The Maturity Structure of Debt, Monetary Policy and Expectations Stabilization*

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Abstract

This paper identifies a channel by which changes in the size and composition of government debt might generate macroeconomic instability in a standard New Keynesian model. The mechanism depends on failures of Ricardian equivalence because of learning dynamics. Under rational expectations, the model has the prediction that Ricardian equivalence holds, and the scale and composition of public debt held by households is irrelevant to the determination of inflation and output. Under learning, holdings of the public debt are perceived as net wealth, with the resulting expenditure effects shown to be destabilizing, depending on both the scale and composition of the public debt. Very short and long average debt maturities are conducive to stability, while short-to-medium average maturities tend to generate instability in the sense that much more aggressive monetary policy is required to prevent divergent learning dynamics. More heavily indebted economies are more sensitive to adjustments in maturity structure. This suggests there might be considerations, aside from the presumed stimulus from large-scale asset purchases via lower longer-term interest rates, that are relevant to evaluating recent proposals for further quantitative easing in the United States.

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1 Introduction

The US financial crisis of 2007-2009 has engendered extraordinary policy responses in many economies around the world. In the US, the scale and scope of fiscal stimulus is unprecedented in postwar history. And considerable monetary accommodation has been afforded by the reduction of the federal funds rate to a target range of 0 to 1/4 percent, together with a substantial expansion in the scale and scope of credit policies aimed at mitigating financial market dislocation and providing much needed liquidity. More recently, the Federal reserve has announced a further program of quantitative easing to the tune of $600 billion, intended to lower longer-term interest rates relevant to investment and spending decisions of households and firms — achieved by a shortening of the maturity structure through large-scale asset purchases.

An important question then is how does the maturity structure of debt affect interest rates and economic decisions? In standard representative agent theory, if the conditions of Ricardian equivalence hold, then economic decisions are invariant to the maturity structure of debt. A shortening of the maturity structure for a given path of government purchases would lead to a shift in the timing of taxation — but not its present discounted value. More generally, for quantitative easing through adjustments in the scale and composition of the public debt to have effect, it must alter state-contingent equilibrium outcomes for consumption. But standard models have the property that changes in the maturity structure of debt held by households do not affect state-contingent consumption, which is pinned down by endogenously determined output — itself independent of debt-management policy. This is the irrelevance proposition of Eggertsson and Woodford (2003).

This paper proposes a model which does not satisfy these irrelevance properties because of violations of Ricardian equivalence. In a standard New Keynesian model of the kind frequently used for monetary policy evaluation, we suppose agents have incomplete knowledge about the structure of the economy. Households and firms are optimizing, have a completely specified belief system, but do not know the equilibrium mapping between observed state variables and market clearing prices. By extrapolating from historical patterns in observed data they approximate this mapping to forecast exogenous variables relevant to their decision problems, such as prices and policy variables. Because agents must learn from historical data, beliefs need not be consistent with the objective probabilities implied by the economic model. The analysis is centrally concerned with conditions under which agents can learn the underlying rational expectations equilibrium of the model. Such convergence is referred to as "expectations stabilization" or "stable expectations". A situation of unstable expectations is referred to as expectations-driven instability.

Under learning dynamics, even though the model is one that would satisfy Ricardian equivalence under rational expectations, there will be departures from this benchmark because agents make forecasting errors about future tax obligations and future real interest rates. Holdings of the public debt are perceived as net wealth — compare Barro (1974). Because changes in the maturity structure imply changes in the timing of taxation, and the nature of this taxation is imperfectly understood by households, the perceived wealth embodied in holdings of the public debt necessarily change over time. Management of the public debt can have relevance for spending and pricing decisions of households and firms.

An advantage of the adopted approach is that it cleanly identifies one specific channel through which debt-management policy might be relevant to macroeconomic outcomes. The model has the property that absent imperfect information the only relevant instrument of policy is the one-period interest rate.[1] Changes in the maturity structure of debt are irrelevant to the conduct of interest-rate policy. The model permits analysis of the questions: are there potentially additional consequences, over and above the presumed stimulus from lower longer-term interest rates, that might be relevant to evaluating the merits of quantitative-easing policy? Furthermore, does imperfect knowledge about the conduct of fiscal policy limit the effectiveness of traditional interest-rate policy?

A central finding is that short-to-medium maturity debt structures are conducive to macroeconomic instability. Economies with very short (less than one year) and longer-maturity debt structures are considerably more stable. Attempts to shorten the maturity structure through large-scale asset purchases may lead to expectations-driven instability, requiring much more aggressive monetary policy for expectations stabilization. The scale of average indebtedness also matters: more heavily indebted economies are more sensitive to adjustments in the maturity structure. This is because the average level of debt scales expenditure effects from holdings of the public debt, and these effects are destabilizing.[2] And while this paper does not consider the specific circumstances of monetary policies that are constrained by the zero lower bound on nominal interest rates, such unusual periods are certainly times when it might be thought that expectations are particularly susceptible to drift — for reasons elucidated by Sargent and Wallace (1975). The paper shows that drifting expectations might have inadvertent consequences.

[1] Assuming that fiscal policy is passive in the language of Leeper (1991) so that equilibrium is Ricardian.

[2] See Eusepi and Preston (2010b) for a detailed discussion.

Because instability arises solely because of departures from Ricardian equivalence, a specific policy recommendation emerges: it is important that households correctly understand the various activities of the fiscal authority, and specifically, that policy is conducted in such a way to ensure the intertemporal solvency of the government accounts. If the fiscal accounts are understood to be intertemporally solvent, Ricardian equivalence obtains. In this case, the model under both rational expectations and learning are isomorphic, in so far as both have the same requirements on monetary policy to ensure the stability of expectations.[3] Both spending and pricing decisions are independent of the scale and structure of debt. This is pertinent given recent events which have witnessed not only considerable uncertainty about the scale, scope and duration of fiscal stimulus — but also about specific details of the future funding of these policies through the tax system. Clearly communicating that the intended future conduct of tax policy is consistent with intertemporal solvency of the fiscal accounts is conducive to economic stability. Viewed through the lens of this model, the arguments of Leeper (2009) for developments in communication and transparency in fiscal policy, that mirror those seen in the theory and practice of monetary policy, appear to have considerable merit.

More specific results of the paper are as follows. Keynesian expenditure effects operating through the public debt are shown to constrain the class of simple monetary policy rules that are consistent with expectations stabilization. In general, satisfaction of the Taylor principle fails to protect the economy against expectations-driven fluctuations when monetary policy is implemented according to rules of the kind proposed by Taylor (1993, 1999). Monetary policy must respond more aggressively to inflation to ensure stability of expectations, even though the model has the property that the Taylor principle ensures determinacy of rational expectations equilibrium. Combining these insights with the results of Preston (2008) suggests more sophisticated procedures for interest-rate policy, such as the targeting-rule approach of Giannoni and Woodford (2002), Giannoni and Woodford (2010) and Svensson and Woodford (2005), might be preferable.

The magnitude of departure from the Taylor principle is shown to depend on various features of household preferences and firm technology. Of particular import is households preparedness to substitute consumption and leisure intertemporally. When the elasticity of intertemporal substitution of consumption is high, the destabilizing wealth effects arising from

[3] Under rational expectations, stability of expectations refers to a unique bounded equilibrium — indeterminacy permitting arbitrary sunspot equilibra clearly being undesirable from the perspective of stabilization policy.

holdings of the public debt tend to be small. The same is true when the Frisch elasticity of labor supply is high. High substitution economies, by their very nature, imply limited importance of wealth effects, though the channels of influence are distinct for the two dimensions of preferences. High intertemporal elasticities of consumption substitution are shown to imply an high interest-rate elasticity of consumption demand. Current and future interest-rate policy have significant restraining effects on demand. High Frisch elasticities minimize the impact of wealth effects on demand directly through the endogenous adjustment of labor supply. As a result of these general equilibrium effects, variations in wealth are less important for such households.

Economies with higher nominal rigidities in price-setting tend to be more unstable. There are two conflicting effects. Nominal price rigidities tend to make inflation more predictable — a stabilizing influence — but also restrict an important equilibrium mechanism: changes in goods prices in part determine the real value of holdings of the public debt. Restricting price movements permits these wealth effects to persist. This latter mechanism tends to dominate leading to instability. Regardless, for given assumptions about preferences and technology, more indebted economies tend to be less stable.

A final result concerns the assumed belief structure of agents under learning. An important equilibrium restriction is a no-arbitrage condition which restricts prices of the two assets in our model: one-period debt and long-maturity debt. It is shown, out of rational expectations equilibrium, there are two ways to impose no-arbitrage on agents' forecasts which imply different state-contingent evolutions of the economy. One approach employs all first-order conditions for household optimality and is referred to as "anchored financial expectations". The other approach relaxes one condition for optimality and is referred to as "unanchored financial expectations". The results described above are for the case of anchored financial expectations. We show that in general these effects are amplified when financial expectations are unanchored, and are particularly severe at long average debt maturities. Substantially more aggressive monetary policy is required for expectations stabilization.

Unanchored financial expectations have properties similar in spirit to irrational bubbles. Even though there are no profit opportunities to exploit from arbitrage, the prices of multiple-maturity bond portfolios become divorced from fundamentals — which are shown under anchored financial expectations to be a specific expected present discounted valuation of one-period interest rates. To the extent that quantitative easing programs lead to speculation about asset prices over the term structure, this might present a second consideration in the design of debt management policies.

This paper is most closely related to two recent analyses of monetary policy under learning dynamics. Eusepi and Preston (2010b) propose a model to study the interactions of fiscal and monetary policy. It forms the basis of the present analysis with three important differences. That paper only considers one-period debt; does not solve for fully optimal decisions rules because households are assumed to forecast future period income directly without taking into account the endogeneity from labor supply; and assumed the central bank had imperfect information when determining interest-rate policy. These features are all demonstrated to be important, but both papers have in common the central mechanism that debt policy matters because of departures from Ricardian equivalence. Sinha (2010) applies the framework of Eusepi and Preston (2010b) to think about issues in asset pricing. In particular, it is demonstrated that learning dynamics can resolve some extant puzzles in pricing of the yield curve, and specifically the finding of Campbell and Shiller (1991) of rejections of the expectations hypothesis.

2 A Simple Model

The following section details an extension of the model proposed by Eusepi and Preston (2010b) to include multiple-maturity debt.[4] The model is similar in spirit to Clarida, Gali, and Gertler (1999) and Woodford (2003) used in many recent studies of monetary policy. The major difference is the incorporation of near-rational beliefs delivering an anticipated utility model as described by Kreps (1998) and Sargent (1999). The analysis follows Marcet and Sargent (1989a) and Preston (2005b), solving for optimal decisions conditional on current beliefs.

2.1 Monetary and Fiscal Authorities

Monetary Policy. The central bank is assumed to implement monetary policy according to the family of interest-rate rules of the form

$$\frac{1+i_t}{1+\bar{\imath}} = \left(\frac{1+i_{t-1}}{1+\bar{\imath}}\right)^{\rho_i} \left(\frac{P_t}{P_{t-1}}\right)^{\phi_\pi} \left(\frac{Y_t}{\bar{Y}}\right)^{\phi_y} \varepsilon_{i,t} \tag{1}$$

where i_t is the period nominal interest rate; P_t a price index of the available goods in the economy; Y_t aggregate output; and $\varepsilon_{i,t}$ is a monetary policy shock. For any variable k_t denote the steady-state value as $\bar{k}$. The policy parameters satisfy ϕ_π, $\phi_y \geq 0$ and $0 \leq \rho_i \leq 1$.

[4]The model was first developed in Eusepi and Preston (2007).

Interest-rate policy exhibits inertia and responds to deviations of inflation and output from steady-state levels. The analysis eschews the study of optimal policy to give emphasis to the interaction of monetary policy with various dimensions of fiscal policy.

Fiscal Policy. The fiscal authority finances government purchases by issuing two kinds of public debt and levying lump-sum taxes. Government purchases, G_t, are exogenously determined and satisfy

$$\ln\left(G_t\right) = \left(1-\rho_G\right)\ln\left(\bar{G}\right) + \rho_G \ln\left(G_{t-1}\right) + \varepsilon_{G,t} \tag{2}$$

where $0 < \rho_G < 1$ and $\varepsilon_{G,t}$ is white noise. There are two types of government debt: one-period government debt, B_t^s, in zero net supply with price P_t^s; and a more general portfolio of government debt, B_t^m, in non-zero net supply with price P_t^m. The former debt instrument satisfies $P_t^s = (1+i_t)^{-1}$. Following Woodford (2001) the latter debt instrument has payment structure $\rho^{T-(t+1)}$ for $T > t$ and $0 < \rho < 1$. The value of such an instrument issued in period t in any future period $t+j$ is $P_{t+j}^{m-j} = \rho^j P_{t+j}^m$. The asset can be interpreted as a portfolio of infinitely many bonds, with weights along the maturity structure given by $\rho^{T-(t+1)}$. Varying the parameter ρ varies the average maturity of debt.[5] Imposing the restriction that one-period debt is in zero net supply, the flow budget constraint of the government is given by

$$P_t^m B_t^m = B_{t-1}^m \left(1 + \rho P_t^m\right) + G_t P_t - T_t. \tag{3}$$

Defining outstanding government liabilities in period t as $L_t = B_{t-1}^m \left(1 + \rho P_t^m\right)$ permits the flow budget constraint to be written as

$$\tilde{L}_{t+1} = \left(\frac{1+\rho P_{t+1}^m}{P_t^m}\right)\left(\tilde{L}_t \frac{P_{t-1}}{P_t} - S_t\right) \tag{4}$$

defining also the structural surplus as

$$S_t = T_t/P_t - G_t \tag{5}$$

and $\tilde{L}_t = L_t/P_{t-1}$ a measure of real government liabilities in period t.

