Technology diffusion and increasing income inequality

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Abstract

This paper considers the contribution of technology diffusion to income inequality via two channels: labor income and capital income. We propose a real business cycle type model in which ownership of productive capital is concentrated in the hands of a subset of agents (capital owners). Labor income of capital owners increases in capital stock and the share of economy using the new technology, and it decreases in the relative supply of capital owners' labor. Capital income is a positive function of capital share of total income and returns to capital. Our welfare analysis shows that capital owners largely benefit from technological change. Workers also gain from technological change but their gains are lower. From the welfare point of view, both channels are important. The labor income source, however, is essential as its elimination causes huge welfare losses to capital owners. The capital income channel does not seem to affect workers' welfare considerably while the labor channel does. When eliminated, the labor income channel generates additional welfare gains for workers.

Keywords: *Income Inequality, Technology Diffusion, Real Business Cycles.*

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1 Introduction

The US economy experienced a continuous increase in income inequality over the past three decades. During the same period, a major technological change took place. Information and Communication Technologies (henceforth ICT) spread into the economy. Macroeconomic literature provided the link between the two trends based on the so called skill-biased technological change[1]. Skill premium reflects the difference between the wages of skilled relative to unskilled workers. The increase in skill premium observed during the last three decades is a result of capital-skill complementarity enhanced by ICT (see Krussel et al. 2000). Via both, higher productivity and higher skilled labor demand, capital-skill complementarily boosted skilled workers' wages. Higher labor income in turn led to higher total income inequality between skilled and unskilled workers.

While skill-premium has been the major cause of raise in income inequality over the last three decades, recent empirical literature demonstrates that other income components also contributed to the observed trend. More specifically, Atkinson et al. (2011) show that, in addition to labor income, capital income has also significantly contributed to the increase in income inequality. This is due to the fact that, in addition to the skill-capital complementarity effect, the ITC has a direct effect on capital intensity. In fact, new technology enhances capital intensity and thus increases returns to capital. As a result, capital owners' (investors') income from capital rises.

In this paper, we model this additional channel of technology diffusion. The proposed framework encompasses both capital-skill complementarity (labor income) channel and capital income channel. The framework for our analysis is a real business cycle model in which ownership of productive capital is concentrated in the hands of a subset of agents (capital owners). The remaining fraction of agents (workers) provides labor.

The observed increase in capital share of income is modeled by increase in capital intensity along with technology diffusion. Capital owners are entrepreneurs and they exhibit entrepreneurial skill-complementarity (enhanced by new technology) similar to capital-skill complementarily proposed by Krussel et al. (2000).

The economy is assumed to undergo technology diffusion process. A standard way of modeling technology in RBC framework is to introduce random shocks affecting the economy growth trend. We model technological change in a novel way, motivated by microeconomics literature. In fact, empirical studies, as Comin et al. (2008), show that technology spreads into economy in an S-shaped pattern. We model technology diffusion accordingly, as the number of households owing a Personal Computer (PC)

[1] Numerous studies show how technological change can explain increase in income inequality. See for instance Krussel et al. (2000), Acemoglu (2002), He and Liu (2008).

as a share of the total population (in the age of using a computer).[2]

As new technology involves economic uncertainty, we introduce expectations formation that is consistent with such an environment. First, we assume that agents in economy are endowed with perfect foresight. Second, we assume that agents have backward-looking expectations. This means that agents use past data to form their expectations about the future. This is a sort of myopic, random walk expectations that has been also used by Heathcote et al. (2010). Although, this way of forming expectations seems more realistic in an uncertainty environment as technological shift, backward-looking expectations are still not satisfying. This is the case because the degree of uncertainty about the future economic outcomes decreases as the share of economy employing new technology increases. We introduce the third case, so called hybrid expectations which approximate the mechanism of learning about new technology.

The proposed model provides a set of theoretical insights. In line with the mechanism advanced by Krussel et al. (2000), the entrepreneurial-skill premium increases in capital stock and decreases in entrepreneurial labor supply. In addition, this premium goes up as a larger share of economy uses new technology. Capital income of entrepreneurs increases as capital intensity improves along with technology diffusion process.

The importance of both channels in the recent income inequality increase and their welfare implications are examined via quantitative analysis which proceeds in three main parts. First, we carry out a benchmark calibration of the proposed model. Second, we simulate the calibrated model and confront its predictions with the data. Third, we use our model to carry out welfare analysis.

The model is calibrated to match the observed increase in both: capital's share of income and top decile income share. Next, we simulate the model and confront its predictions with the data. We find that the proposed framework produces quantities that match well the evolution of macroeconomic variables of interest: wage, labor supply and consumption ratios.

We use this framework to carry out welfare analysis. In the benchmark case, we find that capital owners largely benefit from the technological change. Their consumption boosts as a result of increase in income from labor and capital. The benchmark model simulation indicates that workers gain from technological change as well, although their gains are lower than the ones of entrepreneurs.

The way the agents form their expectations matter for the welfare analysis. Under perfect foresight, the welfare gains are the highest for capital owners and the lowest for workers. In case of backward looking expectations, the opposite holds.

Elimination of one of the income channels at the time allows us to understand the

[2] Similarly, one could compute a PC diffusion path for the firms.

importance and the workings of each of them. First, we remove the capital income channel by assuming a slow, long-run equilibrium growth in capital stock. This counterfactual exercise demonstrates that the capital income channel principally benefits capital owners. Its elimination considerably deteriorates capital owners' welfare while hardly affects the one of workers.

The effect of labor income channel is examined via Cobb-Douglas production technology which eliminates capital-entrepreneurial skill complementarity effect. We show that labor income channel has a major impact on welfare of both groups of agents due to the feedback from labor to capital market. In the case of benchmark model, higher entrepreneurial labor supply increases output and capital returns which in turn boost further investment. The increase in total income of capital owners translates into welfare gains. Workers' welfare also improves due to improved wages and stronger labor supply.

The remainder of the paper is organized as follows. The second section exposes a set of stylized fact related to the increase in income inequality in the US economy, during the past thirty years. The third section describes the proposed model and its solution. In the fourth section, we introduce quantitative analysis including the baseline calibration and numerous simulations of the model. In the fifth section, we carry out welfare analyses and the sixth section concludes.

2 Stylized facts

In this section, we describe the evolution of the macroeconomic quantities related to the increase in income inequality in the US economy, during the past thirty years. The model will be calibrated to match some of these quantities and we will verify its validity through the analysis of the others.

We focus here on two groups of population. In the model, we call them capital owners and workers. For simplicity, we assume that capital owners, who represent 10% of the whole population, hold the entire capital of the economy. This assumption is not unrealistic as the top 10 % richest hold roughly 80% of the US total wealth (see Wolff, 2006). The remaining fraction of the population, called workers, is assumed to participate solely to the labor market. The model aims at studying inequality between both groups.

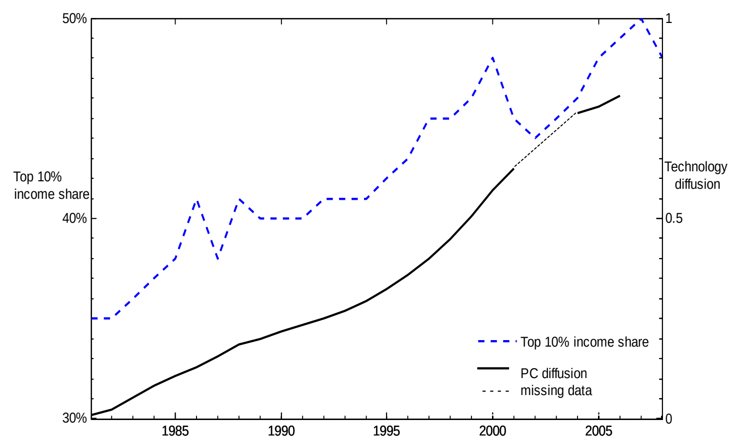
Inequality is usually measured empirically as the ratio between the 90th and 10th percentile. In this paper, we calibrate the model to the aggregate economy and thus we need an index that encompasses the entire population. The inequality is therefore measured by the ratio between the top 10% and the remaining fraction of the economy. Within the model it corresponds to the ratio between the capital owners to the workers. This measure of inequality is similar to the one used by Atkinson et al. (2011).

Some of the survey based macroeconomic data, as consumption inequality for instance, are not directly available to us and are subject to much criticism (see Aguiar and Bils, 2011). In these cases, we will use the findings reported in other, more empirically focused studies and we will treat them with caution.

2.1 Income inequality and ICT diffusion

The US economy experienced a continuous increase in income inequality over the past three decades.[3] During the same period, the US economy underwent a major technological change. Information and Communication Technologies (henceforth ICT) spread into the economy.

Figure 1: PC diffusion in the US and the top decile income share between 1980 and 2008



The figure plots top decile income share and technology diffusion curve approximated by a share of households owing a computer in the US between 1980 and 2008 in the US. 2002 and 2003 data points are missing for the PC diffusion measure. Income share data is provided by Atkinson et al. (2011) and technology diffusion comes from NBER: Cross-country Historical Adoption of Technology (CHAT) available on the NBER website: http://www.nber.org/data/chat/

Figure 1 jointly demonstrates these two trends. First, the dashed line with the corresponding left y-axis shows the evolution of the top decile income share in the US economy. This share increased from roughly 35% in 1980 to 50% in 2008. The

[3] See Atkinson, Piketty and Saez (2011) for empirical evidence on income inequality increase in the US.

solid line and its corresponding right y-axis plots the technology diffusion curve. This curve is approximated by the number of households owing a PC as a share of the total population (in the age of using a computer).

Empirical evidence suggests that technology spreads according to S-shaped curve (see Comin at. al). Figure 1 shows that in the beginning of the 1980s, only a small fraction of population used the PCs and this share increased rapidly during the 1990s and 2000s. The data sample ends in 2007 and suggests that the ICT diffusion is still in progress as the fraction of the population that adopted it is still below 1. In fact, technology is perceived as entirely diffused into an economy, when the fraction of its users reaches a long run level of 1.

The government responded to the increase in income inequality by raising personal transfer receipts. They are payments by governments and businesses to individuals and nonprofit institutions serving individuals. Figure 2 shows their evolution during the last forty years. Transfer receipts as a share of GDP accounted for 10% in 1980 and almost 16% of total personal income at the national level in 2010.

