables without the superscript i and subscript t denote aggregate and steady state values, respectively.

is not identical among households. Depending on its type, a given public service could benefit certain households more than proportionally. It could disproportionately benefit the poor due to their lack of access to its private substitutes; or the rich due to their greater access to its complements. Internet provision by the public sector may disproportionately benefit those individuals who own laptop, for instance. On the other hand, a provision of public transport or free meals in school may benefit the poor more than proportionally as they are the ones who could lack these basic inputs.

Based on these premises, from (2.3), the ith individual human capital function is given by, when $\delta = 1$,

$$h_{t+1}^i = \left(l_t^i\right)^{\omega} h_t^i \left(\frac{\left(g_t^i\right)^{1-\xi}\left(s_t^i\right)^{\xi}}{h_t^i}\right)^{\theta} \tag{5.27}$$

and

$$g_t^i = \frac{g_t}{\left(h_t^i\right)^{\kappa}} \tag{5.28}$$

where[15]

$$g_t = y_t\tau = \epsilon_t\tau\left(h_t\right)^{\alpha} e^{\frac{1}{2}\sigma_t^2(\alpha-1)\alpha} \tag{5.29}$$

From (5.28), the parameter κ features the redistributive nature of the public service. The direction and magnitude of κ determines the type and the intensity of the disproportionate impact of the provision of the public service on the individuals' human capital formation, respectively (see Getachew, 2010). The case $\kappa \in (0,1)$ ($\kappa \in (-1,0)$) is related to public services that benefit the poor (the rich) more than proportional whereas $\kappa = 0$ implies a proportional effect. Equation (5.29) shows that the government has a balanced budget. Also, the idiosyncratic shock is washed in the aggregate. The parameter $1-\xi$ represents the government intensity in the education sector which we call government bias in education. For example, if $\xi = 1$, the government plays no role in education sector which means that the government bias is zero.[16]

[15] With respect to the aggregation of individuals income, note first that $\mathrm{E}\left[y_t^i\right] = \mathrm{E}\left[\epsilon\varphi_t^i\left(h_t^i\right)^{\alpha}\right] = \epsilon\,\mathrm{E}\left[\left(h_t^i\right)^{\alpha}\right]\mathrm{E}\left[\varphi_t^i\right] = \epsilon\,\mathrm{E}\left[\left(h_t^i\right)^{\alpha}\right]$ because φ_t^i is i.i.d. and $\mathrm{E}\left[\varphi_t^i\right] = 1$. Then,

$$\begin{aligned}\ln \mathrm{E}\left[\left(h_t^i\right)^{\alpha}\right] &= \mathrm{E}\left[\ln\left(h_t^i\right)^{\alpha}\right] + \frac{1}{2}\operatorname{var}\left[\ln\left(h_t^i\right)^{\alpha}\right] = \alpha\,\mathrm{E}\left[\ln\left(h_t^i\right)\right] + \frac{1}{2}\alpha^2\sigma_t^2 \\ &= \alpha\left(\ln \mathrm{E}\left[h_t^i\right] - \frac{1}{2}\sigma_t^2\right) + \frac{1}{2}\alpha^2\sigma_t^2 = \alpha\ln h_t + \frac{1}{2}\sigma_t^2\left(\alpha-1\right)\alpha^2\end{aligned}$$

[16] Basu and Bhattarai (2011) calibrate this government bias in the education sector for a range of countries and find

5.2 Equilibrium under the public funding

Households solve optimization problems in two steps. They first solve for optimal human capital investment, consumption and efforts assuming the public spending is given exogenously. Then, they solve for their preferred tax rate τ_t^i.