Tax policy is determined by a family of rules for the structural surplus of the form

$$S_t = \bar{S}\left(\frac{\tilde{L}_t}{\bar{L}}\right)^{\tau_l} \varepsilon_{\tau,t} \tag{6}$$

where the policy parameter satisfies $\tau_L \geq 0$ and $\varepsilon_{\tau,t}$ is white noise. Such rules are consistent with empirical work by Davig and Leeper (2006).

[5] An elegant feature of this structure is that it permits discussion of debt maturity with the addition of single state variable.

2.2 Microfoundations

Households: The economy is populated by a continuum of households which seeks to maximize future expected discounted utility

$$\hat{E}_t^i \sum_{T=t}^{\infty} \xi_T \beta^{T-t} \left[\frac{C_T(i)^{1-\sigma}}{1-\sigma} - \phi \frac{H_T(i)^{1+\gamma}}{1+\gamma} + \nu(G_t) \right] \tag{7}$$

where utility depends on a consumption index, $C_T(i)$; the amount of labor supplied to the production of goods, $H_T(i)$; the level of government purchases; and a preference shock ξ_T which satisfies

$$\ln(\xi_t) = (1-\rho_\xi)\ln(\bar{\xi}) + \rho_G \ln(\xi_{t-1}) + \varepsilon_{\xi,t} \tag{8}$$

where $0 < \rho_\xi < 1$ and $\varepsilon_{\xi,t}$ is white noise. The consumption index, $C_t(i)$, is the Dixit-Stiglitz constant-elasticity-of-substitution aggregator of the economy's available goods and has associated price index, P_t, written, respectively, as

$$C_t(i) \equiv \left[\int_0^1 c_t^i(j)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} \quad \text{and} \quad P_t \equiv \left[\int_0^1 p_t(j)^{1-\theta} dj \right]^{\frac{1}{1-\theta}} \tag{9}$$

where $\theta > 1$ is the elasticity of substitution between any two goods and $c_t^i(j)$ and $p_t(j)$ denote household i's consumption and the price of good j. The discount factor is assumed to satisfy $0 < \beta < 1$. The remaining preference parameters satisfy $\sigma,\ \gamma,\ \phi > 0$ and the function $v(\cdot)$ has curvature properties $v_G > 0$ and $v_{GG} < 0$.

$\hat{E}_t^i$ denotes the beliefs at time t held by each household i, which satisfy standard probability laws. Section 3 describes the precise form of these beliefs and the information set available to agents when forming expectations. Households and firms observe only their own objectives, constraints and realizations of aggregate variables that are exogenous to their decision problems and beyond their control. They have no knowledge of the beliefs, constraints and objectives of other agents in the economy: in consequence agents are heterogeneous in their information sets in the sense that even though their decision problems are identical, they do not know this to be true.

Asset markets are assumed to be incomplete with households having access only to the aforementioned debt instruments for insurance purposes. The household's flow budget constraint is

$$P_t^s B_t^s(i) + P_t^m B_t^m(i) \leq (1 + \rho P_t^m) B_{t-1}^i(i) + B_{t-1}^s(i) + W_t H_t(i) + P_t \Gamma_t - T_t - P_t C_t(i) \tag{10}$$

where $B_t^s(i)$ and $B_t^m(i)$ are household i's holdings of each of the debt instruments; W_t the nominal wage; and Γ_t dividends from holding shares in an equal part of each firm. Initial bond holdings $B_{-1}^m(i)$ and $B_{-1}^s(i)$ are given and identical across agents. Defining household wealth in period t as

$$A_t(i) = (1 + \rho P_t^m) B_{t-1}^m(i) + B_{t-1}^s(i)$$

the No-Ponzi constraint can be written

$$\lim_{T\to\infty} \hat{E}_t^i R_{t,T} A_T(i) / P_T \geq 0$$

where $R_{t,T} = \prod_{s=ts}^{T-1} \left(\frac{1+\rho P_{s+1}^m}{P_t^m} \frac{P_s}{P_{s+1}} \right)$ for $T \geq 1$ and $R_{t,t} = 1$.[6]

The first-order conditions for consumption, holdings of each bond and labor supply imply the following three restrictions

$$\frac{1}{1+i_t} = \hat{E}_t^i \left[\beta \frac{\xi_{t+1}}{\xi_t} \frac{\left(C_{t+1}^i\right)^{-\sigma}}{\left(C_t^i\right)^{-\sigma}} \frac{P_t}{P_{t+1}} \right] \tag{11}$$

$$P_t^m = \hat{E}_t^i \left[\beta \frac{\xi_{t+1}}{\xi_t} \frac{\left(C_{t+1}^i\right)^{-\sigma}}{\left(C_t^i\right)^{-\sigma}} \frac{P_t}{P_{t+1}} \left(1 + \rho P_{t+1}^m\right) \right] \tag{12}$$

and

$$\phi H_t(i)^\gamma = C_t(i)^{-\sigma} \frac{W_t}{P_t} \tag{13}$$

must hold in every period t. Optimality also requires that (10) holds with equality along with satisfaction of the transversality condition

$$\lim_{T\to\infty} \hat{E}_t^i R_{t,T} A_T(i) / P_T = 0. \tag{14}$$

Firms. There is a continuum of monopolistically competitive firms. Each differentiated consumption good is produced according to the linear production function

$$Y_t(j) = Z_t H_t(j) \tag{15}$$

where Z_t denotes an aggregate technology shock satisfying

$$\ln(Z_t) = (1 - \rho_Z) \ln\left(\bar{Z}\right) + \rho_Z \ln(Z_t) + \varepsilon_{Z,t}$$

[6] In general, the No-Ponzi condition does not ensure satisfaction of the intertemporal budget constraint under incomplete markets. Given the assumption of identical preferences and beliefs and aggregate shocks, a symmetric equilibrium will have the property that all households have non-negative wealth. A natural debt limit of the kind introduced by Aiyagari (1994) would never bind.

where $0 < \rho_z < 1$ and $\varepsilon_{Z,t}$ an i.i.d. disturbance. Each firm faces a demand curve $Y_t(j) = (P_t(j)/P_t)^{-\theta} Y_t$, where Y_t denotes aggregate output, and solves a Rotemberg-style price-setting problem. A price $p_t(j)$ is chosen to maximize the expected discounted value of profits

$$\hat{E}_t^j \sum_{T=t}^{\infty} Q_{t,T} \Pi_T(j)$$

where

$$\Pi_T(j) = p_t(j)^{1-\theta} P_T^\theta Y_T - p^{-\theta} P_T^\theta Y_T W_T / A_T - \chi \left(p_T(j)/p_{T-1}(j) - 1\right)^2 \tag{16}$$

denotes period T profits and $\chi > 0$ scales the quadratic cost of price adjustment. Given the incomplete markets assumption it is assumed that firms value future profits according to the marginal rate of substitution evaluated at aggregate income $Q_{t,T} = \beta^{T-t} P_t Y_T / (P_T Y_t)$ for $T \geq t$.[7]

The first-order condition for firm optimality is

$$\begin{aligned} \chi \left(\frac{p_t(j)}{p_{t-1}(j)} - 1 \right) \frac{P_t}{p_{t-1}(j)} &= \hat{E}_t^j \left[Q_{t,t+1} \chi \left(\frac{p_{t+1}(j)}{p_t(j)} - 1 \right) \frac{p_{t+1}(j)}{p_t(j)} \frac{P_t}{p_t(j)} \right] \\ &\quad + \theta \left(\frac{p_t(j)}{P_t} \right)^{-\theta} Y_t \left[\frac{w_t}{A_t} \left(\frac{p_t(j)}{P_t} \right)^{-1} - \frac{\theta - 1}{\theta} \right] \end{aligned} \tag{17}$$

for each firm $j \in [0,1]$ where $w_t = W_t / P_t$ is the real wage. This completes the description of the model.

2.3 Market clearing and Equilibrium

The analysis considers a symmetric equilibrium in which all households and firms are identical. Given that households have identical initial asset holdings and preferences and face common constraints, they make identical state-contingent decisions. Firms face a common profit maximization problem and set a common price. Equilibrium requires all goods and asset markets to clear. The former requires the aggregate restriction

$$\int C_t(i)\, di + G_t = Y_t. \tag{18}$$

The latter requires

$$\int B_t^s(i)\, di = 0 \text{ and } \int B_t^m(i)\, di = B_t^m \tag{19}$$

with $B_{-1}^s(i) = 0$ and $B_{-1}^m(i) = B_{-1}^m(j) > 0$ for all households $i, j \in [0,1]$. Equilibrium is then a sequence of prices $\{P_t, P_t^m, i_t, W_t\}$ and allocations $\left\{C_t, Y_t, H_t, B_t^m, B_t^s, T_t, \Gamma_t, S_t, \tilde{L}_t, A_t\right\}$ satisfying (1), (3), (4), (5), (6), (10), (11), (12), (13), (14), (15), (17), (18) and (19).

[7] The precise details of this assumption are not important to the ensuing analysis so long as in the log-linear approximation future profits are discounted at the rate β^{T-t}.

2.4 Log-linear Approximation: Implications

Subsequent analysis employs a log-linear approximation in the neighborhood of a non-stochastic steady state. To assist interpretation of model properties under learning some implications of the log-linear approximation are discussed in detail.

2.4.1 Asset Markets and No-Arbitrage

A log-linear approximation to (11) and (12) imply

$$\hat{C}_t(i) = \hat{E}_t^i\left[\hat{C}_{t+1}(i) - \sigma^{-1}\left(\hat{\imath}_t - \hat{\pi}_t + \hat{\xi}_{t+1} - \hat{\xi}_t\right)\right]$$

$$\hat{C}_t(i) = \hat{E}_t^i\left[\hat{C}_{t+1}(i) + \sigma^{-1}\left(\hat{P}_t^m - \rho\beta\hat{P}_{t+1}^m + \hat{\pi}_{t+1} - \hat{\xi}_{t+1} + \hat{\xi}_t\right)\right]$$

for each household $i \in [0,1]$, where $\hat{k}_t = \ln\left(k_t/\bar{k}\right)$ is the log deviation from steady state for any variable k_t with the exceptions

$$\hat{\imath}_t = \ln\left(\frac{1+i_t}{1+\bar{\imath}}\right) \text{ and } \hat{\pi}_t = \ln\left(\frac{P_t}{P_{t-1}}\right).$$

Combining these relations gives the no-arbitrage condition

$$\hat{\imath}_t = -\hat{E}_t^i\left(\hat{P}_t^m - \rho\beta\hat{P}_{t+1}^m\right) \tag{20}$$

which represents an equilibrium restriction on the expected movements of asset prices. Household optimality requires this restriction to be satisfied in all periods of their decision horizon. When describing beliefs under learning dynamics, it is important that this restriction be satisfied by each agent's forecasting model. Absent such an assumption, households will, for arbitrary beliefs about the future evolution of asset prices, forecast arbitrage opportunities, leading to substantial shifts in portfolio and, therefore, equilibrium prices and quantities—even though in any given period equilibrium ensures the absence of arbitrage. This might question the appropriateness of a first-order approximation.

Solving the no-arbitrage restriction forward and using transversality determines the price of the bond portfolio as

$$\hat{P}_t^m = -\hat{E}_t\sum_{T=t}^{\infty}(\rho\beta)^{T-t}\,\hat{\imath}_T. \tag{21}$$

The multiple-maturity debt portfolio is priced as the expected present discounted value of all future one-period interest rates, where the discount factor is given by $\rho\beta$. This expression makes evident that the average maturity of the portfolio is given by $(1-\beta\rho)^{-1}$. A central focus of the analysis will be the consequences of variations in average maturity for expectations stabilization. For completeness, one-period debt is priced as

$$\hat{P}_t^s = -\hat{\imath}_t. \tag{22}$$

Analyzing the relative movements of $\hat{P}_t^s$ and $\hat{P}_t^m$ provides insights on the dynamics of the yield curve.[8]

2.4.2 Households

Optimal Labor Supply. To a log-linear approximation, aggregating individual labor supply (13) over the continuum gives

$$\gamma \hat{H}_t = -\sigma \hat{C}_t + \hat{w}_t \tag{23}$$

where

$$\int_0^1 \hat{H}_t(i)\, di = \hat{H}_t \text{ and } \int_0^1 \hat{C}_t(i)\, di = \hat{C}_t.$$

This is a standard labor supply equation determining aggregate hours as a function of the level of real wages and aggregate consumption. The parameter $\gamma > 0$ has the interpretation of the inverse Frisch elasticity of labor supply.