Figure 2: Ratio of personal current transfer receipts to GDP in the US between 1970 and 2009

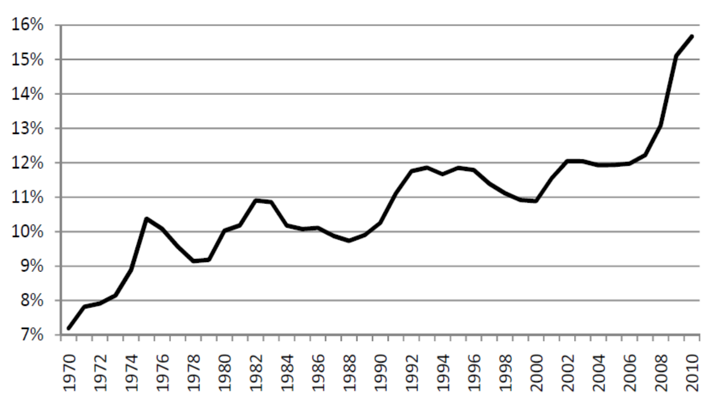
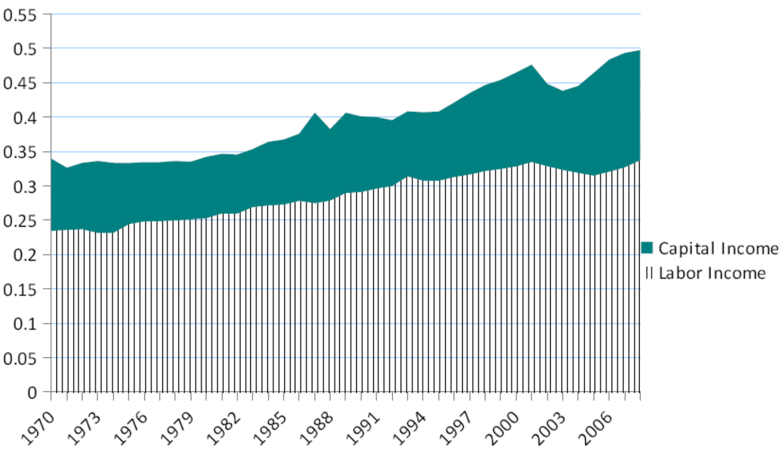


Figure plots ratio of personal current transfer receipts to GDP defined as payments by governments and businesses to individuals and nonprofit institutions.

2.2 Channels of income inequality increase

A clear message from Figure 1 is that the income inequality increased along with the diffusion of ICT. Technology influence on income inequality operates trough two main channels. First, technological progress increases productivity and wages of skilled workers by so called skill premium. Second, it raises capital intensity of aggregate production process and thus capital share of income. The US data shows that both of these effects are primarily beneficial to a small fraction of population.

Figure 3: Top decile income share and its composition between 1970 and 2008



The solid area shows capital share of income. Stripped area shows labor share of income. Income share data is described in Atkinson et al. (2011) and can be found at http://184.168.89.58/sketch/#PAGE_THE_DATABASE

Figure 3 plots top decile income share as in Figure 1 and its composition. The striped area of the figure corresponds to the labor income and demonstrates that its share increased during the past thirty years from 25% to 33%. The adoption of ICT was accompanied by stronger demand for skilled labor. In response to this demand, the skilled labor supply increased as well by around 30 %.

The solid area shows capital share of income which also increased during this period. It represented 8.9% of income of the top decile in 1980 and 16% in 2008. In this paper, we use an aggregate macroeconomic model and therefore we calibrate it to the capital share of income of the entire economy. As the top 10 % richest own roughly 80% of the financial wealth (see Wolff 2006), the capital share of income in the economy as a whole raised along with the one of the top decile. This increase is demonstrated in Figure 4.

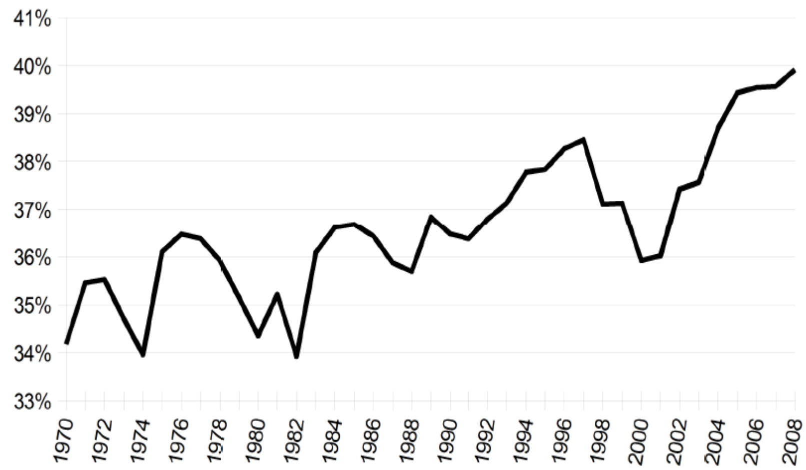
Capital share of total US income increased from roughly 34% in 1980 to 40% in 2008.

2.3 Summary statistics

Table 1 summarizes the stylized facts we described in this section.

The upper panel of the table shows the changes in ratios described earlier and that will be used to calibrate the model. These are the top decile income share increase,

Figure 4: Capital share of total US income between 1970 and 2008



The data is from Bureau of Economic Analysis/Haver Analytics

capital share of income raise, and personal transfer receipts. The lower panel of the table shows the changes in ratios observed in the data. The first one corresponds to the increase in consumption inequality. The consumption data categorized according to the income classes are only available through the interviews collected in Consumer Expenditure Survey's (CES). Analyzes based on these survey data argue that consumption inequality increased much slower than income inequality. Krueger and Perri (2006), for instance, claim that while there was an increase in income inequality of 30%, the consumption inequality increased only moderately, by 17%. Other authors claim, however, that the survey based CES data are subject to measurement errors. Several studies attempted to correct for these measurement errors and found higher consumption inequality (see Aguiar and Bils 2011, Attanasio et al. 2005 and Parker et al. 2009). For our calibration exercise, we chose as a benchmark the study by Aguiar and Bils (2011) who calculate the figures based on the most recent data. They find that consumption inequality mirrors income inequality. In the sensitivity analysis, we will also simulate the model generating the lower consumption inequality increase as in Krueger and Perri (2006).

The second row of the lower panel of Table 1 reports an increase in the ratio of hours worked, as found in Heathcote et al. (2010). Finally, the last row of the table displays the increase in wages ratio which was roughly 30% during the analyzed period.

Table 1: Stylized facts

Income shares		
	1980-2007	
Top 10% Income share	36% → 49%	
Capital share of income	34% → 39%	
Personal transfer receipts	10% → 16%	
Macroeconomic outcomes		
Wages ratio	45%	Authors' calculations
Hours ratio	40%	Heathcote (2011)
Consumption ratio	17% − 30%	Aguiar and Bils (2011)

3 Model

The model consists of workers, capital owners, and competitive firms. The firms are owned by the capital owners. There are n times more workers than capital owners, with the total number of capital owners normalized to one. Workers and capital owners both supply labor to the firms.

3.1 Workers

The workers have preferences as proposed by Greenwood, Hercowitz, and Huffman (1988) and they maximize their expected lifetime utility of consumption

$$\max_{c_t^w} E_0 \sum_{t=0}^{\infty} \beta^t \frac{\left[c_t^w - \frac{D^w}{\gamma^w} H_t \left(\ell_t^w\right)^{\gamma^w}\right]^{1-\alpha} - 1}{1-\alpha},$$

where E_t represents the mathematical expectation operator, β is the subjective time discount factor, c_t^w is the individual worker's consumption, ℓ_t^w is individual workers' labor supply. The disutilty of non-leisure time is governed by the functional form $(D^w/\gamma^w)\ H_t \left(\ell_t^w\right)^{\gamma^w}$, where $D^w > 0$, and $\gamma^w > 0$. This specification implies that foregone leisure is adjusted to reflect trend growth according to $H_t = \exp(\mu\, t)$, which may be interpreted as the reduced form of a more-elaborate specification that incorporates home production.[4] As $\gamma^w \to \infty$, the model reduces to one with inelastic worker labor supply. The intertemporal elasticity of substitution in labor supply for workers is given by $1/\left(\gamma^w - 1\right)$.

[4]The linearity in H_t ensures that workers' time allocations are stationary along the model's balanced growth path. See Greenwood, Rogerson, and Wright (1995, p. 161).

Workers are assumed to incur a transaction cost for saving or borrowing small amounts which prohibits their participation in financial markets. As a result, they simply consume their labor income each period augmented by a lump sum transfer

$$c_t^w \ = w_t^w \ell_t^w + T_t,$$

where c_t^w is the individual worker's consumption, w_t^w is the competitive workers' wage, ℓ_t^w is workers' labor supply and T_t, is a lump sum transfer. In the data, these personal transfer receipts are benefits paid by governments and businesses to individuals and nonprofit institutions. For modeling simplicity, we assume that they correspond to an amount that a government would obtain from businesses (capital owners) and grant to the individual workers each period. Therefore, they will also affect capital owners disposable income.

3.2 Capital Owners

Capital owners are entrepreneurs and own the firms which pay out dividends. They employ workers and supply entrepreneurial labor to their own firms as well. Their decision problem is to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{\left[c_t^c - \frac{D^c}{\gamma^c} H_t \left(\ell_t^c\right)^{\gamma^c}\right]^{1-\alpha} - 1}{1-\alpha}, \tag{1}$$

subject to the budget constraint

$$c_t^c \ + \ i_t \ + w_t^c \ell_t^c = \ y_t + w_t^c \ell_t^c - n\, w_t^w \, \ell_t^w - nT_t,$$

$$c_t^c \ + \ i_t \ = \ y_t - n\, w_t^w \, \ell_t^w - nT_t, \tag{2}$$

Capital owners' derive income from labor $w_t^c \, \ell_t^c$ and from capital $r_t k_t$. They need to pay workers' wage bill $n\, w_t^w \, \ell_t^w$ and transfers to the individual workers nT_t. Their own wage bill $w_t^c \ell_t^c$ is an expenditure and an income at the same time so it cancels out in their budget constraint to give (2).

The Lagrangian function and the corresponding first order conditions of capital owners are

$$L = \sum_{t=0}^{\infty} \beta^t \frac{\left[c_t^c - \frac{D^c}{\gamma^c} H_t \left(\ell_t^c\right)^{\gamma^c}\right]^{1-\alpha} - 1}{1-\alpha} + \varphi \left[y_t + w_t^c \ell_t^c - n\, w_t^w\, \ell_t^w - nT_t\right], \quad (3)$$

$$\frac{\partial L}{\partial k_{t+1}} = 0 \Longrightarrow \frac{i_t}{y_t} = \beta E_t \frac{\varphi_{t+1}}{\varphi_t} \frac{y_{t+1}}{y_t} \left[\lambda s_{t+1}^k + (1-\lambda) \frac{i_{t+1}}{y_{t+1}}\right], \quad (4)$$

$$\frac{\partial L}{\partial \ell_t^c} = 0 \Longrightarrow \varphi = y_t^{-\alpha} \left[\frac{c_t^c - \frac{D^c}{\gamma^c} H_t \left(\ell_t^c\right)^{\gamma_c - 1} \ell_t^c}{y_t}\right]^{-\alpha} \quad (5)$$

where (3) is the Lagrangian function with Lagrangian multiplier, φ. (4) describes the first order condition with respect to capital which has a forward looking form. (5) is the first order condition of capital owners with respect to entrepreneurial labor supply.