Substituting (5.28) into (5.27), we obtain individuals' human capital accumulation function that considers government funding,

$$h_{t+1}^i = \left(l_t^i\right)^{\omega} \left(h_t^i\right)^{1-\theta(1+(1-\xi)\kappa)} \left(g_t\right)^{\theta(1-\xi)} \left(s_t^i\right)^{\xi\theta} \tag{5.30}$$

Thus, in the first step, the ith household optimization problem, from (2.4), (2.5), and (5.30), is given by

$$\max_{l_t^i, s_t^i, \tau} \ln\left(y_t^i (1-\tau) - s_t^i - b\left(l_t^i\right)^{\eta}\right) + \beta \ln \left(l_t^i\right)^{\omega} \left(s_t^i\right)^{\xi\theta} \left(g_t\right)^{(1-\xi)\theta} \tag{5.31}$$

The first order condition gives,

$$s_t^i = y_t^i \frac{\xi\eta\theta\beta(1-\tau)}{\eta+\beta\omega+\xi\theta\beta\eta} \tag{5.32}$$

$$l_t^i = \left(\frac{\beta\omega(1-\tau)}{b(\eta+\beta\omega+\xi\theta\beta\eta)} y_t^i\right)^{\frac{1}{\eta}} \tag{5.33}$$

Simply, eqs. (5.32) and (5.33) are a generalization of (2.9) and (2.10), respectively, when $\xi \neq 1$. The fraction of income that goes to saving decreases with government bias $1-\xi$ and τ. But individuals' effort $\left(\left(l_t^i\right)^{\frac{1}{\eta}}\right)$ as a fraction of y_t^i increases in $1-\xi$ but τ.

Then, in the second step, the optimization problem is computed by substituting (5.29), (5.32) and (5.33) into (5.31),

$$\max_{\tau} \ln\left(\left(1-\tau_t^i\right)\right)^{\beta\left(\frac{\omega}{\eta}+\xi\theta\right)+1} + \ln\left(\tau_t^i\right)^{(1-\xi)\theta\beta} \tag{5.34}$$

The optimization yields,

$$\tau_t^i = \tau = \frac{(1-\xi)\theta\beta}{\frac{\beta\omega}{\eta}+1+\theta\beta} \tag{5.35}$$

Therefore, each individual's preferred tax rate is identical. It is obvious that τ increases if the "government bias" $(1-\xi)$ is large. The preferred tax rate is lower if the adjustment cost is high (i.e., θ

that it is generally higher for rich industrialized countries compared to poor countries.

is small). Also, higher ω and $\frac{1}{\eta}$ lower the preferred tax rate because parents would rather prefer to supply more efforts (as a substitute for capital investment) if ω and $\frac{1}{\eta}$ are large.

From (2.1), (2.2), (5.29), (5.30), (5.32), and (5.33), the optimal human capital accumulation under the public funding which is associated to the ith individual is given by,

$$h_{t+1}^{i} = \phi_2 \left(h_t^i\right)^{\varrho_3} \left(\varphi_t^i\right)^{\varrho_4} \left(h_t\right)^{\varrho_5} e^{\frac{1}{2}\sigma_t^2 \varrho_6} \tag{5.36}$$

where

$$\phi_2 \equiv \left(\frac{\beta}{b\left(\eta+\beta\omega+\xi\theta\beta\eta\right)}\right)^{\frac{\omega}{\eta}+\xi\theta} \left(\frac{\omega}{b}\right)^{\frac{\omega}{\eta}} \left(\xi\eta\theta\right)^{\xi\theta} \left(1-\tau\right)^{\frac{\omega}{\eta}+\xi\theta} \tau^{\theta-\xi\theta} \epsilon^{\theta+\frac{\omega}{\eta}} \tag{5.37}$$

and

$$\begin{aligned}
\varrho_3 &\equiv 1-\theta\left(1-\xi\alpha+\kappa\left(1-\xi\right)\right)+\frac{\omega}{\eta}\alpha \\
&= \alpha\left(\frac{\omega}{\eta}+\xi\theta\right)+1-\theta(1+\kappa\left(1-\xi\right)) &(5.38)\\
\varrho_4 &\equiv \frac{\omega}{\eta}+\xi\theta &(5.39)\\
\varrho_5 &\equiv \theta\alpha\left(1-\xi\right) &(5.40)\\
\varrho_6 &\equiv \left(\alpha-1\right)\alpha\theta\left(1-\xi\right)<0 &(5.41)\\
\varrho_7 &\equiv \varrho_3+\varrho_5 = 1-\theta\left(1-\alpha+\kappa\left(1-\xi\right)\right)+\frac{\omega}{\eta}\alpha &(5.42)
\end{aligned}$$

5.3 Intergenerational Mobility and the Inequality dynamics under public funding of education

As before, the intergenerational mobility and the inequality dynamics are mirror images of each other. The mobility equation is summarized by (5.36). Based on (5.36), the inequality dynamics evolves as follows:

$$\sigma_{t+1}^2 = \left(\varrho_3\right)^2 \sigma_t^2 + \left(\varrho_4\right)^2 \upsilon^2 \tag{5.43}$$

Both eqs. (5.36) and (5.43) are also a generalizations of (3.11) and (4.15), when $\xi \neq 1$. They are similarly stable as long as $0 < \varrho_3 < 1$. Most of the parameters in (5.43) are familiar except the policy parameters, ξ and κ. And, of course, all the results here are similar to the earlier results when $\xi = 1$.