Optimal Consumption. The optimal decision rule for household consumption is a joint implication of the optimality conditions for consumption, labor supply, the flow budget constraint and transversality. Consumption is allocated according to

$$\begin{aligned} \hat{C}_t(i) &= \bar{s}_C^{-1}\delta\left(\hat{b}_{t-1}(i) - \hat{\pi}_t + \rho\beta\hat{P}_t^m\right) \\ &\quad + \hat{E}_t^i \sum_{T=t}^{\infty} \beta^{T-t}\left[\bar{s}_C^{-1}(1-\beta)x_T - \beta\left(\sigma^{-1} - \bar{s}_C^{-1}\delta\right)(\hat{\imath}_T - \hat{\pi}_{T+1}) + \sigma^{-1}\beta\left(\hat{\xi}_T - \hat{\xi}_{T+1}\right)\right] \end{aligned} \tag{24}$$

where

$$x_T = \left(\frac{\theta-1}{\theta}\right)\left(1+\gamma^{-1}\right)\hat{w}_T + \theta^{-1}\hat{\Gamma}_T - s_\tau \hat{\tau}_T$$

denotes period after-tax income of the household, which depends upon the real wage and dividends, with the latter satisfying to a first order

$$\hat{\Gamma}_t = \hat{Y}_t - (\theta-1)\left(\hat{w}_t - \hat{Z}_t\right), \tag{25}$$

[8] While the analysis of this paper does not pursue such properties, Sinha (2010) shows, in a related model, that learning dynamics can explain apparent rejections for the expectations hypothesis identified by Campbell and Shiller (1991).

and where

$$\delta = \bar{S}/\bar{Y} = \beta^{-1}(1-\beta)\bar{b}/\bar{Y};\ \bar{s}_C = \sigma\gamma^{-1}(\theta-1)\theta^{-1} + \bar{C}/\bar{Y};\quad s_\tau = \bar{\tau}/\bar{Y};\ \tau_t = T_t/P_t;$$
$$\hat{\tau} = \ln(\tau_t/\bar{\tau});\ b_t(i) = B_t^m(i)/P_t;\ \bar{b}(i) = \bar{b} = \bar{B}^m/\bar{P};\ \hat{b}_t(i) = \ln\left(b_t(i)/\bar{b}\right)$$

are the steady-state structural surplus-to-income ratio which in turn is proportional to the steady-state debt-to-income ratio; a parameter that is a composite of preference parameters and the steady-state consumption-to-income ratio; the steady-state tax-to-income ratio; the definition of real taxes; the log-deviation from steady-state tax; the real level of taxes; the quantity of bonds held by household i; its associated steady-state value; and its log-deviation from steady state.

Optimal consumption decisions depend on current wealth — determined by the quantity of bonds held and their valuation — and on the expected future path of after-tax income, the real interest rate, and preference shocks. The optimal allocation rule is analogous to permanent income theory, with differences emerging from allowing variations in the real interest rate, which can occur due to variations in the nominal interest rate or inflation, and preference shocks which affect the desired timing of consumption.[9]

Two fiscal policy parameters affect consumption. The steady-state structural surplus-to-income ratio, δ, affects consumption decisions in three ways: i) it determines after-tax income after applying the definition of the structural surplus described below — see relation (34); ii) it reduces the elasticity of consumption demand with respect to real interest rates; and iii) it indexes wealth effects on consumption spending that result from variations in the real value of government debt holdings. Because the steady-state structural surplus-to-output ratio is always pre-multiplied by the parameter $\bar{s}_C^{-1}$ the overall scale of each of these effects also depends upon household preferences. Finally, the shifting value of government debt depends on the maturity structure indexed by ρ.

To interpret these effects further it is useful to consider aggregate consumption demand.

[9] One important distinction between this analysis and that developed by Eusepi and Preston (2010b) is the treatment of labor supply. There households directly forecast period income defined as

$$\hat{w}_t\hat{H}_t + \hat{\Gamma}_t,$$

the sum of the wage bill and dividend income. This paper accounts for the endogeneity of the wage bill by substituting out for labor supply decisions to deliver a consumption decision rule that depends only on variables that are truly exogenous to the household's decision problem. A consequence is that consumption decisions depend upon the Frisch elasticity of labor supply — with non-trivial consequence.

Aggregating over the continuum and rearranging provides

$$\begin{aligned} \hat{C}_t &= \bar{s}_C^{-1}\delta\left(\hat{b}_{t-1} - \hat{\pi}_t + \rho\beta\hat{P}_t^m - \hat{E}_t\sum_{T=t}^{\infty}\beta^{T-t}\left[(1-\beta)\,\delta^{-1}s_\tau\hat{\tau}_T - \beta\left(\hat{\imath}_T - \hat{\pi}_{T+1}\right)\right]\right) \quad (26) \\ &+ \hat{E}_t\sum_{T=t}^{\infty}\beta^{T-t}\left[\bar{s}_C^{-1}\left(1-\beta\right)\left(x_T + s_\tau\hat{\tau}_T\right) - \beta\sigma^{-1}\left(\hat{\imath}_T - \hat{\pi}_{T+1}\right) + \sigma^{-1}\beta\left(\hat{\xi}_T - \hat{\xi}_{T+1}\right)\right] \end{aligned}$$

where

$$\int_0^1 \hat{b}_t\left(i\right)di = \hat{b}_t \text{ and} \int_0^1 \hat{E}_t^i di = \hat{E}_t$$

give the total quantity of bonds outstanding and average expectations. Assuming that agents know the equilibrium relation between taxes, government purchases and the structural surplus then provides:[10]

$$\begin{aligned} \hat{C}_t &= \bar{s}_C^{-1}\delta\left(\hat{b}_{t-1} - \hat{\pi}_t + \rho\beta\hat{P}_t^m - \hat{E}_t\sum_{T=t}^{\infty}\beta^{T-t}\left[(1-\beta)\,\hat{s}_T - \beta\left(\hat{\imath}_T - \hat{\pi}_{T+1}\right)\right]\right) \\ &+ \hat{E}_t\sum_{T=t}^{\infty}\beta^{T-t}\left[\bar{s}_C^{-1}\left(1-\beta\right)\left(\tilde{x}_T - s_G\hat{G}_T\right) - \beta\sigma^{-1}\left(\hat{\imath}_T - \hat{\pi}_{T+1}\right) + \sigma^{-1}\beta\left(\hat{\xi}_T - \hat{\xi}_{T+1}\right)\right] \end{aligned}$$

where $\tilde{x}_t = x_t + s_\tau\hat{\tau}_t$ gives the sum of wage and dividend income and $s_G = \bar{G}/\bar{Y}$ the steady-state fraction of government purchases in output. The second line gives the usual terms that arise from permanent income theory. The term pre-multiplied by $\bar{s}_C^{-1}\delta$ in the first line is the intertemporal budget constraint of the government.[11] In a rational expectations analysis of the model, this is an equilibrium restriction known to be equal to zero. Consumption demand is independent of the timing of taxation and the precise details of debt management policy. Ricardian equivalence holds.

Agents might face uncertainty about the intertemporal solvency of the fiscal accounts.[12] And under arbitrary subjective expectations, households may incorrectly forecast future tax obligations and real interest rates, leading to holdings of the public debt being perceived as net wealth: Ricardian equivalence need not hold out of rational expectations equilibrium. The failure of Ricardian equivalence leads to wealth effects on consumption demand, and the magnitude of these effects is indexed by the structural surplus-to-output ratio, or equivalently

[10] Households do not have this information in the model under the learning — it is one of the many rational expectations equilibrium restrictions that agents are attempting to learn.

[11] To see this, take a log-linear approximation to the flow budget constraint of the government (4), and solve the resulting equation forward to yield the desired expression.

[12] The tax rule is such that each household faces the same tax profile. However, agents are not aware of that: in forecasting future tax obligations they consider the possibility that their individual tax profile might have changed.

the debt-to-output ratio as these steady-state quantities are proportional. On average, the more indebted an economy the larger are the effects on demand. Eusepi and Preston (2010b) demonstrate these properties to be important in the design of stabilization policy when there is only one-period debt and the central bank has imperfect information about the current inflation rate. The central objective of this analysis is to show that more general properties of debt management policy matter, even when the central bank correctly observes current inflation.

2.4.3 Firms

The first-order condition for the optimal price decision of firms, to a log-linear approximation satisfies,

$$\hat{p}_t(i) = \alpha \hat{p}_{t-1}(j) + \psi \alpha \hat{E}_t^j \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left[\hat{w}_T - \hat{Z}_T + \hat{P}_T \right]$$

where $\hat{p}_t(j) = \log(p_t(j)/P_t)$. The optimal price depends on past prices as well as expectations about the future path of real wages, the level of technology and the general level of prices. These expectations about future marginal cost conditions are relevant because of costly price adjustment. The degree of nominal rigidity is indexed by $\psi \equiv (\theta - 1)\bar{Y}/\chi > 0$, where $\bar{Y}$ is steady-state output. Larger values of ψ imply smaller costs of adjustment — prices are more flexible. The parameter α satisfies the restrictions $0 < \alpha < 1$ and $\psi = (1 - \alpha\beta)(1 - \alpha)\alpha^{-1}$. In a model with Calvo price adjustment, α would denote the probability of not re-setting the price.

Aggregating price decisions over the continuum of firms gives a generalized Phillips curve

$$\hat{\pi}_t = \psi \left(\hat{w}_t - \hat{Z}_t \right) + \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left[\psi\alpha\beta \left(\hat{w}_{T+1} - \hat{Z}_{T+1} \right) + (1 - \alpha)\beta \hat{\pi}_{T+1} \right] \quad (27)$$

which determines inflation as a function of the current real wage and technology, and the present discounted value of the same and inflation. Given optimal prices, firms stand ready to supply desired output which determines aggregate hours as

$$\hat{H}_t = \hat{Y}_t - \hat{Z}_t. \quad (28)$$

Finally, goods market clearing implies the log-linear restriction

$$\hat{Y}_t = s_C \hat{C}_t + s_G \hat{G}_t \quad (29)$$

where $s_C = \bar{C}/\bar{Y}$ is the steady-state consumption-to-output ratio.

2.5 Monetary and Fiscal Policy

The nominal interest-rate rule satisfies the approximation

$$\hat{\imath}_t = \rho_i \hat{\imath}_{t-1} + \phi_\pi \hat{\pi}_t + \phi_y \hat{Y}_t + \ln \varepsilon_{i,t}. \tag{30}$$

The activities of the fiscal authority are summarized by a log-linear approximation to (3), (4), (6) and the definition of the structure surplus to give:

$$\hat{b}_t = \beta^{-1}\left(\hat{b}_{t-1} - \hat{\pi}_t\right) + (\rho - 1)\hat{P}_t^m - \left(\beta^{-1} - 1\right)\hat{s}_t \tag{31}$$

$$\hat{l}_t = \hat{b}_{t-1} + \beta\rho\hat{P}_t^m \tag{32}$$

$$\hat{s}_t = \tau_l \hat{l}_t + \ln \varepsilon_{\tau,t} \tag{33}$$

$$\hat{\tau}_t = s_\tau^{-1}\left(\delta\hat{s}_t + s_G\hat{G}\right) \tag{34}$$

which describe the evolution of total outstanding bonds; the definition of government liabilities; tax collections specified directly in terms of the structural surplus; and the definition of the structural surplus.

This completes the description of aggregate dynamics. To summarize, the model comprises the twelve aggregate relations (21), (23) and (25)–(34) which determine the evolution of the variables $\left\{\hat{P}_t^m, \hat{\pi}_t, \hat{\imath}_t, \hat{w}_t, \hat{\Gamma}_t, \hat{C}_t, \hat{Y}_t, \hat{H}_t, \hat{b}_t, \hat{s}_t, \hat{l}_t, \hat{\tau}_t\right\}$ given the exogenous processes $\left\{\hat{G}_t, \hat{Z}_t, \hat{\zeta}_t, \ln\varepsilon_{\tau,t}, \ln\varepsilon_{i,t}\right\}$.