3.3 Technology diffusion

The firm's output is produced according to the technology

$$y_t = A \left\{ \theta_t \left[(1-\rho_t)\, k_t^{\psi_k} + \rho_t \left[\ell_t^c \exp(z_t)\right]^{\psi_k} \right]^{\frac{\psi_\ell}{\psi_k}} + (1-\theta_t) \left[n \ell_t^w \exp(z_t)\right]^{\psi l} \right\}^{\frac{1}{\psi_\ell}} \quad (6)$$

$$\psi_k \equiv \frac{\sigma_k - 1}{\sigma_k}, \ \psi_l \equiv \frac{\sigma_l - 1}{\sigma_l},$$

$$z_t = z_{t-1} + \mu + \varepsilon_t, \qquad \varepsilon_t \sim N\left(0, \sigma_\varepsilon^2\right), \quad (7)$$

with z_0 given. The symbol k_t is the firm's stock of physical capital and z_t is a labor-augmenting "productivity shock" that evolves as a random walk with drift. The drift parameter μ determines the trend growth rate of output. The total labor input is given by $\ell_t^c + n\, \ell_t^w$.

The parameters ψ_k and ψ_l depend on the elasticity of substitution between capital and workers' labor, σ_k, and the elasticity of substitution between capital and entrepreneurial labor, σ_l, respectively. By assuming that $\sigma_\ell > \sigma_k$, we introduce capital-skill complementarity as in Krussel et al (2000). This means that we suppose that the elasticity of substitution between physical capital and workers' labor is higher than the one between capital and entrepreneurial labor. Put differently, capital owners' and physical capital are complementary. Since capital owners own the firms, naturally their skills are coupled with the installed capital. One should think here of the examples of entrepreneurs who emerged thanks to their ICT-enhanced skills, as for instance, Bill Gates or Steve Jobs. Their skill is somewhat different from the one proposed by Krussel et al. (2000). We call it simply entrepreneurial skill and production function in (6) with $\sigma_\ell > \sigma_k$ generates capital-entrepreneurial skill complementarity.

When $\sigma_k = \sigma_l = 1$ (or $\psi_k = \psi_l = 0$), we recover the usual Cobb-Douglas production technology where there is no capital-entrepreneurial skill complementarity. When $\sigma_k \to 0$ and $\sigma_l \to 0$ (or $\psi_k \to -\infty$ and $\psi_l \to -\infty$), the production technology takes a Leontief form such that capital and both types of labor become perfect compliments. When $\sigma_k \to \infty$ and $\sigma_l \to \infty$ (or $\psi_k \to 1$ and $\psi_l \to 1$), capital and both types of labor become perfect substitutes.

The marginal products of production factors are as follows

$$w_t^w = MPL^w = \frac{y_t\left(1 - s_t^k\right)}{n\ell_t^w}, \tag{8}$$

$$w_t^c = MPL^c = \frac{y_t\left(s_t^c - s_t^k\right)}{\ell_t^c}, \tag{9}$$

$$r_t = MPK = \frac{s_t^k y_t}{k_t}, \tag{10}$$

Workers' wage w_t^w, capital owners' wage w_t^c and return on capital r_t are equal to their respective marginal products. The symbol s_t^k is used to represent capital's share of total income (or output) and $\left(1 - s_t^k\right)$ represents labor's share. s_t^c is capital owners' share of income and $(1 - s_t^c)$ corresponds to workers' share of income. Note that the model accounts for two sources of income for capital owners. In addition to labor, capital owners' income is generated by capital. Their pre-tax share of income can be decomposed into two sources: $s_t^c = \frac{r_t k_t}{y_t} + \frac{w_t^c \ell_t^c}{y_t}$, capital income $r_t k_t$ and labor income $w_t^c \ell_t^c$ where $s_t^c > s_t^k = \frac{r_t k_t}{y_t}$. Pre-tax income of workers is derived only from one source, labor income $\frac{w_t^w \ell_t^w}{y_t} = s_t^c - s_t^k$[5].

Resources devoted to investment augment the firm's stock of physical capital according to the law of motion

$$k_{t+1} = B\, k_t^{1-\lambda}\, i_t^{\lambda}, \qquad \begin{matrix} B > 0 \\ \lambda \in (0,1) \end{matrix} \tag{11}$$

with k_0 given and λ being adjustment cost parameter.

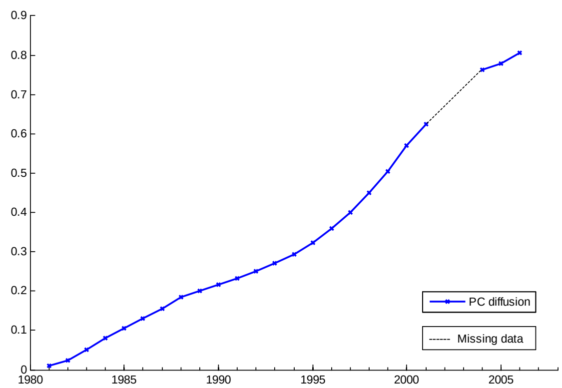
Technology diffuses through the economy via shifting production function in (6). A standard way of modeling technology in RBC framework is to introduce random shocks affecting the economy growth trend as in (7). Although we also assume that the economy follows a long run growth trend defined by μ, we model technological change in a novel way[6]. In fact, empirical studies, as Comin et al. (2008), show that

[5] We calculate here the shares of income before taxes to be consistent with the calibration which matches the pre-tax data.

[6] In the calibration we set $\varepsilon_t = 0$ so that we have $z_t = z_{t-1} + \mu$.

technology spreads into economy in an S-shaped pattern as demonstrated in Figure 5. We model technology diffusion accordingly.

Figure 5: PC diffusion in the US between 1980 and 2007



The figure plots technology diffusion curve approximated by a share of households owing a computer in the US between 1980 and 2007. 2002 and 2003 data points are missing for the US. The data is provided by NBER: Cross-country Historical Adoption of Technology (CHAT) available on the NBER website: http://www.nber.org/data/chat/

Figure 5 shows an example of the adoption curve of Personal Computers (PC) in the US. The figure displays a number of households owing a computer as a share of the total population (in the age of using a computer). The diffusion process is modeled as

$$\pi_t = \pi_{t-1} + \kappa \pi_{t-1} (1 - \pi_{t-1}) \tag{12}$$

where π_0 is given. π_t stands for a share of economy that employs new technology and κ defines the speed of diffusion. Empirically, the technology diffusion process is usually modeled as logistic function (see for instance Comin et al. 2008). (12) is more parsimonious as it has only one free parameter, speed of diffusion, κ.

Technology shifts several parameters of the production function (6). First, new technology enhances marginal product of capital and capital intensity follows a low of motion

$$\theta_t = \theta_1 \exp (\delta_\theta \pi_t), \tag{13}$$

where δ_θ stands for growth in capital intensity.

Second, marginal productivity of entrepreneurial labor and thus its income share is also a function of the share of economy using new technology and it follows a low of motion

$$\rho_t = \rho_1 \exp\left(\delta_\rho \pi_t\right), \tag{14}$$

where δ_ρ corresponds to the growth in marginal product of entrepreneurial labor. At the initial steady state, where the economy uses only old technology, $\pi_t = \pi_0 = 0$, capital intensity and marginal product of entrepreneurial labor equal the ones of the old technology, $\theta_t = \bar{\theta}_1$ and $\rho_t = \bar{\rho}_1$. At the new steady state, when only new technology is employed, $\pi_t = \pi_1 = 1$, capital intensity and share of entrepreneurial labor correspond to the ones of new technology, $\theta_t = \bar{\theta}_2$ and $\rho_t = \bar{\rho}_2$.[7]

3.4 Solution of the model

To facilitate a solution for the equilibrium allocations, the first-order condition (4) must be rewritten in terms of stationary variables. Because labor supply is elastic in this model, the combined entity of the firm and capital owner must take three decisions. It has to choose the amount of labor it wants to supply and it needs from workers. It has to decide the fraction of available income to be devoted to investment, with the remaining fraction devoted to consumption. In the framework of GHH preferences these two problems can be solved separately. First, the amount of optimal workers' and entrepreneurial labor supplies can be recovered from (8) and (9). Second, they can be used in first order condition (4) to specify the decision rule on consumption and investment allocations.

If we define the investment-consumption ratio as $x_t \equiv i_t/y_t$, then the economy's resource constraint $y_t = c_t^c + i_t + n\, w_t^w\, \ell_t^w + nT_t$ can be used to derive the following expressions for the equilibrium allocation ratios

$$\frac{c_t^c}{y_t} = s_t^c - x_t - \tau_t,\ \tau_t = \frac{nT_t}{y_t} \tag{15}$$

$$\tau_t = \bar{\tau}_1 \exp(\delta_\tau \pi_t), \tag{16}$$

$$\frac{c_t^w}{y_t} = \frac{1 - s_t^c}{n} + \frac{\tau_t}{n}, \tag{17}$$

where s_t^c is the capital owners' share of total income, given below. The personal receipts-income ratio τ_t follows the low of motion in (16) which is also an increasing function of technology diffusion share π_t. This is in line with the empirical evidence

[7] As the values of θ_t and ρ_t are not observable, we will calibrate them to the capital share of income and top decile share of income in the US economy.

that as ICT spread into economy, the income inequality increased, and as illustrated in Figure 2, government raised individual transfers.

Defining the normalized capital stock as $k_{n,t} \equiv k_t / \exp(z_t)$, and using equation (11), the law of motion for $k_{n,t}$ is

$$k_{n,t+1} = k_{n,t}^{1-\lambda} B A^{\lambda} x_t^{\lambda} \exp(-\mu - \varepsilon_{t+1}) \left\{ \theta_t \left[(1-\rho_t) k_{n,t}^{\psi_k} + \rho_t \left(\ell_t^c\right)^{\psi_k} \right]^{\frac{\psi_\ell}{\psi_k}} + (1-\theta_t) \left(n\, \ell_t^w\right)^{\psi_\ell} \right\}^{\frac{\lambda}{\psi_k}}, \tag{18}$$

where we have made use of $z_t - z_{t+1} = -\mu - \varepsilon_{t+1}$.