The distributional effects of policy depends on the interplay between adjustment cost (θ) and the fiscal policy parameters. The pro-poor government role in education is summarized by the term $\kappa(1-\xi)$. Absent adjustment cost ($\theta = 1$), the intergenerational mobility coefficient ϱ_3 reduces to $\alpha\left(\frac{\omega}{\eta}+\xi\right)-\kappa(1-\xi)$. A pro-poor government policy (higher $\kappa(1-\xi)$) increases the intergenerational mobility. Adjustment cost weakens this positive effect. This happens because the same adjustment cost parameter (θ) dampens the complementarity between private and public spending on education as seen from (5.27).

It is not surprising that a big pro-poor government speeds up intergenerational mobility when policy is pro-poor ($\kappa > 0$). However, surprisingly a pro-rich public service programme may also help the intergenerational mobility the absolute value of κ has an upper bound. Particularly, according to (5.43), a greater size of such a pro-rich government programme (i.e.,greater $1-\xi$) promotes intergenerational mobility if $|\kappa| < \alpha$. This is summarized in the following proposition.

Proposition 4 *A provision of public funds (higher $1-\xi$) for individual human capital production speeds up (slows down) intergenerational mobility if $\kappa > -\alpha$ ($\kappa < -\alpha$). The size of the adjustment cost θ determines the effectiveness of policy. The greater the θ, the more effective is a policy instrument.*

Proof. *Based on (5.38), note that $\frac{d\varrho_3}{d(1-\xi)} = -\theta(\alpha+\kappa)$ which is negative if and only if $\alpha+\kappa > 0$.* ■

Although this result seems apparently counter-intuitive, the reasons can be understood if one carefully interprets the parameters κ and α. The parameter κ is the extent of the rich-bias in the public service programme. In other words, the greater the absolute value of κ, more pro-rich the public service programme is. On the other hand, poor can also be disadvantaged if α is higher. Note that α basically represents the degree to which households are able to exploit their relative initial advantage and thus can be interpreted as a convergence parameter. The smaller the α, the larger the difference in marginal productivities of human capital (h_t^i) of the poor and the rich, which means a greater degree of convergence. Thus while a small α helps mobility, a large $|\kappa|$ impedes it. As long as the latter is kept low meaning $|\kappa| < \alpha$, a greater pro-rich government programme (a lower ξ) boosts intergenerational mobility. As usual the adjustment cost (θ) slows the process of convergence

With public fund involved, individuals' next period human capital decreases in today's wealth inequality. We see in (5.36) inequality (σ_t^2) affects individuals human capital accumulation (h_{t+1}^i), which is in contrast to (3.11). This could not be a surprise. Since now a public resource is used for individual activities, inequality could be channeled through it to affect individual households. The households' human capital accumulation is partly funded through the provision of the government resource (g_t) which, in turn, depends on the level of aggregate income y_t. Because σ_t^2 has a negative influence over y_t it would also have a negative effect on h_{t+1}^i.

5.4 Other macroeconomic variables under the public funding

Aggregate capital

Aggregating (5.36), we obtain the economy wide capital dynamics:

$$\begin{aligned} \ln h_{t+1} &= \ln \phi_2 \left(h_t\right)^{\varrho_7} + \frac{\upsilon^2}{2}\left(\varrho_4\right)\left(\varrho_4 - 1\right) \\ &\quad + \frac{1}{2}\sigma_t^2 \left(\left(\varrho_3\right)\left(\varrho_3 - 1\right) + \varrho_6\right) \end{aligned} \tag{5.44}$$

Similar to the private funding, aggregate inequality dynamics of the economy is determined by (4.15) and (4.26). The system is stable and monotonically converges as long as $0 < \varrho_7 < 1$. Moreover, because $0 < \varrho_3 < 1$ and $\varrho_6 < 0$, σ_t^2 is negatively correlates to h_{t+1}, in line with Proposition 4.