3 Belief Formation

Beliefs. This section describes the learning dynamics and the criterion to assess convergence of beliefs. The benchmark assumptions on beliefs are laid out before returning to a discussion of some specific implications of the assumption of no-arbitrage under learning dynamics. The optimal decisions of households and firms require forecasting the evolution of future prices — nominal interest rates, real wages, dividends, taxes and inflation — and exogenous shocks. In the benchmark case, agents are assumed to use a linear econometric model of the form

$$\begin{bmatrix} \hat{\pi}_t \\ \hat{\imath}_t \\ \hat{w}_t \\ \hat{\Gamma}_t \\ \hat{s}_t \\ b_t \end{bmatrix} = \Omega_{t,0} + \Omega_{t,1} \begin{bmatrix} \hat{\pi}_{t-1} \\ \hat{\imath}_{t-1} \\ \hat{w}_{t-1} \\ \hat{\Gamma}_{t-1} \\ \hat{s}_{t-1} \\ b_{t-1} \end{bmatrix} + \Omega_{t,2} \begin{bmatrix} \hat{G}_{t-1} \\ \hat{Z}_{t-1} \\ \hat{\zeta}_{t-1} \end{bmatrix} + e_t \tag{35}$$

where $\Omega_{t,0}$ is a matrix with dimension (6×1) ; $\Omega_{t,1}$ a matrix with dimension (6×6); $\Omega_{2,1}$ a matrix of dimension (6×3); and e_t a vector of regression errors. The belief structure is over-parameterized relative to the minimum-state-variable rational expectations solution, which depends only on the states $\left\{\hat{b}_{t-1}, \hat{\imath}_{t-1}, \hat{G}_{t-1}, \hat{Z}_{t-1}, \hat{\zeta}_{t-1}\right\}$. While the rational expectations solution does not contain a constant, it has a natural interpretation under learning of capturing uncertainty about the steady state. For simplicity it is assumed that agents know the autoregressive coefficients of the exogenous processes for government purchases, technology and preference shocks.[13]

Beliefs updating and forecasting. Each period, as additional data become available, agents update the coefficients of their parametric model given by (35) using a recursive least-squares estimator. Letting $\Omega = \begin{bmatrix} \Omega_0 & \Omega_1 & \Omega_2 \end{bmatrix}$ be the matrix of coefficients to estimate, $u_t = \left(\hat{\pi}_t, \hat{\imath}_t, \hat{w}_t, \hat{\Gamma}_t, \hat{s}_t, \hat{b}_t\right)$ and $q_t = \left(1, u_t, \hat{G}_t, \hat{Z}_t, \hat{\zeta}_t\right)$, the algorithm can be written in recursive terms as

$$\hat{\Omega}_t = \hat{\Omega}_{t-1} + g_t^{-1} R_t^{-1} q'_{t-1} \left(u'_t - \hat{\Omega}_{t-1} q'_{t-1}\right)' \tag{36}$$

$$R_t = R_{t-1} + g_t^{-1} \left(q'_{t-1} q_{t-1} - R_{t-1}\right) \tag{37}$$

where g_t is a decreasing sequence and where $\hat{\Omega}_t$ denotes the current-period's coefficient estimate.[14] Agents update their estimates at the end of the period, after making consumption, labor supply and pricing decisions. This avoids simultaneous determination of the parameters defining agents' forecast functions and current prices and quantities. To compare the model under learning with the predictions under rational expectations, we assume that agents' expectations are determined simultaneously with consumption, labor supply and pricing decisions,

[13] The assumption that autocorrelation coefficients are known to agents are not too important for the results of the paper. The E-stability conditions are independent of this assumption because given observations on each disturbance, asymptotically the autocorrelation coefficients are recovered with probability one using linear regression. The assumption is more relevant for the simulations. Assuming these parameters are known serves to understate variation for a given primitive shocks.

[14] It is assumed that $\sum_{t=1}^{\infty} g_t = \infty$, $\sum_{t=1}^{\infty} g_t^2 < \infty$ — see Evans and Honkapohja (2001).

so that agents observe all variables that are determined at time t, including $\hat{b}_t$. For example, the one-period-ahead forecast for $\hat{\pi}_t$ is

$$\hat{E}_t \hat{\pi}_{t+1} = \hat{\Omega}^{\pi}_{0,t-1} + \hat{\Omega}^{\pi}_{1,t-1} \begin{bmatrix} \hat{\pi}_t \\ \hat{\imath}_t \\ \hat{w}_t \\ \hat{\Gamma}_t \\ \hat{s}_t \\ b_t \end{bmatrix} + \hat{\Omega}^{\pi}_{2,t-1} \begin{bmatrix} \rho_G \hat{G}_t \\ \rho_G \hat{Z}_t \\ \rho_\zeta \hat{\zeta}_t \end{bmatrix}$$

where $\hat{\Omega}^{\pi}_{0,t-1}$, $\hat{\Omega}^{\pi}_{1,t-1}$ and $\hat{\Omega}^{\pi}_{2,t-1}$ are the previous-period's estimates of belief parameters that define the period t forecast function. They observe the same variables that a 'rational' agent would observe. The only difference is that they are attempting to learn the 'correct' coefficients that characterize optimal forecasts.

True Data Generating Process. Using (35) to substitute for expectations in (26), (27) and (21) and solving with the intratemporal conditions of the model delivers the actual data generating process

$$\begin{aligned} u_t &= \Gamma_1\left(\hat{\Omega}_{t-1}\right) q'_{t-1} + \Gamma_2\left(\hat{\Omega}_{t-1}\right)\varepsilon_t && (38) \\ \hat{\Omega}_t &= \hat{\Omega}_{t-1} + gR_t^{-1} q'_{t-1}\left(\left[\left(\Gamma_1\left(\hat{\Omega}_{t-1}\right) - \hat{\Omega}_{t-1}\right) q'_{t-1} + \Gamma_2\left(\hat{\Omega}_{t-1}\right)\varepsilon_t\right]\right)' && (39) \\ R_t &= R_{t-1} + g\left(q'_{t-1} q_{t-1} - R_{t-1}\right) && (40) \end{aligned}$$

where $\Gamma_1\left(\hat{\Omega}\right)$ and $\Gamma_2\left(\hat{\Omega}\right)$ are nonlinear functions of the previous-period's estimates of beliefs. The actual evolution of u_t is determined by a time-varying coefficient equation in the state variables q_t and the exogenous i.i.d. disturbances $\varepsilon_t = (\varepsilon_{G,t}, \varepsilon_{Z,t}, \varepsilon_{\zeta,t}, \varepsilon_{i,t}, \varepsilon_{\tau,t})$, where the coefficients evolve according to (39) and (40). The evolution of u_t depends on $\hat{\Omega}_{t-1}$, while at the same time $\hat{\Omega}_t$ depends on u_t. Learning induces self-referential behavior. The dependence of $\hat{\Omega}_t$ on u_t is related to the fact that outside the rational expectations equilibrium $\Gamma_1\left(\hat{\Omega}_{t-1}\right) \neq \hat{\Omega}'_{t-1}$ and similarly for Γ_2. This self-referential behavior emerges because each market participant ignores the effects of their learning process on prices and income, and this is the source of possible divergent behavior in agents' expectations.

Expectations Stability. The data generating process implicitly defines the mapping between agents' beliefs, $\hat{\Omega}$, and the actual coefficients describing observed dynamics, $\Gamma_1\left(\hat{\Omega}\right)$. A rational expectations equilibrium is a fixed point of this mapping. For such rational expectations equilibria we are interested in asking under what conditions does an economy with learning dynamics converge to each equilibrium. Using stochastic approximation methods, Marcet and Sargent (1989b) and Evans and Honkapohja (2001) show that conditions for convergence are characterized by the local stability properties of the associated ordinary differential equation

$$\frac{d\left(\hat{\Omega}\right)}{d\tau} = \Gamma_1\left(\hat{\Omega}\right) - \hat{\Omega}, \tag{41}$$

where τ denotes notional time. The rational expectations equilibrium is said to be expectationally stable, or E-Stable, when agents use recursive least squares if and only if this differential equation is locally stable in the neighborhood of the rational expectations equilibrium.[15]

Restrictions from No-Arbitrage. The belief structure laid out above has the property that the no-arbitrage condition (20) is satisfied in all periods of the household's decision horizon. Agent's beliefs determine a forecast of the future sequence of one-period interest rates $\{\hat{\imath}_T\}$ from which the multiple-maturity bond portfolio is priced using (21). Because the bond pricing equation is an implication of the no-arbitrage condition, relation (20) is necessarily satisfied at all dates.

An alternative approach would be to suppose households forecast future bond prices directly using some econometric model, which combined with (20) would determine a no-arbitrage consistent forecast path for the one-period interest rate. While these two approaches are equivalent under rational expectations, they will in general differ under arbitrary assumptions on belief formation. Indeed, consider augmenting the beliefs structure (35) with an additional dependent variable in the price of the multiple-maturity bond portfolio. Then absent the restrictions imposed by a rational expectations equilibrium analysis — which would impose the specific restriction that beliefs are consistent with no-arbitrage — the augmented belief structure will permit very general relationships between the bond price and the period interest rate — there is no reason to suppose that the implied forecast would satisfy (20).

Given this observation, the benchmark analysis proceeds assuming the belief structure (35), using (21) to price multiple-maturity debt. This belief structure is referred to as "anchored financial market expectations" because it exploits all conditions for household optimality — specifically transversality in determining the price of the bond portfolio. The alternative belief structure is also analyzed and referred to as "unanchored financial market expectations" since it exploits one less first-order condition from household optimality. These two approaches

[15]Standard results for ordinary differential equations imply that a fixed point is locally asymptotically stable if all eigenvalues of the Jacobian matrix $D\left[\Gamma\left(\Omega\right) - \left(\Omega\right)\right]$ have negative real parts (where D denotes the differentiation operator and the Jacobian is understood to be evaluated at the relevant rational expectations equilibrium).

give different conclusions about the stability properties of the model under simple rules for monetary and fiscal policy.

4 Benchmark Implications

To anchor ideas and provide a comparative benchmark, it is useful to state model properties under rational expectations.

Proposition 1 *Under rational expectations the following conditions are necessary and sufficient for a unique bounded equilibrium:*

$$\kappa\left(\phi_{\pi}-1+\rho_{i}\right)+(1-\beta)\,\phi_{y}>0$$

and

$$1<\tau_{l}<\frac{1+\beta}{1-\beta}$$

where

$$\kappa=\left(\gamma+\sigma s_{C}^{-1}\right)(1-\alpha)(1-\alpha\beta)\,\alpha^{-1}.$$

This is a familiar result in New Keynesian monetary economics. Uniqueness of rational expectations equilibrium requires that interest-rate policy be sufficiently aggressive, as characterized by the first restriction, referred to as the Taylor principle by Woodford (2003). A further requirement is that fiscal policy is Ricardian in the sense that for all sequences of prices, tax policy is conducted in such a way that ensures intertemporal solvency of the government accounts. This is guaranteed by the second restriction on tax policy. Note also that such equilibria have the property that inflation, output and nominal interest rates all evolve independently of debt policy, and in particular, the average maturity of debt.[16]

In general the model prohibits an analytic characterization of E-Stability. For one special case an analytic characterization is possible.

Proposition 2 *Under learning dynamics, assuming there is only one-period debt, $\rho=0$, and that monetary policy is not inertial, $\rho_i=0$, the following conditions are necessary and sufficient for E-Stability:*

$$\kappa\left(\phi_{\pi}-1\right)+(1-\beta)\,\phi_{y}>0$$

and

$$1<\tau_{l}<\frac{1+\beta}{1-\beta}$$

where

$$\kappa=\left(\gamma+\sigma s_{C}^{-1}\right)(1-\alpha)(1-\alpha\beta)\,\alpha^{-1}.$$

[16] See Leeper (1991) for a seminal discussion on the importance of fiscal policy for monetary equilibria.

A sketch of the proof can be found in the technical appendix of Eusepi and Preston (2010b). In the case of one-period debt and no inertia in interest-rate policy the conditions for expectational stability are isomorphic to those for determinacy of rational expectations equilibrium. This coincidence in requirements for non-inertial policy rules is only true for one-period debt. As the maturity structure of debt increases from one period — that is, as ρ is increased from zero — the equivalence result breaks down. Details of debt management and fiscal policy matter for expectations stability.

5 Expectational Stability: Anchored Financial Expectations

The analysis proceeds numerically, with the discussion of E-Stability organized around defining characteristics of each agent in the model: i) decision making of firms: the degree of nominal rigidities in price setting; ii) decision making of households: the Frisch elasticity of labor supply and risk aversion which control the intertemporal substitution of leisure and consumption; iii) debt management policy: the average maturity of debt and the average level of indebtedness; and iv) variations in the class of monetary policy rule. Some extensions to these basic results are then offered with focus on the role of wealth effects on labor supply and the question of the importance of anchored financial expectations.

No attempt is made to fit the model to data. The intention is to consider fairly conventional parameter values and understand how E-Stability depends on plausible variations of these parameters. The benchmark parameterization of the model follows, assuming a quarterly model, with departures noted at they arise. Household decisions: the discount factor is $\beta = 0.99$; the inverse Frisch elasticity of labor supply $\gamma = 2$; the inverse elasticity of intertemporal substitution of consumption $\sigma = 2$; and the elasticity of demand across differentiated goods $\theta = 5$. Firm decisions: nominal rigidities are determined by $\alpha = 0.75$.[17] Monetary policy: $\phi_\pi = 1.5$ and $\phi_y = \rho_i = 0$ so that there is no inertial component of policy and no response to the state of aggregate demand. Fiscal policy: the structural surplus is adjusted in response to outstanding debt: $\tau_b = 1.5$; the average maturity of debt and the average level of indebtedness (in terms of debt over *annual* output) are determined by $\rho = 0.976$ and $\bar{b}/\left(4\bar{Y}\right) = 2$. The latter two parameters are chosen to approximate the maturity structure and indebtedness of Japan. The configuration of monetary and fiscal policy are consistent with the conditions for a unique bounded rational expectations equilibrium. The great ratios are taken to be $s_C = 0.8$ and $s_G = 0.2$. The autoregressive coefficients for the exogenous processes for technology,

[17] Recall the parameter χ is determined by the choice of α.