We can also derive the following expressions

$$\frac{y_t}{k_t} = A k_{n.t}^{-1} \left\{ \theta_t \left[(1-\rho_t) k_{n,t}^{\psi_k} + \rho_t \left(\ell_t^c\right)^{\psi_k} \right]^{\frac{\psi_\ell}{\psi_k}} + (1-\theta_t) \left(n \ell_t^w\right)^{\psi_l} \right\}^{\frac{1}{\psi_l}} \tag{19}$$

$$\ell_t^c = \left[\frac{\frac{A}{D^c} \theta_t \rho_t \left[(1-\rho_t) k_{n,t}^{\psi_k} + \rho_t \left(\ell_t^c\right)^{\psi_k} \right]^{\frac{\psi_l - \psi_k}{\psi_k}}}{\left\{ \theta_t \left[(1-\rho_t) k_{n,t}^{\psi_k} + \rho_t \left(\ell_t^c\right)^{\psi_k} \right]^{\frac{\psi_\ell}{\psi_k}} + (1-\theta_t) \left(n\, \ell_t^w\right)^{\psi_\ell} \right\}^{\frac{\psi_l - 1}{\psi_l}}} \right]^{\frac{1}{\gamma^c - \psi_k}} \tag{20}$$

$$\ell_t^w = \left[\frac{\frac{A}{D^w} \left[(1-\theta_t) n^{\psi_l - 1} \right]^{\frac{\psi_l - \psi_k}{\psi_k}}}{\left\{ \theta_t \left[(1-\rho_t) k_{n,t}^{\psi_k} + \rho_t \left(\ell_t^c\right)^{\psi_k} \right]^{\frac{\psi_\ell}{\psi_k}} + (1-\theta_t) \left(n\, \ell_t^w\right)^{\psi_\ell} \right\}^{\frac{\psi_l - 1}{\psi_l}}} \right]^{\frac{1}{\gamma^w - \psi_l}} \tag{21}$$

where (19) is the ratio of output to capital stock. (20) is a non-linear equation describing optimal entrepreneurial labor supply. Note that ℓ_t^c depends on two state variables: capital stock $k_{n,t}$, technology diffusion stage π_t, and decision variable, workers' labor supply ℓ_t^w. Workers' labor supply defined by (21) is a non-linear function of state variables: $k_{n,t}$ and π_t and decision variable ℓ_t^c. We can jointly solve for optimal labor supplies ℓ_t^c and ℓ_t^w from (20) and (21) depending on π_t in (12) and $k_{n,t}$ in (18).

We also derive the shares of income depending on the state and decision variables

$$s_t^k = \frac{r_t k_t}{y_t} = \frac{\theta_t\ (1-\rho_t)\left[(1-\rho_t)\,k_{n,t}^{\psi_k} + \rho_t\ (\ell_t^c)^{\psi_k}\right]^{\frac{\psi_l-\psi_k}{\psi_k}} k_{n,t}^{\psi_k}}{\theta_t\ \left[(1-\rho_t)\,k_{n,t}^{\psi_k} + \rho_t\ (\ell_t^c)^{\psi_k}\right]^{\frac{\psi_\ell}{\psi_k}} + (1-\theta_t)\,(n\,\ell_t^w)^{\psi_\ell}} \tag{22}$$

$$s_t^c = \frac{\theta_t\ \left[(1-\rho_t)\,k_{n,t}^{\psi_k} + \rho_t\ (\ell_t^c)^{\psi_k}\right]^{\frac{\psi_\ell}{\psi_k}}}{\theta_t\ \left[(1-\rho_t)\,k_{n,t}^{\psi_k} + \rho_t\ (\ell_t^c)^{\psi_k}\right]^{\frac{\psi_\ell}{\psi_k}} + (1-\theta_t)\,(n\,\ell_t^w)^{\psi_\ell}} \tag{23}$$

(22) and (23) define capital owners and capital shares of income. Those depend on two state variables: $k_{n,t}$, and π_t, and labor supplies ℓ_t^c and ℓ_t^w. When $\sigma_l = \sigma_k = 1$ (or $\psi_k = \psi_l = 0$), we have the Cobb-Douglas case where $s_t^k = \theta_t$.

Using (5) and definitions of stationary variable, $x_t \equiv i_t/y_t$, the first-order condition (4) can be rewritten as follows

$$x_t = \beta E_t \left[\frac{\frac{\gamma_c - 1}{\gamma_c} s_{t+1}^c - (x_{t+1} + \tau_{t+1}) + \frac{s_{t+1}^k}{\gamma_c}}{\frac{\gamma_c - 1}{\gamma_c} s_t^c - (x_t + \tau_t) + \frac{s_t^k}{\gamma_c}}\right]^{-\alpha} \left[\frac{y_{t+1}}{y_t}\right]^{\phi} [\lambda s_{t+1}^k + (1-\lambda)\,x_{t+1}], \tag{24}$$

with $\phi \equiv 1 - \alpha$. An expression for the output growth in terms of stationary variables can be obtained from production function (6)

$$\frac{y_{t+1}}{y_t} = \exp(\mu + \varepsilon_{t+1}) \left\{\frac{\theta_{t+1}\ \left[(1-\rho_t)\,k_{n,t+1}^{\psi_k} + \rho_{t+1}\ (\ell_t^c)^{\psi_k}\right]^{\frac{\psi_\ell}{\psi_k}} + (1-\theta_{t+1})\,(n\ell_t^w)^{\psi_l}}{\theta_t\ \left[(1-\rho_t)\,k_{n,t}^{\psi_k} + \rho_t\ (\ell_t^c)^{\psi_k}\right]^{\frac{\psi_\ell}{\psi_k}} + (1-\theta_t)\,(n\ell_t^w)^{\psi_l}}\right\}^{\frac{1}{\psi_l}}, \tag{25}$$

Substituting (25) into equation (24) yields the following transformed version of the first-order condition in terms of stationary variables

$$\frac{x_t \left\{\theta_t\ \left[(1-\rho_t)\,k_{n,t}^{\psi_k} + \rho_t\ (\ell_t^c)^{\psi_k}\right]^{\frac{\psi_\ell}{\psi_k}} + (1-\theta_t)\,(n\,\ell_t^w)^{\psi_\ell}\right\}^{\frac{\phi}{\psi_k}}}{\left[\frac{\gamma_c - 1}{\gamma_c} s_t^c - (x_t + \tau_t) + \frac{s_t^k}{\gamma_c}\right]^{\alpha}} = \tag{26}$$

$$= \beta E_t \frac{\exp(\phi\mu + \phi\varepsilon_{t+1}) \left\{\theta_{t+1}\ \left[(1-\rho_t)\,k_{n,t+1}^{\psi_k} + \rho_{t+1}\ (\ell_t^c)^{\psi_k}\right]^{\frac{\psi_\ell}{\psi_k}} + (1-\theta_{t+1})\,(n\ell_t^w)^{\psi_l}\right\}^{\frac{\phi}{\psi_k}}}{\left[\frac{\gamma_c - 1}{\gamma_c} s_{t+1}^c - (x_{t+1} + \tau_{t+1}) + \frac{s_{t+1}^k}{\gamma_c}\right]^{\alpha}},$$

(26) includes three decision variables x_t, ℓ_t^c and $\ell_t^w$and two state variables, $k_{n,t}$ and π_t, with corresponding laws of motion given by equations (18) and (12) and optimal

labor supply allocations described by (20) and (21). (26) is a function of the state and decision variables $\mathrm{f}(x_t,\, \ell_t^c, \ell_t^w, k_{n,t},\, \pi_t) \;=\; E_t\, \mathrm{h}\big(x_{t+1},\, \ell_{t+1}^c, \ell_{t+1}^w, k_{n,t+1},\, \pi_{t+1}\big)$. We use numerical methods to solve for transition dynamics along the technology diffusion process. More precisely, we find decision variable x_t at each period t so that $\mathrm{f}(\cdot) - E_t\, \mathrm{h}(\cdot) = 0$, as a function of the state variables $k_{n,t},\, \pi_t,\, k_{n,t+1},\, \pi_{t+1}$.

3.5 Expectations

The first order condition in (26) is a forward looking equation which includes the expectations operator E_t. First, we assume that agents in economy are endowed with perfect foresight so that E_th(t+1)=h(t+1), where h(t+1)=$\mathrm{h}\big(x_{t+1},\, \ell_{t+1}^c, \ell_{t+1}^w, k_{n,t+1},\, \pi_{t+1}\big)$. However, in reality, new technology involves economic uncertainty and agents' foresight is not perfect. Therefore, we assume the second case, i.e. agents have backward looking expectations $\hat{E}_t$h(t+1)=h(t-1), with h(t-1)=$\mathrm{h}\big(x_{t-1},\, \ell_{t-1}^c, \ell_{t-1}^w, k_{n,t-1},\, \pi_{t-1}\big)$. This means that agents use past data to form their expectations about the future. This is a sort of naive, random walk expectations.

Although, this way of forming expectations seems more realistic in an uncertainty environment as technological shift, backward-looking expectations are still not satisfying. This is the case because the degree of uncertainty about the future economic outcomes decreases as the share of economy employing new technology increases. Put differently, the firms that employ the new technology know more about its speed of diffusion and its return. We introduce the third case, so called hybrid expectations which approximate the mechanism of learning about new technology. These expectations are formed as follows

$$\tilde{E}_t \mathrm{h}(\mathrm{t}+1) = \omega_t^f \underbrace{\mathrm{h}^*(\mathrm{t}+1)}_{\substack{\text{perfect} \\ \text{foresight}}} + \left(1 - \omega_t^f\right) \underbrace{\mathrm{h}(\mathrm{t}-1)}_{\substack{\text{backward-} \\ \text{looking}}} \tag{27}$$

were ω_t^f denotes weight given to the perfect foresight, h*(t+1)=$\mathrm{h}\big(x_{t+1},\, \ell_{t+1}^c, \ell_{t+1}^w, k_{n,t+1},\, \pi_{t+1}\big)$ and h(t-1) is defined as before. Hybrid expectations denoted by operator $\tilde{E}_t$ in (27) are the weighted average of the perfect foresight $\mathrm{h}^*(\mathrm{t}+1)$ and the backward looking expectations $\mathrm{h}(\mathrm{t}-1)$. As the new technology spreads into economy, the weight attributed to the perfect foresight, ω_t^f, increases; $\omega_t^f = \pi_t$. Note also that the perfect foresight expectations function changes $\mathrm{h}^*(\mathrm{t}+1) \neq \mathrm{h}(\mathrm{t}+1)$. This is the case because, the dynamics of economy are affected by agents using backward looking expectations and the perfect foresight needs to account for it.