Steady state distribution and capital

According to (5.43) and (5.44), the steady state wealth distribution and capital under public funding are given by, respectively,

$$\sigma^2 = \frac{\left(\varrho_4\right)^2}{1 - \left(\varrho_3\right)^2}\upsilon^2 \tag{5.45}$$

and

$$\begin{aligned} \ln h &= \ln \phi_2^{\frac{1}{1-\varrho_7}} + \frac{1}{2\left(1 - \varrho_7\right)}\left(\upsilon^2 \varrho_4 \left(\varrho_4 - 1\right) + \sigma^2\left(\varrho_3\left(\varrho_3 - 1\right) + \varrho_6\right)\right) \\ &= \ln \left(\phi_2\right)^{\frac{1}{1-\varrho_7}} + \left(\frac{1}{2}\frac{\varrho_4}{1 - \varrho_7}\upsilon^2\left(\varrho_4\left(\frac{1 - \varrho_3 + \varrho_6}{1 - \left(\varrho_3\right)^2}\right) - 1\right)\right) \end{aligned} \tag{5.46}$$

The level of the steady state capital is lower with higher σ^2 which is similar to the result we find earlier.

6 Conclusion

This paper has developed models that analyze the distributional effect of adjustment cost within an incomplete market and a heterogeneous economy. The source of endogenous inequality is missing credit and insurance markets. When individuals cannot perfectly ensure themselves from future income uncertainty and, the credit market is imperfect, inequality persists. The dynamics of aggregate variables and inequality are jointly determined. Imperfection in capital markets and existence of diminishing returns to private investment imply a suboptimal level of individual investment in the inegalitarian society. The presence of a higher adjustment cost for human capital slows down the intergenerational mobility and results in persistent inequality across generations. The result is robust in alternative environments where individuals' human capital accumulation is partly funded using public resources. Moreover, other productivity parameters such as the output elasticity of human capital, labour supply elasticity of home production and the productivity of child care are important determinant of the intergenerational mobility and inequality dynamics. A public redistributive policy favouring poor helps the intergenerational mobility. A pro-rich redistributive policy can also help this mobility as long the rich bias is kept in a limit. The adjustment cost generally slows the down the convergence process even in an environment where the government is actively following such a redistributive policy.

A Incomplete Depreciation

As a robustness check, we provide here the general solution for the case that capital do not fully depreciate, $\delta \neq 1$. Consider only the basic model with inelastic labor supply and without home production. Thus, from (2.3), (2.4) and (2.5), individual ith optimization problem is:

$$\max_{s_t^i} \ln\left(y_t^i(1-\tau) - s_t^i\right) + \beta \ln h_t^i \left(1 - \delta + \frac{s_t^i}{h_t^i}\right)^{\theta} \tag{A.47}$$

The optimal saving of the ith individual is then given by

$$s_t^i = \frac{\theta\beta}{1+\theta\beta} y_t^i - \frac{1-\delta}{1+\theta\beta} h_t^i \tag{A.48}$$

Substituting this into (2.3), and considering (2.1) and (2.2), we obtain the ith individual optimal human capital accumulation function,

$$\ln h_{t+1}^{i}=\theta\ln\left(\frac{\theta\beta}{1+\theta\beta}\right)+\ln h_{t}^{i}+\theta\ln\left(1-\delta+\epsilon\varphi_{t}^{i}\left(h_{t}^{i}\right)^{\alpha-1}\right) \tag{A.49}$$

In contrast to (3.11) and (5.36), the last term to the right of (A.49) is nonlinear and correlates to the second term from the last. We therefore use its log-linear approximate in deriving the inequality dynamics and aggregate variables, following Campbell (1994).