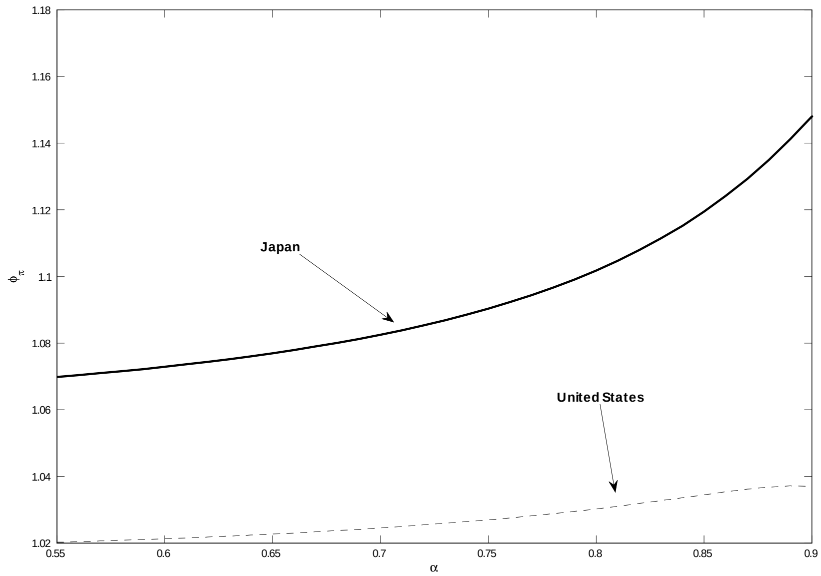

Figure 1: Stability regions over different degrees of nominal rigidity in (ϕ_π, α) space. Stable regions are up and to the left of the contours indexed by average indebtedness b.

preferences and government purchases are $\rho_Z = \rho_G = \rho_\zeta = 0.6$.[18]

The analysis now considers variations in these benchmark assumptions that are relevant to the question of expectations stability.

5.1 Nominal Rigidities

Figure 1 plots E-Stability regions in monetary policy and nominal rigidities space. Regions upwards and to the left of each contour indicate regions of expectational stability. The two contours depict two economies with differing levels of average debt. The dashed line gives an economy with average indebtedness in annual terms equal to that of the US: $\bar{b}/\left(4\bar{Y}\right) = 0.7$, and the solid line equal to that of Japan: $\bar{b}/\left(4\bar{Y}\right) = 2$.

Two important properties are evident. First, for a given level of indebtedness, economies with greater nominal rigidities tend to be less stable. As the parameter α rises a more aggressive monetary policy is required for stability. Second, these effects are stronger the greater the level indebtedness. However, in either dimension the effects are fairly small in the sense that monetary policy need not be much more aggressive than mandated by the Taylor principle: which under the maintain assumptions is $\phi_\pi > 1$ in the model with rational expectations.

[18] The autoregressive coefficients do not play an important role in the stability analysis.

Even with a steady-state debt-to-output ratio of 200 percent and an exceptionally high degree of nominal rigidity, response coefficients greater than 1.15 are consistent with E-Stability.

That the degree of nominal rigidity matters for stability might seem surprising when compared with the requirements for determinacy. The difference emerges because of departures from Ricardian equivalence. As holdings of the public debt are perceived by households to be net wealth, an important part of equilibrium determination are changes in the valuation of debt. The real value of debt can change for two reasons: the price of the debt portfolio, $\hat{P}_t^m$, can change, and the implied real wealth of the portfolio in terms of goods can change due to variation in the general level of goods prices. With greater nominal rigidity, goods prices exhibit less variation, making departures from Ricardian equivalence a more important determinant of aggregate demand. This contributes to instability.

A final observation is that the effects documented here on the role of average indebtedness on stability are weaker than those presented in Eusepi and Preston (2010b). One important source of difference is the assumed monetary policy. In that paper, the monetary authority adjusted nominal interest rates in response to a forecast of inflation rather than actual inflation. This informational constraint proves critical: having monetary policy respond to expectations renders the equilibrium more susceptible to self-fulfilling dynamics. And in that case, fiscal variables can take on a more prominent role — this point is returned to later.

5.2 Intertemporal substitution of consumption

Figure 2 provides an analogous plot to figure 1, with different contours now indexed by the inverse elasticity of intertemporal substitution of consumption. For a given degree of nominal rigidity, as the elasticity of substitution declines the region of stability contracts — a more aggressive monetary policy is required for stability. There are two channels through which this parameter affects stability. First, it directly reduces the interest-rate elasticity of consumption demand, which was earlier demonstrated to be equal to $\beta\left(\sigma^{-1} - \bar{s}_C^{-1}\delta\right)$ — recall (24). As households become less willing to substitute consumption intertermporally, aggregate demand management through interest-rate policy becomes less effective and the wealth effects deriving from fiscal policy become relatively more important. Second, changes in σ also affect the relation between output and wages, given by

$$\hat{w}_t = \left(\gamma + \sigma s_C^{-1}\right)\hat{Y}_t.$$

On the one hand, this steepens the Phillips curve as the wealth effects on labor supply become stronger, making prices more responsive to output changes. This is a source of stability, as

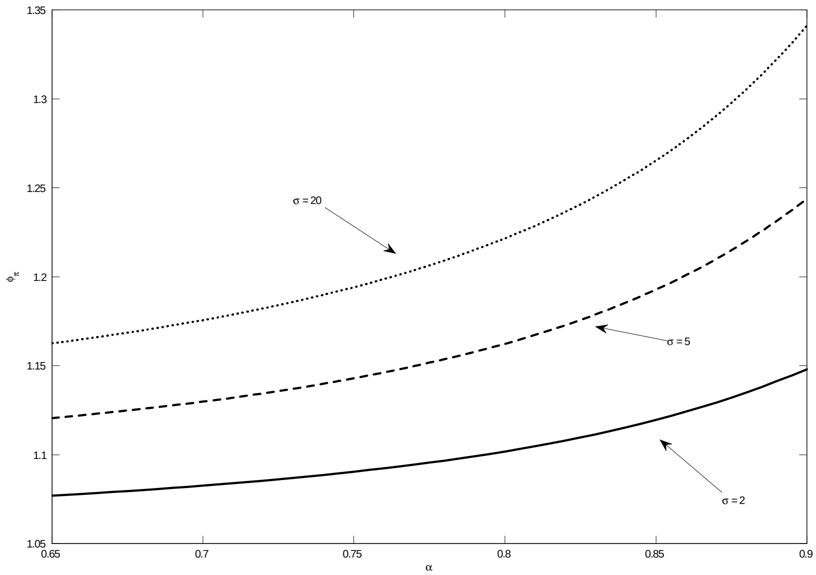

Figure 2: Stability regions over different degrees of nominal rigidity in (ϕ_π, α) space. Stable regions are up and to the left of the contours indexed by average indebtedness σ.

discussed above. On the other hand, by inducing a higher response of wages to output, it amplifies the wealth effects from future expected wages in the consumption decision rule. This imparts destabilizing effects, as wages comove with output and inflation in response to changes in aggregate demand. On net, for the chosen calibration, the destabilizing effects dominate.

5.3 Frisch Elasticity of Labor Supply

Figure 3 plots E-Stability regions in monetary policy and inverse Frisch elasticity space. It is immediate that variations in the preparedness of households to supply labor in response to movements in the real wage have a non-trivial impact for monetary policy: as labor supply become less elastic, the more aggressive must be monetary policy to ensure expectational stability.

To understand this result, note that the Frisch elasticity affects three key model coefficients: i) the scale of Keynesian expenditure effects arising from departures from Ricardian equivalence as captured in the parameter $\bar{s}_C^{-1}$; ii) the wage elasticity of consumption demand

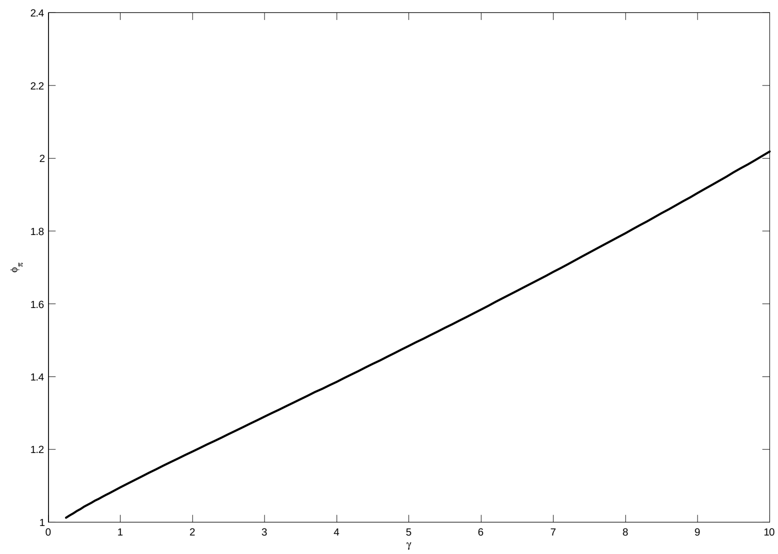

Figure 3: Stability regions over different degrees of Frisch elasticity in (ϕ_π, γ) space. The stable region is up and to the left.

given by

$$\bar{s}_C^{-1}\left(\frac{\theta-1}{\theta}\right)\left(1+\gamma^{-1}\right)=\frac{\left(\frac{\theta-1}{\theta}\right)(1+\gamma)}{\sigma\left(\frac{\theta-1}{\theta}\right)+\gamma s_C};$$

and iii) the slope of the Phillips curve. Consider the first two effects over the range of elasticities $\gamma \in (0, \infty)$. The corresponding range for the scale of wealth effects from debt is $\left(0, s_C^{-1}\right)$ and the range of wage elasticity of consumption increases from its minimum value σ^{-1} to its maximum $\left(\frac{\theta-1}{\theta}\right) s_C^{-1}$, assuming an intertemporal elasticity of consumption substitution that is always less than unity: consistent with a broad range of micro and macro empirical evidence — see Hall (2009) for one recent discussion. In the limit case of an infinite Frisch elasticity of labor supply, $\gamma = 0$, there are no Keynesian expenditure effects on aggregate demand from holdings of the public debt. Consistent with this effect is that the wage elasticity of consumption demand is also smallest in this case — variations in the real wage lead to smaller variations in consumption demand. Both effects are conducive to stability. At the other extreme, if labor supply is perfectly inelastic, then the Keynesian expenditure effects are maximal as is the cross elasticity. Both contribute to instability.[19] General equilibrium effects on labor supply

[19]The observation that inelastic labor supply leads to instability is consistent with Eusepi and Preston (2010b). That paper did not account for the endogeneity of labor supply in the optimal decision rule for consumption: agent's forecast period income defined as the sum of total wage income and dividend income directly. Here households forecast the wage rate, as distinct to the wage bill, and dividends separately. However,

are therefore important to propagation of wealth effects from debt holdings.

5.4 The Maturity Structure of Debt

Figure 4 plots the interaction between monetary policy and the average maturity of debt. The inverse consumption elasticity of intertemporal substitution and the inverse Frisch elasticity are assumed to take values $\sigma = 20$ and $\gamma = 5$. As revealed, these values give greater role to the average level of indebtedness in determining E-Stability. It is immediate that the average maturity of debt also imposes constraints on the design of monetary policy and that these constraints are non-monotonic. When $\rho = 0$, so that only one-period debt is issued, the Taylor principle is necessary and sufficient for both determinacy of rational expectations equilibrium and also E-Stability. However, as the average maturity of debt rises, monetary policy is required to be more aggressive to ensure stability under learning — recall that the determinacy conditions are invariant to the specification of fiscal policy. This effect peaks at an average maturity of about 2 years when the response of nominal interest rates is around 1.7 and then declines.[20] Long maturities tend to be more conducive to stability, and in the limit of infinite-maturity debt the Taylor principle is restored as sufficient for E-Stability.

That the maturity structure of debt matters for expectational stability presents a strikingly different prediction to a rational expectations analysis of the model where the maturity structure is irrelevant to macroeconomic dynamics. To the extent that expectations stabilization is a priority of monetary and fiscal policy then either very short or long maturities are desirable. Average maturities of debt in the neighborhood of $1 - 7$ years present a more stringent constraint, requiring a more aggressive monetary policy.

The source of non-monotonicity is not obvious, but is located in the valuation effects of the multiple-maturity debt portfolio. To gain intuition, substitute the arbitrage equation (21) and the monetary policy rule[21], $\hat{\imath}_t = \phi_\pi \hat{\pi}_t$, into the aggregate consumption decision rule (26) to give

$$\begin{aligned} \hat{C}_t &= \bar{s}_C^{-1}\delta\hat{b}_{t-1} + \hat{E}_t^i \sum_{T=t}^{\infty} \beta^{T-t} \left[\bar{s}_C^{-1}(1-\beta)x_T - \beta\sigma^{-1}(\phi_\pi\hat{\pi}_T - \hat{\pi}_{T+1}) + \sigma^{-1}\beta\left(\hat{\xi}_T - \hat{\xi}_{T+1}\right)\right] \\ &\quad + \bar{s}_C^{-1}\delta\hat{E}_t\left[\beta\sum_{T=t}^{\infty}\beta^{T-t}(\phi_\pi\hat{\pi}_T) - \beta\rho\sum_{T=t}^{\infty}(\beta\rho)^{T-t}(\phi_\pi\hat{\pi}_T) - \sum_{T=t}^{\infty}(\beta)^{T-t}\hat{\pi}_T\right]. \end{aligned} \tag{42}$$

in the case of competely inelastic supply of labor hours variation are clearly unimportant to period income.

[20] These effects remain relevant for higher elasticities of intertemporal substitution. For example, when $\sigma = 5$ the peak policy response is above 1.35 with the broad features of the maturity structure unchanged but shifted downwards.

[21] With this particular policy rule which responds to current inflation, E-Stability does not depend on agents' knowledge of the rule.