3.6 Steady state

This model has two steady states. The first steady state corresponds to the equilibrium where economy uses old technology only: $\pi_t = \pi_0 = 0$. In this steady state, capital intensity and marginal product of entrepreneurial labor equal the ones of the old technology, $\theta_t = \bar{\theta}_1$ and $\rho_t = \bar{\rho}_1$. At the second steady state, when only new technology is employed, $\pi_t = \pi_1 = 1$, capital intensity and marginal product of entrepreneurial labor correspond to the ones of new technology, $\theta_t = \bar{\theta}_2$ and $\rho_t = \bar{\rho}_2$. We can solve the first order condition in (4) to obtain steady state values for decision variable $\bar{x}$. By setting $\theta_t = \bar{\theta}$ and $\rho_t = \bar{\rho}$, $k_{n,t} = k_{n,t+1} = \bar{k}_n$, $s_t^k = s_{t+1}^k = \bar{s}^k$, $s_t^c = s_{t+1}^c = \bar{s}^c$, $\ell_t^c = \bar{\ell}^c$, $\ell_t^w = \bar{\ell}^w$ the first order condition in (4) generates the equilibrium decision rule

$$\bar{x} = \frac{\beta\lambda\bar{s}^k \exp(\phi\mu)}{1-\beta\exp(\phi\mu)(1-\lambda)} \tag{28}$$

(28) shows that, at steady state, investment-output ratio increases in long run growth of economy μ and capital share of income $\bar{s}^k$.

$$\bar{s}^k = \frac{\bar{\theta}\,(1-\bar{\rho})\left[(1-\bar{\rho})\,\bar{k}_n^{\psi_k} + \bar{\rho}\,\left(\bar{\ell}^c\right)^{\psi_k}\right]^{\frac{\psi_l-\psi_k}{\psi_k}}\bar{k}_n^{\psi_k}}{\bar{\theta}\,\left[(1-\bar{\rho})\,\bar{k}_n^{\psi_k} + \bar{\rho}\,\left(\bar{\ell}^c\right)^{\psi_k}\right]^{\frac{\psi_\ell}{\psi_k}} + \left(1-\bar{\theta}\right)\left(n\,\bar{\ell}^w\right)^{\psi_\ell}} \tag{29}$$

Steady state capital share of income in (29) increases in capital intensity $\bar{\theta}$ (under Cobb-Douglas technology $\bar{s}^k = \bar{\theta}$). We expect investement-output ratio to be higher at the second steady state, as new technology is characterized by higher capital intensity $\bar{\theta}_2 > \bar{\theta}_1$.

Steady state labor supplies are as follows

$$\bar{\ell}^c = \left[\frac{\frac{A}{D^c}\bar{\theta}\,\bar{\rho}\left[(1-\bar{\rho})\,\bar{k}_n^{\psi_k} + \bar{\rho}\,\left(\bar{\ell}^c\right)^{\psi_k}\right]^{\frac{\psi_l-\psi_k}{\psi_k}}}{\left\{\bar{\theta}\,\left[(1-\bar{\rho})\,\bar{k}_n^{\psi_k} + \bar{\rho}\,\left(\bar{\ell}^c\right)^{\psi_k}\right]^{\frac{\psi_\ell}{\psi_k}} + \left(1-\bar{\theta}\right)\left(n\,\bar{\ell}^w\right)^{\psi_\ell}\right\}^{\frac{\psi_l-1}{\psi_l}}}\right]^{\frac{1}{\gamma^c-\psi_k}} \tag{30}$$

$$\bar{\ell}^w = \left[\frac{\frac{A}{D^w}\,\left[\left(1-\bar{\theta}\right)n^{\psi_l-1}\right]^{\frac{\psi_l-\psi_k}{\psi_k}}}{\left\{\bar{\theta}\,\left[(1-\bar{\rho})\,\bar{k}_n^{\psi_k} + \bar{\rho}\,\left(\bar{\ell}^c\right)^{\psi_k}\right]^{\frac{\psi_\ell}{\psi_k}} + \left(1-\bar{\theta}\right)\left(n\,\bar{\ell}^w\right)^{\psi_\ell}\right\}^{\frac{\psi_l-1}{\psi_l}}}\right]^{\frac{1}{\gamma^w-\psi_l}} \tag{31}$$

and steady state capital low of motion follows

$$\bar{k}_n^\lambda = BA^\lambda\bar{x}^\lambda\exp(-\mu)\left\{\bar{\theta}\,\left[(1-\bar{\rho})\,\bar{k}_n^{\psi_k} + \bar{\rho}\,\left(\bar{\ell}^c\right)^{\psi_k}\right]^{\frac{\psi_\ell}{\psi_k}} + \left(1-\bar{\theta}\right)\left(n\,\bar{\ell}^w\right)^{\psi_k}\right\}^{\frac{\lambda}{\psi_l}}, \tag{32}$$

3.7 Inequality increase in the model

In this section, we analyze the effects of technology diffusion on inequalities between two groups of agents: entrepreneurs and workers, implied by the model. We are principally interested in the impact of different channels on the wage ratio between two fractions of population. For this purpose, we decompose the overall wage effect into three elements. The first one corresponds to capital-entrepreneurial skill complementarity, similar to the one introduced by Krussel et al. (2000). Second, called labor supply effect, has also been analyzed by Krussel et al. (2000). Finally, we examine how so called technology effect affects the wage ratio. The last effect corresponds to the exogenous mechanism we introduced via technology diffusion process (12) which affects capital intensity in (13) and marginal product of entrepreneurial labor in (14).

Combining (8) and (9) with (22) and (23), we derive the expression for the ratio of entrepreneurial to workers' wages

$$\frac{w_t^c}{w_t^w} = \underbrace{\left[(1-\rho_t)\, k_{n,t}^{\psi_k} + \rho_t\,(\ell_t^c)^{\psi_k}\right]^{\frac{\psi_\ell-\psi_k}{\psi_k}}}_{\substack{\text{skill}\\ \text{effect}}} \quad \underbrace{\frac{(\ell_t^c)^{\psi_k-1}}{(\ell_t^w)^{\psi_\ell-1}}}_{\substack{\text{supply}\\ \text{effect}}} \quad \underbrace{\frac{\theta_t\rho_t}{(1-\theta_t)}}_{\substack{\text{technology}\\ \text{effect}}} \tag{33}$$

Wage ratio in (33) can be decomposed into three effects. The first one, called skill effect, is similar to the capital-skill complementarity effect introduced by Krussel et al. (2000). This effect mainly depends on the elasticities of substitution between capital and workers labor supply, σ_ℓ, and between capital and entrepreneurial labor supply, σ_k. The empirical values indicate that $\sigma_\ell > 1$ ($\psi_\ell > 0$) and $\sigma_k < 1$ ($\psi_k < 0$), hence $(\psi_\ell - \psi_k) > 0$. Therefore, an increase in normalized capital stock, $k_{n,t}^{\psi_k}$, has a positive effect on the wages of capital owners w_t^c, and negative effect on workers' wages w_t^w. This is the case because faster growth in capital stock increases the relative demand for entrepreneurial labor. Technological change generates capital stock increase via new investments. Skill effect is therefore the first channel generating wages inequality increase. Increase in elasticity of substitution between capital and workers' labor supply, σ_ℓ, raises the wage ratio $\frac{w_t^c}{w_t^w}$ and increase in elasticity between capital and entrepreneurial labor supply, σ_k, decreases it.

The second component in wage ratio in (33) corresponds to the labor supply effect. When supply of entrepreneurial labor ℓ_t^c increases ($\psi_k < 0$), the ratio $\frac{w_t^c}{w_t^w}$ drops and when supply of workers' labor ℓ_t^w increases ($1 > \psi_\ell > 0$), this ratio goes up.

The third, novel, component of the wage ratio (33), technology effect, is a function of two parameters, capital intensity θ_t and income share of entrepreneurial labor ρ_t. Their increase generates higher wage ratio $\frac{w_t^c}{w_t^w}$. As new technology improves capital intensity so that $\bar{\theta}_2 > \bar{\theta}_1$ and entrepreneurial labor share of income goes up $\bar{\rho}_2 > \bar{\rho}_1$, we expect the technology effect to generate an increase in the wage ratio (33).

To summarize, the first, skill effect generates an increase in the wage inequality. Capital-entrepreneurial skill complementarity effect generates incentives for capital owners to increase investment. The second effect suggest that an increase of relative supply of entrepreneurial labor decreases this inequality. Thus, labor supply effect generates incentives for capital owners to work less. Finally, technology effect raises the wages inequality. The overall effect on the wage inequality depends on the relative contributions of each of the components in (33) which will be analyzed through quantitative analysis.

In addition to labor income, entrepreneurs in this model derive their income from capital. The model is thus designed to study the evolution and the impact of return to capital on the share of income of entrepreneurs (capital owners).

$$r_t = \frac{s_t^k y_t}{k_t} \tag{34}$$

Return to capital, as in (34) increases in capital share of income $s_t^k y_t$ and output y_t and decreases in capital stock k_t. We know that all the three quantities go up during technology diffusion.

As we are interested in the welfare effects, we also examine the impact of technology diffusion driven income inequality increase on consumption ratio

$$\frac{c_t^c}{c_t^w} = \frac{\left(s_t^c - \frac{i_t}{y_t} - \tau_t\right) n}{(1 - s_t^c) + \tau_t} \tag{35}$$

$$s_t^c = \frac{r_t k_t}{y_t} + \frac{w_t^c \ell_t^c}{y_t}, \tag{36}$$

(35) is the ratio of capital owners' consumption c_t^c to workers' consumption c_t^w. An increase in labor and capital incomes of capital owners raises their consumption as long as investment grows at the lower pace than the sum of the two $\left(s_t^c = \frac{r_t k_t}{y_t} + \frac{w_t^c \ell_t^c}{y_t}\right)$. Personal transfer receipts τ_t lower the ratio in (35). The contribution of the capital and labor income channels as well as the effect of the government measures on the consumption patterns of both groups will be assessed in subsequent quantitative analysis.

4 Quantitative analysis

The quantitative analysis proceeds in three main parts. First, we carry out a benchmark calibration of the proposed model. Second, we simulate the calibrated model and confront its predictions with the data. Numerous simulations allow us to understand the intuition for obtained results. Third, once we are convinced that our model appropriately describes the inequality dynamics, we employ it to carry out welfare analysis.

4.1 Model Calibration

A time period in the model is taken to be one year. Some of the parameters are chosen to match various empirical targets. The others are calibrated outside the model. They are summarized in Table 2, in the appendix. A process of trial and error is then used to select the parameter values which are used for the nonlinear model simulations.