Taylor's first order approximation of (A.49) gives

$$\widetilde{h}_{t+1}^{i}\approx\nu+\left(1+\left(\alpha-1\right)\theta\lambda\right)\widetilde{h}_{t}^{i}+\theta\lambda\widetilde{\epsilon}_{t}^{i} \tag{A.50}$$

where $\lambda\equiv\frac{\ln\epsilon\varphi+(\alpha-1)\ln h+1}{2-\delta+\ln\epsilon\varphi+(\alpha-1)\ln h}$ and $\nu\equiv\ln\left(\left(\frac{\theta\beta}{1+\theta\beta}\right)\left(1-\delta+\epsilon\varphi\left(h\right)^{\alpha-1}\right)\right)^{\theta}$; $\widetilde{h}_{t+1}^{i}\equiv\ln h_{t+1}^{i}-\ln h$ and $\widetilde{\varphi}_{t}^{i}\equiv\ln\varphi_{t}^{i}-\ln\varphi$.

The distribution dynamics related to this is given by:

$$\sigma_{t+1}^{2}=\left(1-\left(1-\alpha\right)\lambda\theta\right)^{2}\sigma_{t}^{2}+\left(\theta\lambda\right)^{2}\upsilon^{2} \tag{A.51}$$

Aggregating (A.50) gives,

$$\begin{aligned}\ln h_{t+1} &= \nu+\left(1-\theta\lambda\left(\alpha-1\right)\right)\ln h_{t}+\frac{1}{2}\upsilon^{2}\theta\lambda\left(\theta\lambda-1\right)\\ &\quad-\frac{1}{2}\sigma_{t}^{2}\left(1-\theta\lambda\left(\alpha-1\right)\right)\left(1-\alpha\right)\theta\end{aligned} \tag{A.52}$$

Of course, there is not much change in the effects of θ in the economy because now $\delta\neq1$. Inequality is higher in an economy with a higher adjustment cost (lower θ). However, a lower depreciation cost δ relates to a greater inequality dynamics since lower depreciation cost relates to a lower optimal allocation of after tax income for new investment (A.48). Moreover, lower depreciation cost relates to a greater gradual accumulation of capital, which apparently relates to greater inequality.

The steady state capital and distribution are given by,

$$\sigma^{2}=\frac{\left(\theta\lambda\right)^{2}\upsilon^{2}}{1-\left(1-\lambda\theta\left(1-\alpha\right)\right)^{2}} \tag{A.53}$$

and

$$h=\nu^{\frac{1}{(\alpha-1)\lambda}}\exp\left\{-\frac{1}{2}\sigma^{2}\left(1-\theta\lambda\left(\alpha-1\right)\right)+\frac{1}{2}\frac{\upsilon^{2}\left(\theta\lambda-1\right)}{\alpha-1}\right\} \tag{A.54}$$

Note that with complete depreciation, inelastically labor, and without home production, ($\delta = 1$, $\omega = 0$ and $b = 0$), (3.11), (4.15), (4.26), (4.25), and (A.50) are reduced to (A.50), (A.51), (A.52), (A.53) and (A.54), respectively.

B Decomposition of Inequality

Start from

$$\ln h^i_{t+1} - \ln h^j_{t+1} = \varrho_1 \left(\ln \varphi^i_t - \ln \varphi^j_t\right) + \varrho_2 \left(\ln h^i_t - \ln h^j_t\right) \tag{B.55}$$

which means

$$\begin{aligned} \ln h^i_{t+1} - \ln h^j_{t+1} &= \varrho_1 \left(\ln \varphi^i_t - \ln \varphi^j_t\right) + \varrho_2 \begin{pmatrix} \varrho_1 \left(\ln \varphi^i_{t-1} - \ln \varphi^j_{t-1}\right) \\ +\varrho_2 \left(\ln h^i_{t-1} - \ln h^j_{t-1}\right) \end{pmatrix} \\ &= \varrho_1 \left(\ln \varphi^i_t - \ln \varphi^j_t\right) + \varrho_2 \varrho_1 \left(\ln \varphi^i_{t-1} - \ln \varphi^j_{t-1}\right) \\ &\quad + (\varrho_2)^2 \left(\ln h^i_{t-1} - \ln h^j_{t-1}\right) \end{aligned} \tag{B.56}$$