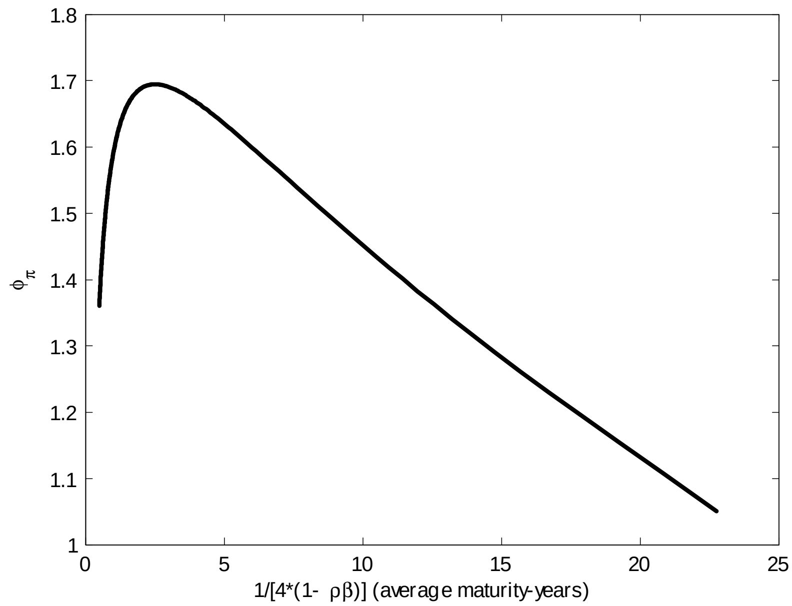

Figure 4: Stability regions over different average maturities of debt in $\left(\phi_\pi, (1-\beta\rho)^{-1}\right)$ space. The stable region lies above the contour.

The second line of this expression gives non-Ricardian wealth effects attached to a given holding of bonds, $\hat{b}_{t-1}$. Wealth effects originate from three sources when higher future inflation is anticipated: the first term captures the positive effects of higher anticipated nominal interest rates and because of the Taylor principle, higher real rates; the second term is a negative wealth effect from anticipated capital losses: higher expected future inflation implies a fall in the price of the bond; and the final term captures the fact that a given portfolio of bonds is worth less in terms of goods when higher inflation is anticipated.

How do these effects depend on the maturity structure? To fix ideas, suppose households anticipate a constant rate of inflation above steady state. When $\rho = 0$ the wealth effects from capital losses are zero. Assuming $\phi_\pi > \beta^{-1}$ the wealth effect from higher nominal interest rates exceeds the loss from higher future inflation eroding the value of wealth in terms of goods. The overall effect on consumption demand is positive. When $\rho \to 1$ the elasticity of consumption with respect to expected nominal rates is restored to $\beta\sigma^{-1}$, which corresponds to the Ricardian economy. Aside from debt, the only non-Ricardian component in the decision rule is the present value of discounted expected inflation (the third term in the final line), which affects the expected real income from holding debt. The overall effect on consumption demand is therefore negative. The wealth effects are therefore maximal for one-period debt, and decline monotonically with maturity to be negative with $\rho = 1$. Note that when the wealth effects are largest, the model is E-Stable — the source of instability and non-monotonicity must be coming from the evolution of debt.

Using (21) and the monetary policy rule, $\hat{\imath}_t = \phi_\pi \hat{\pi}_t$, the evolution of real debt (31) can be written

$$\hat{b}_t = \beta^{-1}\hat{b}_{t-1} - \beta^{-1}\hat{\pi}_t + (1-\rho)\,\phi_\pi\hat{\pi}_t + (1-\rho)\,\rho\beta\hat{E}_t\sum_{T=t}^{\infty}(\rho\beta)^{T-t}\,\phi_\pi\hat{\pi}_{T+1} - \left(\beta^{-1}-1\right)\hat{s}_t. \quad (43)$$

The key source of instability is that for intermediate maturities of debt, expectations about future interest rates (that is about expected inflation) affect the evolution of debt. In Eusepi and Preston (2010b) this is shown to be a source of instability. Because of the quadratic term in ρ, the effects of inflation expectations on debt are hump-shaped in the average maturity of debt. Assuming for simplicity that agents project constant inflation throughout the forecasting horizon, the effects of expected inflation are described by

$$(1-\rho)\,\rho\beta\hat{E}_t\sum_{T=t}^{\infty}(\rho\beta)^{T-t}\,\phi_\pi\hat{\pi}_{T+1} = \frac{(1-\rho)\,\rho\beta}{1-\rho\beta}\phi_\pi\hat{E}_t\pi$$

where $\hat{E}_t\pi$ is the constant expected value of inflation over the forecast horizon. Assuming $\beta = 0.99$, the term $(1-\rho)\,\rho\beta/(1-\rho\beta)$, which measures the effects of expectations on debt, peaks at $\rho \simeq 0.9$, or an average maturity of about 2 years, consistent with the E-Stability region in Figure 4. Interestingly, when $\rho = 1$ there are no valuation effects on the evolution of real debt from bond prices. Combined with earlier observations, this makes clear the source of stability when only infinite maturity consoles are issued. Intuitively, as the maturity structure lengthens a smaller portion of debt is rolled over in any period, while at the same time, the price of that component becomes more volatile. In the limit of infinite-maturity debt the former effect dominates — changes in valuation are irrelevant to the evolution of real debt.

Summing up, for intermediate values of ρ, both the wealth effects coming from holding debt in (42) and the high impact of expected interest rates (inflation) on the dynamics of real debt in (43) combine to foster instability. For sufficiently small values of ρ the economy is more stable, despite higher wealth effects from holding public debt, because expected rates do not have a large influence on the dynamics of real debt. For ρ sufficiently high, both wealth effects and the effects of expected interest rates on real debt dynamics vanish, making the equilibrium more stable.

5.5 Details of Monetary Policy

It is often argued that monetary policy ought to be specified in terms of a reaction function in which nominal interest rates respond to expectations of next-period inflation rather than realizations of current-period inflation. Indeed, there is a variety of empirical evidence supporting central bank reaction functions of this kind — see, for example, Clarida, Gali, and Gertler (1998) and Clarida, Gali, and Gertler (2000). To this end, consider a rule of the form

$$\hat{\imath}_t = \phi_\pi \left((1 - \omega)\, \hat{\pi}_t + \omega \hat{E}_t \hat{\pi}_{t+1} \right) \tag{44}$$

where $0 \leq \omega \leq 1$ is a policy parameter weighting the relative of importance of contemporaneous inflation versus inflation expectations in the policy rule. It is assumed that in implementing this interest-rate rule, the central bank responds to observed private-sector inflation expectations. An alternative, but equivalent assumption, is that the central bank has the same forecasting model of inflation as households and firms.

One additional assumption is required for interesting results. It is assumed that households understand that monetary policy is determined according to equation (44). Absent this assumption, rules of this kind engender considerable instability. This was first noted by Preston (2006) in the case of $\omega = 1$ in a model almost identical to that proposed here, assuming an economy with no debt, no government purchases and no taxation. Eusepi and Preston (2010a, 2010b) develop that analysis further and interpret the assumption of knowledge of the monetary policy rule as central bank communication. As details of the monetary policy strategy are known, households can make policy-consistent forecasts. Eusepi and Preston (2010a) show that this assists stability as aggregate demand management through interest-rate policy is more effective. Agents knowing the rule ensures that projections of nominal interest rates satisfy the Taylor principle. This leads to the appropriate restraint of aggregate demand. Absent this assumption, demand management fails because households project very flat profiles for the real interest rate in response to various disturbances.

Figure 5 plots stability regions for the rule that also responds to inflation expectations. Two contours are plotted for the maximum and minimum values of ω, bounding the transition point from regions of stability to instability that arise for intermediate values. When $\omega = 0$ so that no weight is given to inflation expectations in interest-rate policy, the regions of instability and stability are identical to Figure 4. In contrast, when $\omega = 1$, so that full weight is given to inflation expectations, the stability region contracts.

A notable implication arising from expectations-based instrument rules is that the effect

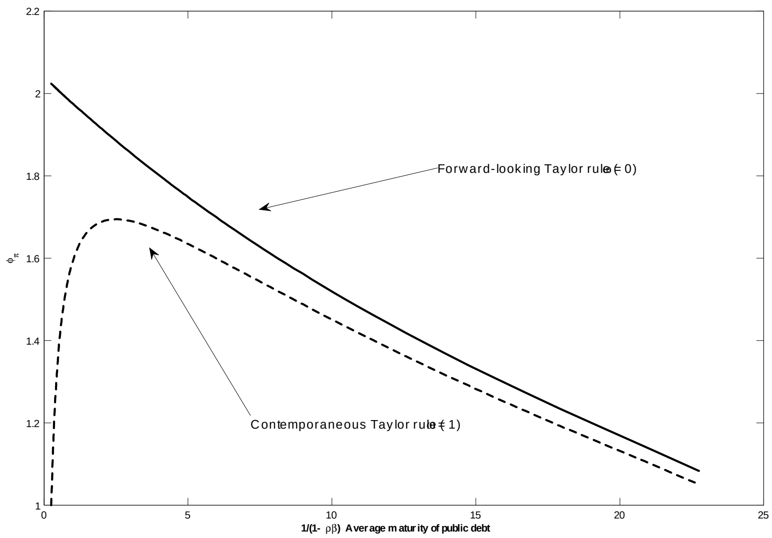

Figure 5: Stability regions over different average maturities of debt in $\left(\phi_\pi, (1-\beta\rho)^{-1}\right)$ space. Stable regions are up and to the right. The two contours correspond to monetary policy rules that respond to contemporaneous inflation and expectations of next-period inflation respectively.

of increasing average maturity of debt is monotonic — by continuity this must be true for a range of values satisfying $\omega < 1$ in the neighborhood of $\omega = 1$. As before, the intuition relies on the dynamics of real debt, with $\omega = 1$ (43) becomes

$$\hat{b}_t = \beta^{-1}\hat{b}_{t-1} - \beta^{-1}\hat{\pi}_t + (1-\rho)\,\phi_\pi \hat{E}_t\hat{\pi}_{t+1} + (1-\rho)\,\rho\beta\hat{E}_t\sum_{T=t}^{\infty}(\rho\beta)^{T-t}\,\phi_\pi\hat{\pi}_{T+2} - \left(\beta^{-1}-1\right)\hat{s}_t.$$

Now even for small values of ρ real debt depends on expected inflation, which is the source of instability at short maturities. The case of $\rho = 0$ is discussed in Eusepi and Preston (2010b), where it is shown, consistently with Figure 5, that the Taylor principle is not sufficient for E-Stability. The region of instability is largest in the case of a debt portfolio comprised only of one-period instruments. As the maturity structure increases, monetary policy can be less aggressive from the perspective of expectations stabilization. This is consistent with the discussion in the previous section. To the extent that central banks will always in practice need to respond to a forecast of inflation, these results suggest that longer-maturity debt is more desirable on the ground of protecting against expectations-driven instability from learning dynamics.[22]

[22] A more realistic timing assumption might be to assume the nominal interest-rate policy is determined as a

6 Extensions of the Basic Results

This section offers two extensions to the core results. The first considers an alternative preference structure to isolate two different kinds of wealth effects in the model: those directly arising from departures from Ricardian equivalence, and those that operate through general equilibrium effects on labor supply. The second considers the role of beliefs in the determination of bond prices — specifically the role of unanchored financial market expectations.

6.1 Wealth Effects on Labor Supply

The previous section identified a central role to the Frisch elasticity in labor supply for E-Stability. The more elastic is labor supply the more stable is the model under learning dynamics. The key channel of stability was shown to be that this parameter regulates the scale of wealth effects arising from departure from Ricardian equivalence on consumption demand. In the case that the Frisch elasticity is infinite, these wealth effects are zero. General equilibrium effects on labor supply are therefore central.

To further our understanding of the interaction between wealth effects and labor supply, we now consider a version of the model in which there are no wealth effects on labor supply decisions. This is achieved by adopting the preference structure proposed by Greenwood, Hercowitz, and Huffman (1988) so that period utility is given by

$$(1-\sigma)^{-1}\left(C_t - \phi\frac{H_t^{1+\gamma}}{1+\gamma}\right)^{1-\sigma} + v\left(G_t\right)$$

where each parameter retains earlier interpretation.

Under this assumption the optimal labor supply condition becomes, up to a log-linear approximation,

$$\gamma \hat{H}_t = \hat{w}_t.$$

This expression makes evident that income effects on labor supply are zero given the absence of the marginal utility of income. Under these preferences the optimal consumption decision rule is

$$\begin{aligned}\hat{C}_t^i &= s_C^{-1}\left(\frac{\theta-1}{\theta}\right)\hat{h}_t + s_C^{-1}\delta\left(\hat{b}_{t-1}^i - \hat{\pi}_t + \rho\beta\hat{P}_t^m\right) + \\ &\quad + \hat{E}_t^i\sum_{T=t}^{\infty}\beta^{T-t}\left[s_C^{-1}\left(1-\beta\right)x_T - \beta\left(\tilde{\sigma}^{-1} - s_C^{-1}\delta\right)\left(\hat{\imath}_T - \hat{\pi}_{T+1}\right) - \tilde{\sigma}^{-1}\beta\left(\hat{\xi}_T - \hat{\xi}_{T+1}\right)\right]\end{aligned}$$

function of $\hat{E}_{t-1}\hat{\pi}_t$. The findings of Eusepi and Preston (2010b) suggest similar results would be expected to obtain.