The number of workers per capital owner is set to $n = 9$ so that capital owners represent the top income decile of households in the model economy. The production function parameter B has been chosen to match the average capital stock-output ratio in the US economy during the last 30 years. Note that this ratio has been scaled by 0.8. This adjustment has been made to account for the share of wealth in the total economy that is held by the top decile.[8] The production function parameter A is chosen so that we obtain the same normalized capital stock at the initial steady state, $\bar{k}_{n,1}^{CD}$, as in case of Cobb-Douglas technology. $\alpha = 2.5$ has been chosen as a benchmark risk aversion. We will analyze a wider range of values for α in the sensitivity analysis. β is calibrated to the mean equity return in the US, $r_t = 0.08$. Both parameters describing elasticities of substitutions have been selected based on the empirical findings (see for instance Krussel et al. 2000). Frisch labor supply elasticities γ^c and γ^w have also been chosen based on earlier empirical estimates. The adjustment cost parameter of the capital accumulation equation (11) , λ, has been chosen to match the empirical mean investment-output ratio in the US economy (also scaled by 0.8). The long run growth of the economy has been set to the US mean consumption growth, $\mu = 2.03\%$. The capital intensity parameter at the initial steady state θ_1 and at the second steady state θ_2 are calibrated to the capital shares of income in 1980 and 2007, in the US economy. To obtain values for the entrepreneurial labor shares of income $\rho_1$and ρ_2 we use the capital shares of income and shares of income of the top decile, all of them in years 1980 and 2007.

4.2 Macroeconomic implications of the model

In this section, we assess the validity of the proposed model. For this purpose, we compare the stylized facts described in the second section with the predictions of the

[8] Top 10% hold roughly 80% of the total wealth in the US (see Wolff, 2006).

calibrated model. Next, using numerous simulations of the model, we provide the intuition for obtained results.

Table 3. Data and model predictions

	Data	Model
	1980-2007	
Top 10% Income share	36% → 49%	36% → 49%
Capital share of income	28% → 31%	28% → 31%
Personal transfer receipts	10% → 16%	10% → 16%
Macroeconomic outcomes	Data	Model
Wages ratio	45%	40%
Hours ratio	40%	40%
Consumption ratio	17% − 30%	45%

Table 3 is composed of two parts. The upper part displays the evolution of quantities that the model is calibrated to. These are top decile income share corresponding to s_t^c in the model, capital share of income s_t^k, and personal transfer receipts τ_t. The lower part of the table domesticates how well the proposed model matches the evolution of macroeconomic inequalities. For this purpose, we calculate the ratios of respective macroeconomic variables between capital owners and the remaining fraction of the population, workers.

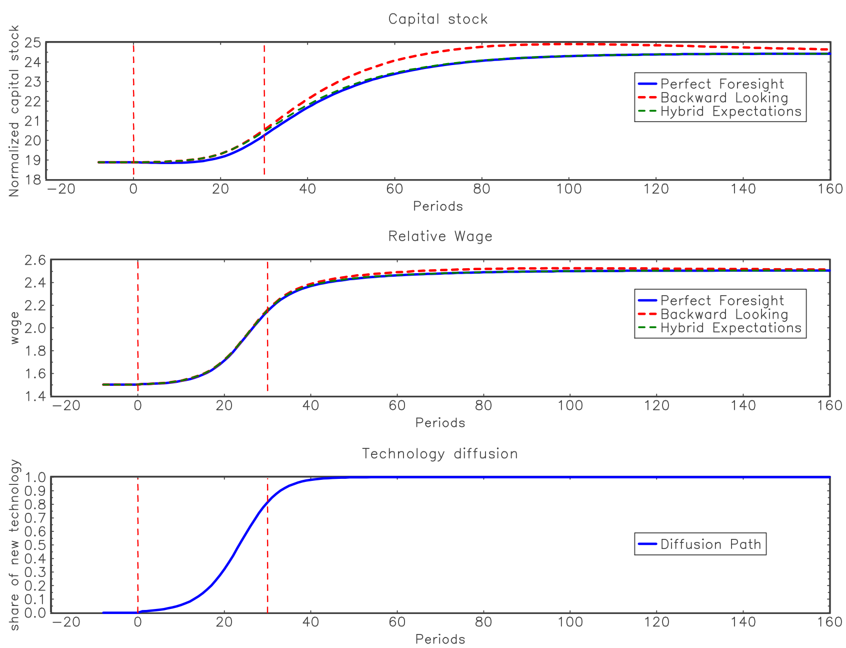
Table 3 shows that the calibrated model matches quite well the evolution of macroeconomic variables of interest. First, it predicts a 40% increase of the wage ratio, $\frac{w_t^c}{w_t^w}$, between 1980 and 2007. The data suggests that this increase was slightly lower, 35%. Second, the model forecasts that the relative supply of entrepreneurial labor went up by 40%, during the same period; the figure also suggested by empirical studies. Finally, our model predicts an increase in consumption ratio $\frac{c_t^c}{c_t^w}$ of 45%, higher than the 30% suggested by the Aguiar and Bils (2011) statistic that was our benchmark in the baseline calibration.

In what follows, we seek to understand the mechanisms which drive income inequality in our model. In particular, using the model simulations, we focus on the understanding of labor and capital incomes' contributions to the observed increase in income inequality.

We first simulate the model using the parameter values from the baseline calibration described in Table 2. Figure 6 shows simulated paths of three variables in skill effect defined in the wage ratio (33).

The upper panel of Figure 6 shows normalized capital stock, $k_{n,t}$, the middle panel plots the wage ratio, $\frac{w_t^c}{w_t^w}$, and the bottom panel displays the share of economy using new technology π_t. The first dotted, vertical line indicates the beginning of the technology diffusion process. It corresponds to the first steady state, where $\pi_t = 0$. The dynamics in the technology process are boosted by an initial technology

Figure 6: Skill effect in the model



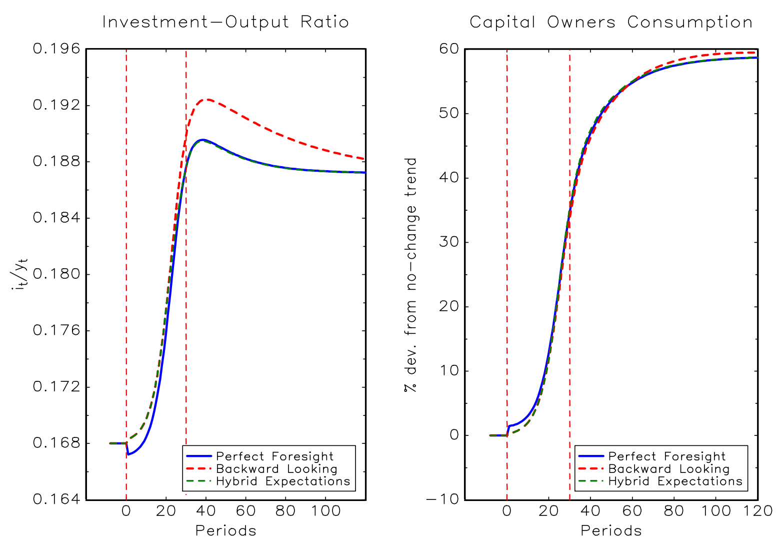
The figure plots normalized capital stock $k_{n,t}$, wage ratio $\frac{w_t^c}{w_t^w}$, and the share of economy that uses new technology, π_t.

impulse, ν_π. Empirically, the initial steady state corresponds to 1980. The second dotted, vertical line in the figure points to the current date, roughly 2010. Thus the simulated period between the two vertical dotted lines corresponds to 30 years. Note that, according to the model, the economy has not converged to the new steady state yet. This is because, as demonstrated in Figure 5 and bottom panel of Figure 6, the share of economy using new technology is 90%, $\pi_t = 0.9$. Hence this is a still on-going process and macroeconomic variables, plotted in the top and the middle panels of Figure 6, have not converge to the second steady state yet.

Figure 6 demonstrates the evolution of the skill effect, as described by formula (33). As the new technology spreads into economy, normalized capital stock, $k_{n,t}$, accumulates and boosts the relative wages of capital owners. The top and the middle panels plot three different lines. The solid line corresponds to the variables' paths under prefect foresight and the dotted line under backward looking expectations. A hardly visible, grey line in between both, indicates hybrid expectations.

Capital stock accumulates faster under backward looking expectations. This is the case because in this model, investment grows slower under perfect foresight than under hybrid and backward looking expectations. Figure 7 illustrates this fact.

Figure 7: Investment-output ratio and consumption of capital owners



The figure plots the evolution of decision variable investment-output ratio $\frac{i_t}{y_t}$ and capital owners' consumption c_t^c.

The left panel of Figure 7 demonstrates that under perfect foresight (solid line) the investment output ratio $\frac{i_t}{y_t}$ initially drops. This is the case because capital owners endowed with the perfect foresight know that technology diffusion will generate a permanent increase in their income. As a result, they adjust upwards their current consumption as shown in the right panel of Figure 7. As the fraction devoted to current consumption goes up, the remaining share devoted to investment is lower. This is a standard result in RBC framework where under perfect foresight current consumption raises at the cost of slower accumulation of investment.

Agents who are not endowed with perfect foresight (backward looking and hybrid expectations) cannot foresee the future benefits of the new technology and they consider the boost in income as transitory and therefore do not adjust their consumption path. Investment-output ratio grows faster as illustrated by the dotted line in the left panel of Figure 7 and capital owners' consumption increases slower (right panel of Figure 7). As a result, the consumption ratio increase during the calibrated thirty years, implied by the model, is the highest in case of perfect foresight and the lowest under backward looking expectations. This result is similar to findings by Heathcote et al. (2010). They show that the welfare gains are significantly smaller in case of myopic agents.

This result is also interesting as it replicates the positive comovements among consumption, investment, and output that we observe in the data.