Iterating once more,

$$\begin{aligned} \ln h^i_{t+1} - \ln h^j_{t+1} &= \varrho_1 \left(\ln \varphi^i_t - \ln \varphi^j_t\right) + \varrho_2 \varrho_1 \left(\ln \varphi^i_{t-1} - \ln \varphi^j_{t-1}\right) \\ &\quad + (\varrho_2)^2 \begin{pmatrix} \varrho_1 \left(\ln \varphi^i_{t-2} - \ln \varphi^j_{t-2}\right) \\ +\varrho_2 \left(\ln h^i_{t-2} - \ln h^j_{t-2}\right) \end{pmatrix} \\ &= \varrho_1 \left(\ln \varphi^i_t - \ln \varphi^j_t\right) + \varrho_2 \varrho_1 \left(\ln \varphi^i_{t-1} - \ln \varphi^j_{t-1}\right) \\ &\quad + (\varrho_2)^2 \varrho_1 \left(\ln \varphi^i_{t-2} - \ln \varphi^j_{t-2}\right) + (\varrho_2)^3 \left(\ln h^i_{t-2} - \ln h^j_{t-2}\right) \end{aligned} \tag{B.57}$$

or,

$$\begin{aligned} \ln h^i_{t+1} - \ln h^j_{t+1} &= (\varrho_2)^0 \varrho_1 \left(\ln \varphi^i_t - \ln \varphi^j_t\right) + (\varrho_2)^1 \varrho_1 \left(\ln \varphi^i_{t-1} - \ln \varphi^j_{t-1}\right) \\ &\quad + (\varrho_2)^2 \varrho_1 \left(\ln \varphi^i_{t-2} - \ln \varphi^j_{t-2}\right) + ... + (\varrho_2)^{t-1} \varrho_1 \left(\ln \varphi^i_1 - \ln \varphi^j_1\right) \\ &\quad + (\varrho_2)^t \varrho_1 \left(\ln \varphi^i_0 - \ln \varphi^j_0\right) + (\varrho_2)^{t+1} \left(\ln h^i_0 - \ln h^j_0\right) \end{aligned} \tag{B.58}$$

or,

$$\begin{aligned}\ln h_{t+1}^{i} - \ln h_{t+1}^{j} &= \varrho_1 \sum_{k=0}^{k=t} \varrho_2^k \left(\ln \varphi_{t-k}^{i} - \ln \varphi_{t-k}^{j}\right) + (\varrho_2)^{t+1} \left(\ln h_0^i - \ln h_0^j\right) \\ &= \varrho_1 \sum_{k=0}^{k=t} \varrho_2^k \Delta_{t-k}^{\varphi} + (\varrho_2)^{t+1} \left(\ln h_0^i - \ln h_0^j\right) \end{aligned} \tag{B.59}$$

from which we get (4.18).

C Dynastic Altruism

In this section, we show that our key results continue to hold in a model with dynastic altruism as in Barro (1985). Each generation lives one period and discounts the future generation's utility by β. The ith agent born at date t has the utility:

$$v_t^i = \ln c_t^i + \beta E_t^i v_{t+1} \tag{C.60}$$

It maximizes the above s.t.

$$c_t^i + s_t^i = (1 - \tau^i) \omega_t^i h_t^{i\alpha} \tag{C.61}$$

where

$$\omega_t^i = \epsilon_t \varphi_t^i$$

with ϵ_t as the aggregate shock and φ_t^i as the individual idiosyncratic shock with the same distributional assumption as before.

$$h_{t+1}^i = h_t^i \left(\frac{\left(g_t^i\right)^{1-\xi} \left(s_t^i\right)^{\xi}}{h_t^i} \right)^{\theta} \tag{C.62}$$

and

$$g_t^i = \frac{g_t}{\left(h_t^i\right)^{\kappa}} \tag{C.63}$$

where

$$g_t = y_t \tau = \tau \left(h_t\right)^{\alpha} e^{\frac{1}{2}\sigma_t^2 (\alpha - 1)\alpha} \tag{C.64}$$