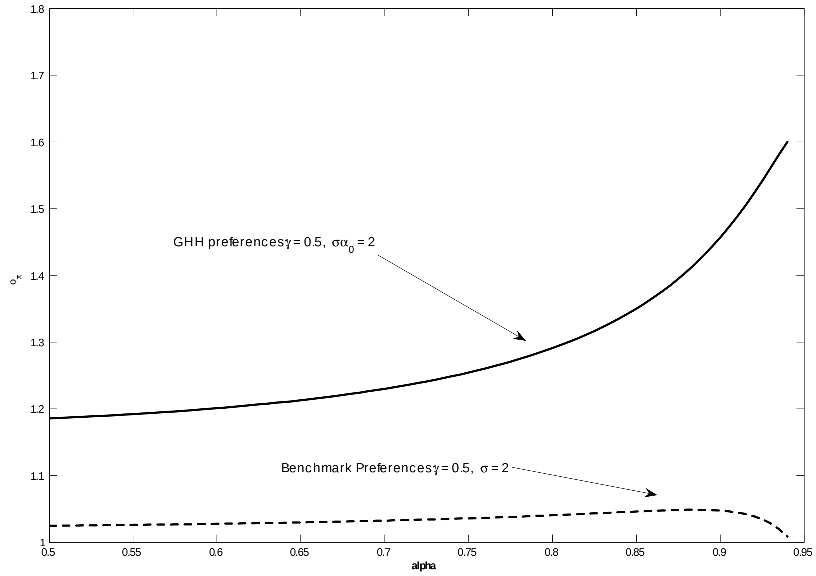

Figure 6: Stability regions over different degrees of nominal rigidity in (ϕ_π, α) space. Stable regions are up and to the left of the contours indexed by average indebtedness b.

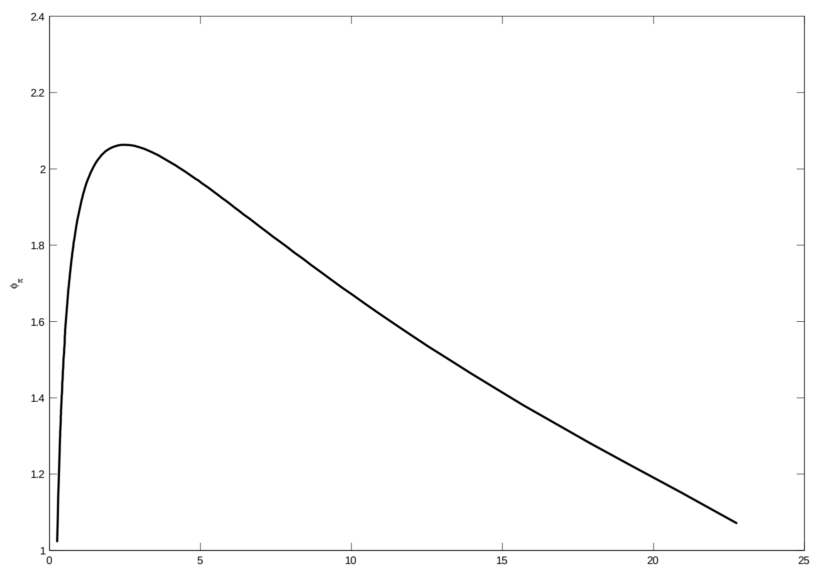

Figure 7: Stability regions over different average maturities of debt in $\left(\phi_\pi, (1-\beta\rho)^{-1}\right)$ space. The stable region lies above the contour.

where

$$x_t = \left(1-\theta^{-1}\right)\hat{w}_T + \theta^{-1}\hat{\Gamma}_T - \frac{\bar{\tau}}{\bar{Y}}\hat{\tau}_T$$

$$\tilde{\sigma} = \sigma\left(1-\bar{s}_C^{-1}\left[\frac{\theta-1}{\theta}\right]\frac{1}{1+\gamma}\right)^{-1}.$$

These relations replace (23) and (26). To a log-linear approximation, remaining model equations are unchanged.

The consumption decision rule takes a similar form as before, though there are important differences. Wealth effects arising from departure from Ricardian equivalence are now indexed by $s_C^{-1}\delta$ instead of $\bar{s}_C^{-1}\delta$. The scale of wealth effects are now independent of preference parameters, including the Frisch elasticity. This is quite distinct from the benchmark preference structure which had the following properties: when the Frisch elasticity was infinite then the scale of wealth effects was zero. When the Frisch elasticity was zero, the scale of wealth effects were $s_C^{-1}\delta$ and therefore identical to those under GHH preferences. Despite this, model implications still differ across these preferences structures. Because of the complementarity between consumption and hours embodied in the utility function, optimal consumption decisions depend on an additional term in the current level of labor supply. This reflects the fact that periods of high labor supply are also periods of high marginal utility — inducing additional consumption.

Figure 6 reproduces figure 1 in the case of both benchmark preferences and those of Greenwood, Hercowitz, and Huffman (1988), showing dependence of monetary policy on the degree of nominal rigidities for stability under learning dynamics. It is immediate that instability is more prevalent for GHH preferences, though the basic tenor of results are unchanged. One small difference arises at very high levels of nominal rigidity. For benchmark preferences the stability region expands for α values over 0.9, while the stability region continues to contract in the case of GHH. Regardless, eliminating the general equilibrium effects of public debt wealth on labor supply increases the likelihood of expectations-driven instability. Endogenous labor supply responses are an important stabilizing influence.

Figure 7 reproduces figure 4 in the case of GHH preferences, where $\tilde{\sigma} = 4.5$ and $\gamma = 0.5$ (in line with RBC estimates). The central conclusion is similar again: this preference structure tends to expand the region of instability. Notice that under GHH preference γ only regulates the elasticity of labor supply. More elastic labor supply in this model is associated with higher instability, as more elastic labor supply induces larger variations in both hours and consumption — due to the complementarity in preferences.

6.2 Financial Market Expectations

Section 5 analyzed the benchmark case in which beliefs were characterized as having "anchored financial market expectations". In that scenario, agent's beliefs determine a forecast of the sequence of future one-period interest rates $\{\hat{\imath}_T\}$ from which the multiple-maturity bond portfolio is priced using (21). Because the bond pricing equation is an implication of the no-arbitrage condition, relation (20) is necessarily satisfied at all dates.

Now consider an alternative approach. Replace the asset pricing equation (21) with (20), leaving other model equations unchanged. Assume that households forecast future bond prices directly using some econometric model, which combined with (20) determine a no-arbitrage consistent forecast path for the one-period interest rate. This approach utilizes one less condition from consumer optimality — it does not employ the transversality condition (14) but nonetheless eliminates profits from arbitrage — and is referred to as "unanchored financial market expectations". It turns out to be important for expectations stabilization.

Formally these assumptions imply two changes to the model. The equations for the bond price and consumption, (21) and (26), are replaced by the no-arbitrage condition (20) which determines the asset price as

$$\hat{P}_t^m = -\hat{\imath}_t + \rho\beta\hat{E}_t\hat{P}_{t+1}^m$$

and the consumption decision rule

$$\begin{aligned}\hat{C}_t \;=\;& \bar{s}_C^{-1}\delta\left(\hat{b}_{t-1} - \hat{\pi}_t + \rho\beta\hat{P}_t^m\right) - \beta\left(\sigma^{-1} - \bar{s}_C^{-1}\delta\right)\hat{\imath}_t \\ & -\hat{E}_t\sum_{T=t}^{\infty}\beta^{T-t}\left[\beta\left(\sigma^{-1} - \bar{s}_C^{-1}\delta\right)\left(\beta\left(\rho\beta\hat{P}_{T+2}^m - \hat{P}_{T+1}^m\right) - \hat{\pi}_{T+1}\right)\right] \\ & +\hat{E}_t\sum_{T=t}^{\infty}\beta^{T-t}\left[\bar{s}_C^{-1}\left(1-\beta\right)x_T + \sigma^{-1}\beta\left(\hat{\xi}_T - \hat{\xi}_{T+1}\right)\right] \qquad (45)\end{aligned}$$

obtained by substitution for nominal interests rates by (20) in (26). All other relations remain unchanged, with the exception of beliefs where the bond price $\hat{P}_t^m$ replaces nominal interest rates $\hat{\imath}_t$ as a dependent variable in (35) to give

$$\begin{bmatrix}\hat{\pi}_t \\ \hat{P}_t^m \\ \hat{w}_t \\ \hat{\Gamma}_t \\ \hat{s}_t \\ b_t\end{bmatrix} = \Omega_{t,0} + \Omega_{t,1}\begin{bmatrix}\hat{\pi}_{t-1} \\ \hat{P}_{t-1}^m \\ \hat{w}_{t-1} \\ \hat{\Gamma}_{t-1} \\ \hat{s}_{t-1} \\ b_{t-1}\end{bmatrix} + \Omega_{t,2}\begin{bmatrix}\hat{G}_{t-1} \\ \hat{Z}_{t-1} \\ \hat{\zeta}_{t-1}\end{bmatrix} + e_t.$$

Several observations can be made about the decision rule (45). When the average maturity of debt is unity, so that $\rho = 0$, then the model is isomorphic to the model under anchored financial expectations. Here the multiple-maturity debt portfolio collapses to one-period bonds, which satisfy $\hat{P}_t^s = \hat{P}_t^m = -\hat{\imath}_t$. Even though agents only have a forecasting model in the bond price this is equivalent to forecasting the period interest rate when there is only one-period debt. As the average maturity structure of debt increases this equivalence breaks down. An implication is that only current interest rates have a direct effect of aggregate demand. In the benchmark model, changing interest rates directly impact beliefs about future interest rates. In contrast, in this model, changing interest rates are only relevant for beliefs to the extent that they affect prices in equilibrium and influence future forecasts of the bond price — not the one-period interest rate itself. This substantially weakens the restraining influence of future interest-rate policy on aggregate demand.

Figure 8 plots stability regions over different maturities of debt. It is immediate that instability is much more prevalent when financial expectations are unanchored. As ρ increases to values that imply fairly modest average maturities of debt, monetary policy must be substantially more aggressive than implied by the Taylor principle. For example, at an average maturity equaling 10 years, the inflation response coefficient in the monetary policy rule must be greater than 25. Importantly, though not shown, these results are invariant to the average level of indebtedness.

The source of difficulty for monetary policy lies in the failure of the standard channel of aggregate demand management. When agents have anchored financial expectations they directly forecast the future path nominal interest rates and inflation. These beliefs together satisfy the Taylor principle. Hence, if inflation expectations rise, leading to an increase in aggregate demand and inflation, households correctly forecast that interest-rate policy will respond in a way that respects the Taylor principle — the real interest rate is projected to rise over the forecast horizon. In contrast, when households have unanchored financial expectations, bond prices are forecasted directly, with a no-arbitrage consistent forecast of future nominal interest rates determined from the no-arbitrage condition (20). In general, there is no reason that this forecast satisfy the Taylor principle. This leads to a failure of aggregate demand management. The only leverage monetary policy has over current demand is current interest rates — the restraining influence of having households anticipate higher future nominal interest rates is lost. It is for this reason that monetary policy must be substantially more aggressive.

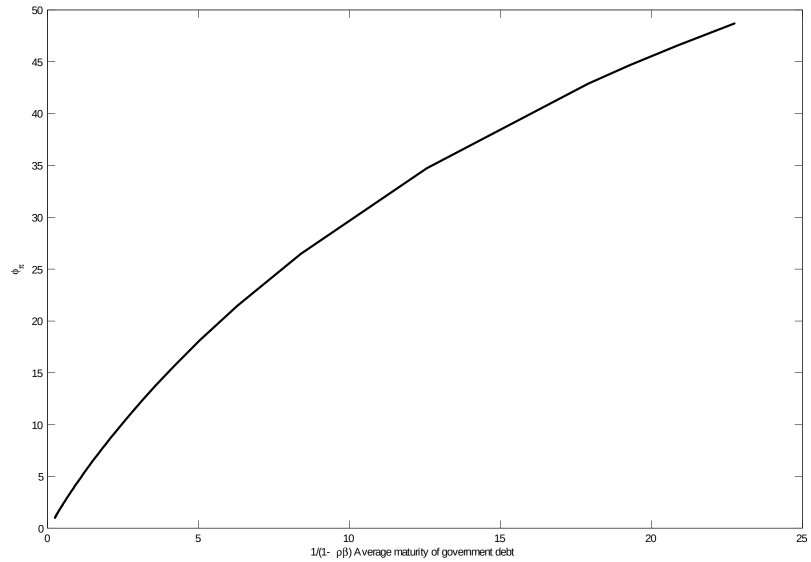

Figure 8: Stability regions over different maturities of debt in $\left(\phi_\pi, (1-\beta\rho)^{-1}\right)$ space. The stable region lies above the contour.