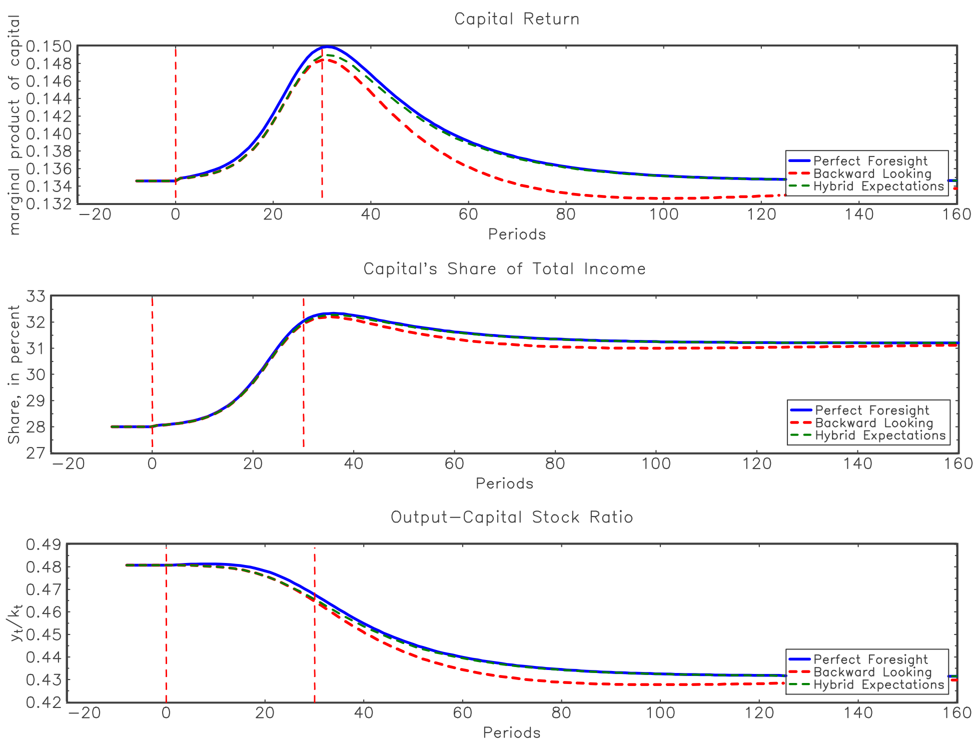
Figure 12 in the appendix shows that both, wages w_t^c and work hours ℓ_t^c of capital owners increase during the transition while those for workers, w_t^w and ℓ_t^w decrease. The labor supply responds strongly to the wages as we assumed high Frisch elasticities in the benchmark calibration (see Table 2). An increase in relative entrepreneurial labor supply $\frac{\ell_t^c}{\ell_t^w}$ decreases the wage ratio in (33). This labor supply effect is however offset by positive impact of the skill and technology components in (33). As a result, the labor income of capital owners increases relative to the workers' and the consumption ratio $\frac{c_t^c}{c_t^w}$ also raises. We will verify how important the labor supply effect is in the consumption inequality increase in the sensitivity analysis. In particular, we will examine the economy with both more elastic labor supply and inelastic labor supply.

The so-called technology component in (33) also generates higher ratio $\frac{w_t^c}{w_t^w}$. Technology diffusion increases capital intensity and entrepreneurial labor share of income via (13) and (14). Increase in both boosts the relative capital owners' wages and thus their labor income.

Return to capital, $r_t = \frac{s_t^k y_t}{k_t}$, also increases during technological change. The upper panel of Figure 8 demonstrates capital's return during simulated period.

Initially, capital return increases. In fact, Figure 8 suggests that the current return to capital is roughly at its peak and it will decreases subsequently to reach a lower level at the second steady state. This prediction of the model stems from two determinants of capital returns. First, capital share of income under CES production function is overshooting, as illustrated in the middle panel of Figure 8. In the long

Figure 8: Return to capital, capital's share of income and output-capital stock ratio during technology diffusion process



The figure plots simulated return to capital r_t, capital's share of income s_t^k, and output-capital stock ratio.

run, it will converge to the lower equilibrium than the current peak.

Second, output-capital stock ratio is decreasing during technological shift; output grows slower than capital stock.

5 Welfare analysis

Empirical evidence shows that there was an increase in income inequality during the last three decades in the US. The increase in income inequality could have been however accommodated by various public and private channels to smooth the consumption of the fraction of the population whose income decreased. For instance, Figure 2 indicates that personal transfer receipts increased during the last thirty years reducing consumption inequality.

In what follows, we focus on consumption inequality evolution and carry out welfare analysis in the framework of the proposed model. Welfare costs (benefits) of technological change are measured by the percentage change in per-period consumption that makes the agent indifferent between the two economies being compared. These two economies are (i) the economy before technological change and (ii) the economy after the change. Details of the welfare computations are contained in the appendix.

5.1 Expectations matter

We first examine the results of the welfare analysis for the baseline model under different expectations formation.

Table 4. Welfare gains form technological change

Expectations	Capital Owners	Workers	Net gains
Perfect foresight	5.36	1.58	3.78
Hybrid expectations	3.60	1.79	1.81
Backward looking	2.53	1.93	0.60

Welfare gains (losses) are measured by the % change in yearly consumption that makes the agent indifferent between the two economies being compared.

Table 4 includes welfare gains for both fractions of population: capital owners and workers. As expected, capital owners largely benefit from the technological change. Their consumption boosts as a result of increase in income from labor and capital. The benchmark model simulation indicates that workers gain from technological change as well, although their gains are lower than the ones of entrepreneurs.

The way the agents form their expectations matter for the welfare analysis. In case of perfect foresight, the gains are the highest for entrepreneurs and the lowest for workers. Technological change generates wealth effect due to increase in permanent income and substitution effect in response to increase in return to capital. The wealth effect dominates because, the return to capital increases only temporarily and decreases in the long run, as demonstrated in the top panel of Figure 8. Under perfect foresight, technological change increases consumption at the expense of investment. Lower investment leads to slower growth in output and wages of workers drop reducing their consumption.

In case of backward looking expectations, the welfare gains for workers are the highest. As entrepreneurs use the past to make the forecast about the future behavior of economic variables, they perceive increase in both income and capital return as temporary and split the additional income between consumption and investment more equally. Faster output growth generates higher workers' consumption.

The welfare gains in the case of hybrid expectations, presented in the third row of Table 4, are in-between the two, the perfect foresight and the backward looking expectations. In the beginning of technology diffusion, only a small fraction of economy is using the new technology and therefore the resulting macroeconomic quantities behave more like in backward-looking cases. Towards the end of technological change process, as most of the economy shifted to new technology, the economy behaves as under perfect foresight.

5.2 Consumption inequality and welfare gains

Table 4 suggests that, in the benchmark case, both groups gain from technological change. We examine the contribution of personal transfers to this result. Put differently, we vary the final consumption inequality growth generated in the model. First, we assume that the government did not increase the transfers, $\tau_t = \bar{\tau} = 10\%$. Second, we assume that its mechanism was more generous, so that the final consumption inequality increased only by 17 %. The latter exercise is motivated by the ongoing debate regarding the evidence on increase in consumption inequality. While Krueger and Perri (2006) argue that consumption inequality increased much less than the one of income, Aguiar and Bils (2011) claim that consumption inequality mirrored income inequality during the last three decades. Our benchmark calibration follows the figures proposed by Aguiar and Bils (2011). In addition, we carry out the simulation with increased personal transfer receipts that can produce the figures suggested by Krueger and Perri (2006).

Table 5. Impact of transfers on welfare gains and losses

Transfers	Capital Owners	Workers	$\Delta \frac{c_t^c}{c_t^w}$
$\tau_1 = 10\%$, $\tau_2 = 16\%$	3.60	1.79	45%
$\tau_t = \bar{\tau} = 10\%$	16.96	−1.90	100%
$\tau_1 = 10\%$, $\tau_2 = 19\%$	2.53	1.93	17%

Welfare gains (losses) correspond to the numbers under hybrid expectations and are measured by the % change in yearly consumption that makes the agent indifferent between the two economies being compared. Table reports results only for hybrid expectations. τ_t corresponds to personal transfer receipts. The last column displays resulting consumption inequality change, $\Delta \frac{c_t^c}{c_t^w}$.

Table 5 reports the results of three different simulations. The first one is a benchmark simulation which generates consumption ratio increase of 45%. The second assumes that government's transfers remained at the initial level, $\bar{\tau} = 10\%$. In the third simulation, we assume that the transfers' growth during the last three decades was high enough to match the modest increase in consumption ratio increase of 17%. The figures reported in the table describe the welfare gains (losses) under hybrid expectations.

Table 5 suggests that transfers introduced in the benchmark calibration are the main driver of the workers' welfare gains. The third row of the table indicates that once these transfers are removed, capital owners' welfare gains become very large and workers' lose from technological change. The resulting increase in the consumption ratio, of 100%, is however unrealistic, according to any measure of consumption inequality. The last column of the table implies that an increase in transfers of roughly 100% is needed to match the consumption inequality of 17%. In this case, both groups gain from technology diffusion.

This exercise demonstrates that appropriate estimation of consumption inequality is crucial to the evaluation of the welfare gains and losses of two groups of population.[9] If consumption inequality mirrored income inequality increase during the last three decades, the proposed model suggests that the capital owners gained twice as much from technological change as the workers did. If however the consumption inequality increase was only moderate during this period, the gains of both groups are almost the same. Based on the current empirical evidence and the workings of our model, we can conclude however that both groups benefited from the technological change.

5.3 Labor income and capital income

In this section we decompose welfare gains according to the source: capital and labor income. In particular, we are interested in the welfare gains of capital owners

[9] Several papers recently contributed to this research agenda. See for instance Aguiar and Bils (2011), Parker et al. (2009).

from capital income, the novel source of income in the model. For this purpose, we eliminate one of the channels at the time. First, we eliminate the capital income channel by varying δ_θ so that the capital share of income s_t^k is the same at both steady states $s_1^k = s_2^k$.

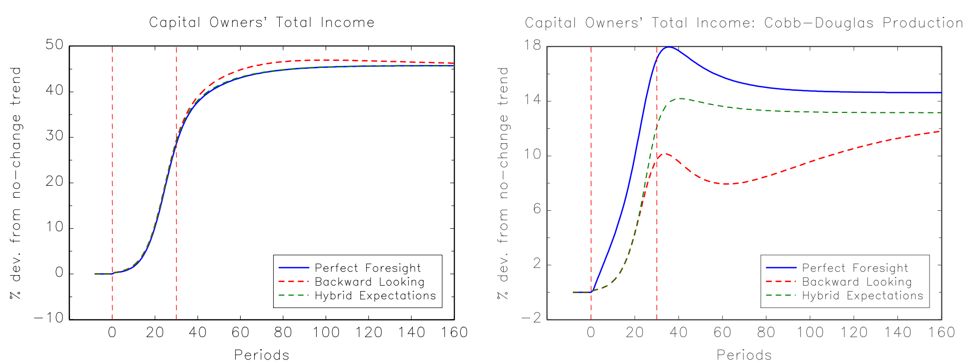
Table 6. Decomposition of capital gains and losses

Channels at work	Changed parameter	Capital Owners	Workers
Labor Income	$\delta_\theta= 0.16$	-16.81	0.92
Capital Income	$\sigma_k= \sigma_l= 1 \Rightarrow$ CD	-36.24	2.37

The first column displays the income channel that is in place in the model simulation. The other channel is eliminated. Welfare gains (losses) correspond to the numbers under hybrid expectations and are measured by the % change in yearly consumption that makes the agent indifferent between the two economies being compared. Table reports results only for hybrid expectations. CD stands for Cobb Douglas.