The adjustment cost function (C.62) can be rewritten after plugging (C.63) as:

$$
\begin{aligned}
h^i_{t+1} &= h^{i1-\theta}_t s^{i\xi\theta}_t \left\{ \frac{g_t}{\left(h^i_t\right)^\kappa} \right\}^{\theta(1-\xi)} \\
&= h^{i(1-\theta)-\theta\kappa(1-\xi)}_t s^{i\xi\theta}_t g^{\theta(1-\xi)}_t
\end{aligned}
$$

which can be rewritten generically as:

$$
h^i_{t+1} = h^{i\Theta_1}_t s^{i\Theta_2}_t g^{\Theta_3}_t \tag{C.65}
$$

where

$$
\begin{aligned}
\Theta_1 &= (1-\theta) - \theta\kappa(1-\xi) \\
\Theta_2 &= \xi\theta \\
\Theta_3 &= \theta(1-\xi)
\end{aligned}
$$

Thus the ith agent maximizes (C.60) s.t.(C.61 and C.65)

The value function for this problem can be written as:

$$
v(h^i_t, \omega^i_t.g_t) = Max_{h^i_{t+1}} \left[\ln\left\{ (1-\tau^i)\omega^i_t h^{i\alpha}_t - \frac{h^{i(1/\Theta_2)}_{t+1}}{h^{i(\Theta_1/\Theta_2)}_t g^{\Theta_3/\Theta_2}_t} \right\} + \beta E_t v(h^i_{t+1}, \omega^i_{t+1}.g_{t+1}) \right]
$$

Conjecture that the value function is loglinear in state variables as follows:

$$
v(h^i_t, \omega^i_t.g_t) = \pi_0 + \pi_1 \ln h^i_t + \ln \omega^i_t + \ln g_t
$$

which after plugging into the value function

$$
\begin{aligned}
& \pi_0 + \pi_1 \ln h^i_t + \ln \omega^i_t + \ln g_t && \text{(C.66)} \\
= \; & Max_{h^i_{t+1}} \left[\ln\left\{ (1-\tau^i)\omega^i_t h^{i\alpha}_t - \frac{h^{i(1/\Theta_2)}_{t+1}}{h^{i(\Theta_1/\Theta_2)}_t g^{\Theta_3/\Theta_2}_t} \right\} + \beta \left\{ \pi_0 + \pi_1 \ln h^i_{t+1} + \ln \omega^i_{t+1} + \ln g_{t+1} \right\} \right]
\end{aligned}
$$

We will use the method of undetermined coefficients to solve for π_i.

Differentiating with respect to h^i_{t+1} and rearranging terms one gets:

$$h_{t+1}^i = \left[\frac{\Theta_2\pi_1\beta}{1+\beta\Theta_2\pi_1}\right]^{\Theta_2} \{(1-\tau^i)\omega_t^i\}^{\Theta_2} g_t^{\Theta_3} h_t^{i(\alpha\Theta_2+\Theta_1)} \tag{C.67}$$

For the dynamics of human capital, the only coefficient that matters is π_1. Thus if we can solve this coefficient we find the decision rule. Plugging (C.67) into (C.66) and comparing the left hand side and right hand side coefficients of the value function we can uniquely solve π_1 as follows:

$$\pi_1 = \frac{\alpha}{1-\beta(\Theta_1+\alpha\Theta_2)}$$

which after plugging into (C.67) we get:

$$h_{t+1}^i = \Omega\omega_t^{i\Theta_2} g_t^{\Theta_3} h_t^{i(\alpha\Theta_2+\Theta_1)} \tag{C.68}$$

where

$$\Omega = \left[\frac{\alpha\beta\Theta_2}{1-\beta\Theta_1}\right]$$

Distributional dynamics

Based on (C.68) the distributional dynamics are given by the following equation:

$$\ln h_{t+1}^i = \text{constant} + \{1-\theta(1-\xi)(1+\kappa)\}\ln h_t^i + \xi\theta(\ln\omega_t^i + \ln g_t) \tag{C.69}$$

which leads to:

$$\sigma_{t+1}^2 = \{1-\theta(1-\xi)(1+\kappa)\}^2\sigma_t^2 + \frac{\Theta_2^2\nu^2}{2}$$

A higher adjustment cost again slows down the intergenerational mobility as the coefficient of h_t^i in (C.69) is decreasing in θ. A redistributive policy of spreading public service to the poor promotes intergenerational mobility because the coefficient of h_t^i is decreasing in $\kappa(1-\xi)$ given that $\kappa > 0$.[17]

[17] For stability of the steady state, one requires that $\kappa < \alpha\xi/1-\xi)$. In other words, there must be an upper bound for the degree of redistribution.

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