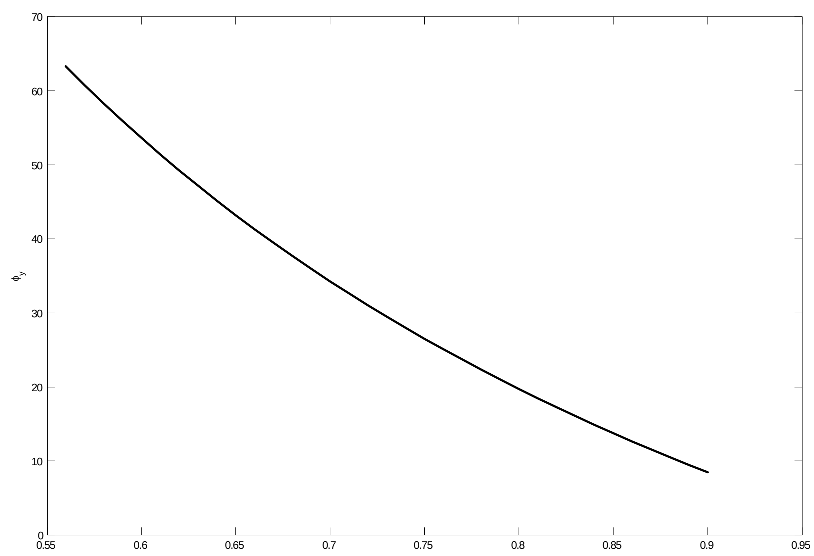

Figure 9: Stability regions over different degrees of nominal rigidity in $\left(\phi_y, \alpha\right)$ space. The stable region lies above the contour.

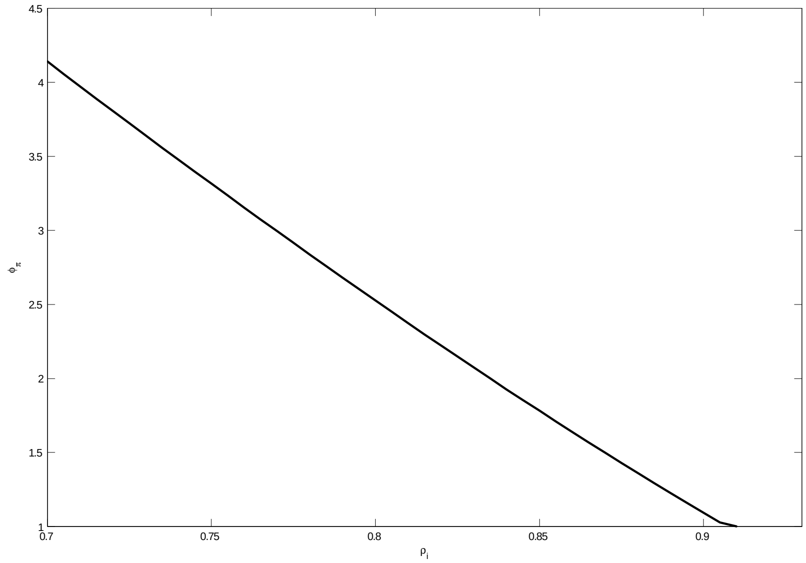

Figure 10: Stability region over different configurations of monetary policy in (ϕ_π, ρ_i) space. The stable region lies above the contour.

This description of beliefs could be interpreted as an irrational bubble. While there are no profit opportunities from arbitrage, the price of multiple-maturity debt is divorced from fundamentals. Under unanchored expectations, the price of long-debt is determined by the no-arbitrage condition given current one-period interest rates and expectations about tomorrow's price of long debt. But the fundamentals-based price for this asset actually depends upon current and all future one-period interest rates as described by (21). To the extent that quantitative easing leads to speculation about asset prices over different maturities on the yield curve, this might present further qualification to our understanding of the consequences and desirability of such policy.

Given this instability, Figures 9 and 10 evaluate the efficacy of alternative monetary policy rules. Figure 9 plots stability regions over different levels of nominal rigidity in terms of the output response coefficient ϕ_y. Aggressive responses to output variation assist stability. But as the degree of price stickiness rises the required output response declines. Comparison of this result with the conclusions from Figure 1 and 2 might suggest contradictory findings. In one case higher nominal rigidities lead to greater instability; in the other lower instability. They can be reconciled as follows. In the case of anchored financial expectations, the steady-state level of debt was critical to E-Stability as it determined the scale of aggregate demand effects arising from departure from Ricardian equivalence. In the case of unanchored financial expectations, average indebtedness and the associated wealth effects are a relatively less important source of instability — in fact, the results are independent of the average level of debt. In some sense, the instability arising from beliefs dwarfs the effects of arising from Keynesian expenditure effects. As a result, nominal rigidities have different effects. In the present case, because instability is so prevalent, price stickiness tends to make inflation more predictable assisting learnability of rational expectations equilibrium — even though it leads to smaller mitigating real wealth effects. In the case of anchored financial expectations, the beneficial effect of more predictable inflation is weighed against greater instability from the wealth effect on consumption demand; with the latter proving dominant.

Figure 10 gives one final perspective on policy design, showing the interaction of inflation response and nominal interest-rate inertia. It is clear that inertial monetary policy rules assist stability under learning dynamics. This is consistent with Bullard and Mitra (2007) which demonstrates in a model of learning where only one-period-ahead expectations matter — discussed in section 8— that conditions of proposition 1 are also necessary and sufficient for stability under learning dynamics. More generally, Preston (2008) demonstrates in a model closely related to this paper, that having nominal interest rate policy respond lagged aggregate data, such as the price level, improves stability under learning dynamics.

7 Implications for Communications Policy

This paper identifies two channels through which expectations can lead to macroeconomic instability. The first concerns the consequences of violations of Ricardian equivalence, paving the way for the size and composition of debt holdings to matter for macroeconomic dynamics. The second concerns the consequences of a specific component of agents beliefs — specifically the notion of unanchored financial expectations. Given that both generate instability through shifting expectations it is worthwhile contemplating whether communications policy might mitigate expectations-driven instability.

Under anchored financial expectations, the sole reason for debt-management policy being relevant is households failure to understand that the intertemporal fiscal accounts of government were solvent with certainty. Because of uncertainty about the future conduct of tax and monetary policy, holdings of the public debt are perceived as net wealth. This implies that in periods, such as those witnessed during the recent financial crisis, there may be advantages to clear communication about the future intended conduct of fiscal policy, and its consistency with intertemporal solvency of the government accounts.

Supposing such declarations are credible and successful, so that household believe the government accounts to be solvent in all future states of the world with absolute certainty and that households understand future policy satisfies the restriction $s_\tau \hat{\tau}_t = \delta \hat{s}_t + s_G \hat{G}_t$ in all periods, yields a consumption decision rule of the form:

$$\hat{C}_t = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[\bar{s}_C^{-1} (1-\beta) \left(\tilde{x}_T - s_G \hat{G}_t \right) - \beta \sigma^{-1} \left(\hat{\imath}_T - \hat{\pi}_{T+1} \right) + \sigma^{-1} \beta \left(\hat{\xi}_T - \hat{\xi}_{T+1} \right) \right].$$

It is immediate that this expression is independent of the scale and composition of the government debt. As a result, the conditions of proposition 2 for E-Stability apply to such an economy: the Taylor principle is necessary and sufficient for E-Stability and independent of debt-management policy. Of course, the details of fiscal policy still matter for out of equilibrium dynamics. Worth noting is that this same decision rule results in economies having zero government debt on average. Economies with "anchored fiscal expectations" and zero average debt are isomorphic in terms of expectations stabilization.

These observations might suggest the identified channel of expectations instability to be of limited importance. However, given the dramatic expansion in fiscal activities, and on-going concern about long-term fiscal sustainability, as reflected in President Obama's establishment of The National Commission on Fiscal Responsibility and Reform, it is not so unreasonable to suppose that households might incorrectly forecast tax and real interest obligations.

But even if the possibility of departures from Ricardian equivalence are not admitted, because it is thought that households surely understand the long-run implications of fiscal policy with absolute certainty even in currently highly uncertainty times, learning dynamics may still be relevant. Section 7 demonstrated that if financial expectations are unanchored, then expectational stability requires substantially more aggressive interest-rate policy. Moreover, the longer is the average maturity of debt, the more acute are the documented instability problems. An interesting property of the model of unanchored financial expectations is that the instability results are completely independent of the average level of indebtedness. They are therefore independent of whether fiscal expectations are anchored or not. Speculative asset trade along the yield curve may well be destabilizing, making short-term debt issuance more desirable. Interestingly, when only one-period debt is issued, the unanchored and anchored financial expectations economies are isomorphic. From an E-Stability perspective this nullifies instability arising from both channels discussed in this paper. In contrast, if only infinite maturity debt is issued, there would no instability from Keynesian expenditure effects, while those from unanchored financial expectations would be pervasive.

8 Alternative Models of Learning

Many recent papers have proposed analyses of learning dynamics in the context of models where agents solve infinite-horizon decision problems, but without requiring that agents make forecasts more than one period into the future. In these papers, agents' decisions depend only on forecasts of future variables that appear in Euler equations used to characterize rational expectations equilibrium. Important contributions include Bullard and Mitra (2002) and Evans and Honkapohja (2003). Of most relevance to the present study is Evans and Honkapohja (2007) which similarly studies the interaction of monetary and fiscal policy, but in a model of learning dynamics in which only one-period-ahead expectations matter to expenditure and pricing plans of households and firms. The following section replicates part of their analysis in the context of the model developed here, and contrasts the resulting findings with those of sections 5. and 6.

Since the optimal decision rules for households and firms presented in section 2 are valid under arbitrary assumptions on expectations formation, they are valid under the rational expectations assumption. Application of this assumption implies the law of iterated expectations to hold for the aggregate expectations operator and permits simplification of relations (10) and (11) in the paper to the following aggregate Euler equation and Phillips curve:[23]

$$\begin{aligned}\hat{C}_t &= E_t\hat{C}_{t+1} - \left(\hat{\imath}_t - E_t\hat{\pi}_{t+1} - \left(1-\rho_\zeta\right)\hat{\zeta}_t\right)\\ \hat{\pi}_t &= \psi\left(\hat{w}_t - \hat{Z}_t\right) + \beta E_t\hat{\pi}_{t+1}.\end{aligned}$$

Under learning dynamics, with only one-period-ahead expectations, it is assumed that aggregate demand and supply conditions are determined by

$$\hat{C}_t = \hat{E}_t\hat{C}_{t+1} - \left(\hat{\imath}_t - \hat{E}_t\hat{\pi}_{t+1} - \left(1-\rho_\zeta\right)\hat{\zeta}_t\right) \quad (46)$$

$$\hat{\pi}_t = \psi\left(\hat{w}_t - \hat{Z}_t\right) + \beta \hat{E}_t\hat{\pi}_{t+1}. \quad (47)$$

One other model equation depends on expectations: the no-arbitrage condition. Remaining within the spirit of the one-period expectation approach, proceed under the assumption that financial expectations are unanchored so that the restriction

$$\hat{\imath}_t = -\hat{E}_t\left(\hat{P}^m_t - \bar{\pi}^{-1}\rho\beta\hat{P}^m_{t+1}\right)$$

[23] See Preston (2005a, 2005b) for a detailed discussion.

determines the price of multiple-maturity debt, given expectations about this price next period, and current nominal interest rates. This replaces

$$\hat{P}_t^m = -\hat{E}_t \sum_{T=t}^{\infty} (\rho\beta)^{T-t}\, \hat{\imath}_T$$

which of course requires forecasts of nominal interest rates into the indefinite future. All remaining equations, being independent of expectations, are as before.

The model is closed with a description of beliefs. Under the reformulated model, wages, dividends, taxes and nominal interest rates need not be forecasted. However, spending decisions now depend on a forecast of next-period aggregate consumption. Assume, for analytical simplicity, that agent's beliefs are given by the model

$$\begin{bmatrix} \hat{C}_t \\ \hat{\pi}_t \\ \hat{b}_t \\ \hat{P}_t^m \end{bmatrix} = \Omega_{t,0} + \Omega_{t,1}\hat{b}_{t-1} + \Omega_{t,2} \begin{bmatrix} \hat{G}_{t-1} \\ \hat{Z}_{t-1} \\ \hat{\zeta}_{t-1} \end{bmatrix} + e_t$$

with appropriate redefinition of the dimensions of $\Omega_{t,0}$, $\Omega_{t,1}$ and $\Omega_{t,2}$.

It is immediate that such an approach represents a fundamentally different approach to decision making. Moreover, the approach is unable to study whether departures from Ricardian equivalence are important to macroeconomic dynamics, as taxes and debt holdings are irrelevant to economic decisions — and specifically the details of debt management policy have no consequences for household spending plans. As such, it fails to provide a fruitful framework for analyzing debt policy.

In the one-period-ahead forecasting model, defined by replacing relations (26), (27) and (21) with (46), (47) and (20) in the benchmark model, the following stability result obtains.

Proposition 3 *Assuming monetary policy is not inertial, necessary and sufficient conditions for expectational stability are*

$$\kappa\,(\phi_\pi - 1) + (1-\beta)\,\phi_y > 0$$

and

$$1 < \tau_l < \frac{1+\beta}{1-\beta}$$

where

$$\kappa = \left(\gamma + \sigma s_C^{-1}\right)(1-\alpha)(1-\alpha\beta)\,\alpha^{-1}.$$

This generalizes one specific result of the Evans and Honkapohja (2006) analysis to a model with nominal rigidities.[24] When only one-period-ahead expectations matter, the Taylor principle is a necessary and sufficient condition — given a Ricardian fiscal policy — to rule out expectations-driven instability. In contrast, in a model of optimal decisions, these conditions obtain only if the average maturity of debt is equal to unity. Absent this property, the analysis of this paper suggests a smaller menu of policies is consistent with expectations stabilization. Debt management policy can represent an important constraint on the design of monetary policy.

9 Conclusion

[TO BE ADDED]

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[24] The proof is available on request.

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