The second row of Table 6 reports result of the simulation. It demonstrates that once the capital income channel is eliminated, entrepreneurs' welfare deteriorates. In fact, without capital income source, technology diffusion generates an important welfare loss for capital owners. Figure 9 plots the growth in total income of capital owners under the benchmark simulation and under the simulation without capital income channel.

Figure 9: Capital owners' income generated by the benchmark model and model without capital income channel (in percentage deviations from no-change trend)



The left panel plots capital owners' income: $r_t k_t + w_t^c \ell_t^c$ generated by the benchmark model with the parameters described in Table 2. The right panel plots capital owners' income generated by the model without capital income channel. In this simulation $\delta_\theta= 0.16$ and the remaining parameter values are the same as in Table 2.

Since we removed one of the channels, capital owners' total income grows slower as demonstrated in the right panel of Figure 9. When both channels are in place, the capital owners' income increases roughly by 46% during the transition to the new technology. When capital income channel is eliminated, this growth drops to 15%, relative to the no-change trend. Capital owners loose several benefits that are directly or indirectly generated by capital income channel. First, directly, as capital intensity increases much less, returns to capital drop and reduce capital income of entrepreneurs, relative to the benchmark model. Second, indirectly, as capital accumulates slower, the demand for entrepreneurial labor slows down as well and reduces labor income of capital owners. Finally, lower capital intensity decreases total productivity and total output (income).

The lower increase in capital owners' income makes it more attractive for them to increase investment at the expense of consumption. Slower increase in income generates substitution effect leading to a reduction in consumption and increase in investment. It is important to note that, although in this version of the model, investment raises at the expense of consumption, its growth is still lower than in the benchmark model.

Elimination of capital intensity channel from the model affects intertemporal decisions between labor supply and leisure of workers. As due to slower increase in total output workers' wages decrease, their labor supply, labor income and consumption drop as well, relative to the benchmark case. As a result, their welfare slightly deteriorates in comparison to the baseline simulation of Table 4.

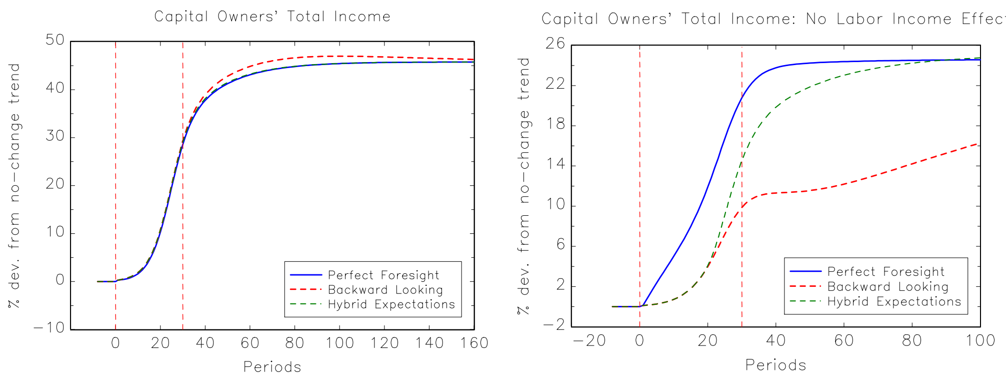
In total, capital income channel principally benefits entrepreneurs. Its elimination considerably deteriorates capital owners' welfare while it affects only modestly the one of workers.

The third row of Table 6 displays the results of simulation of the model with capital income channel only. We eliminated capital-entrepreneurial skill complementarity effect by assuming Cobb-Douglas production technology ($\sigma_\ell = \sigma_k = 1$) and setting $\delta_\rho = 0$ in (14) so that $\rho_t = \bar{\rho}_1$. As a result, the capital owners' share of labor income, $s_t^c - s_t^k$ does not change during the transition period so that the labor income channel is eliminated. The wage ratio becomes

$$\frac{w_t^c}{w_t^w} = \underbrace{\frac{\theta_t \bar{\rho}_1}{(1-\theta_t)}}_{\substack{\text{technology} \\ \text{effect}}} \quad \underbrace{\frac{\ell_t^w}{\ell_t^c}}_{\substack{\text{supply} \\ \text{effect}}} \tag{37}$$

where only technology and supply effects are present. As we assume that θ_t still evolves according to (13), and fixed $\rho_t = \bar{\rho}_1$, the overall technology effect over transition is smaller in (37). As for supply effect, increase in relative labor supply of capital workers reduces their wages by factor $\frac{1}{\rho_t}$, less than in the benchmark case. We find that in this simulation capital owners' labor supply is much lower than in the baseline case defined in equation (33). As a result, their income and total output of the economy grow slower.

Figure 10: Capital owners' income generated by the benchmark model and model with Cobb-Douglas production function (in percentage deviations from no-change trend)



The left panel plots capital owners' income: $r_t k_t + w_t^c \ell_t^c$ generated by the benchmark model with the parameters described in Table 2. The right panel plots capital owners' income generated by the model without entrepreneurial skill ($\sigma_\ell = \sigma_k = 1$).

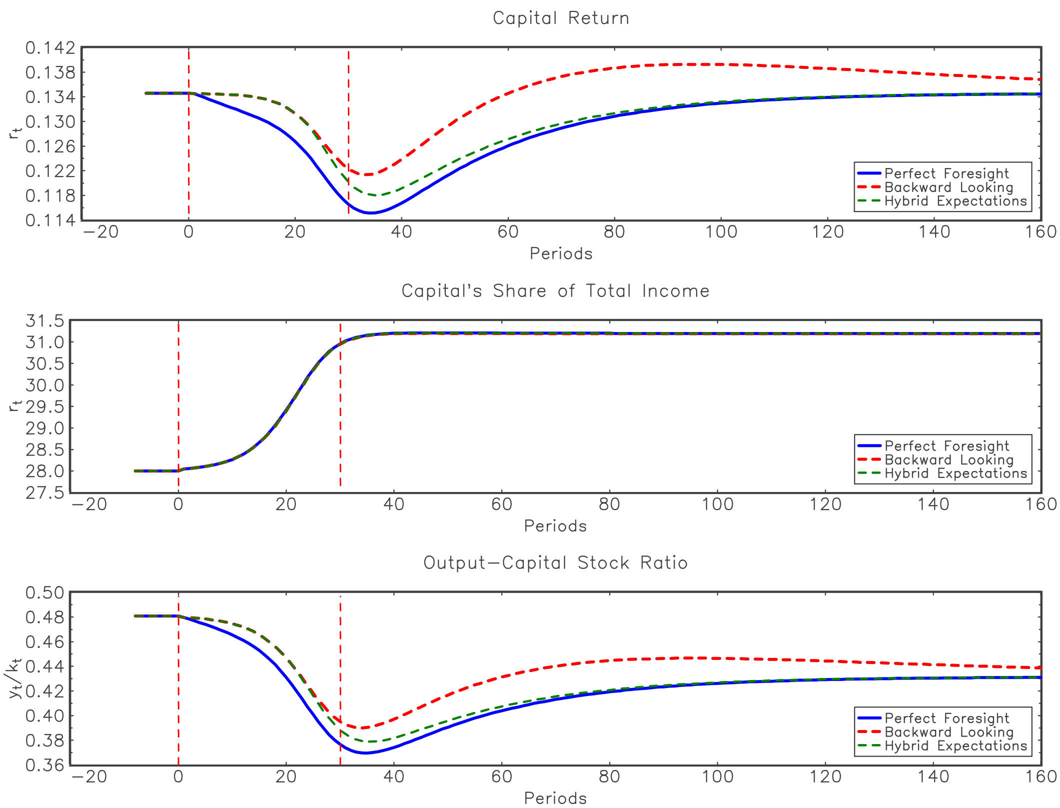
The right panel of Figure 9 plots capital owners' income growth when labor income is removed and shows that it increased roughly by half as much as in the benchmark case, plotted in the left panel of the figure.

The workers' share of income goes up and generates an increase in wages. Because workers' wages increase and those of capital owners drop, relative to the benchmark case, we find that the wage ratio $\frac{w_t^c}{w_t^w}$ increases only by 10% , relative to 40% in the baseline simulation. Similarly, we observe an increase in workers' labor supply and entrepreneurial labor reduction which generate a modest increase of 10% in the worked hours ratio, relative to 40% in the benchmark case.

As capital stock accumulates at the speed similar to the benchmark case and capital share of income remain unchanged, reduced output results in a drop in capital returns, plotted in Figure 11.

To summarize, from the welfare point of view, both income channels are important. Labor income source seems however to be essential as its elimination causes huge welfare losses to capital owners. Capital income channel does not seem to affect considerably workers' welfare while labor channel does. When eliminated, labor income channel generates additional welfare gains for workers. Higher workers' wages provide incentives for workers to supply more labor leading to higher consumption.

Figure 11: Return to capital, capital's share of income and output-capital stock: Cobb-Douglas production technology)



The figure plots simulated return to capital r_t, capital's share of income s_t^k, and output-capital stock ratio in Cobb-Douglas version of the model.

6 Conclusion

In this paper, we propose a model which includes two important channels of technology diffusion which contributed to the increase in income inequality in the US during the last three decades. The first channel corresponds to capital-entrepreneurial skill complementarity (labor income) channel. The second novel in the related literature channel is capital income channel.

The framework for our analysis is a real business cycle model in which ownership of productive capital is concentrated in the hands of a subset of agents (capital owners). The remaining fraction of agents (workers) provides labor. The economy is assumed to undergo technology diffusion process.

We examined theoretically and quantitatively the workings and the importance of the capital and labor income channels in the increase in income inequality and its welfare implications.

Labor income of capital owners increases in capital stock and the share of economy using technology diffusion. It decreases in the relative supply of capital owners' labor. Capital income is a positive function of capital share of total income and return to capital.

We calibrated the proposed model to match the observed increase in both: capital's share of income and top decile income share. Next, we simulated the calibrated model and confronted its predictions with the data. We found that the proposed framework produced quantities that match quite well the evolution of macroeconomic variables of interest: the wage, labor supply and consumption ratios.

We used this framework to carry out welfare analysis.

In the benchmark case, we found that capital owners largely benefitted from the technological change. Their consumption boosted as a result of increase in income from labor and capital. The benchmark model simulation indicates that workers gained from technological change as well, although their gains were lower than the ones of entrepreneurs.

We found that the way the agents form their expectations matter for the welfare analysis. Under perfect foresight, the welfare gains are the highest for capital owners and the lowest for workers. In case of backward looking expectations, the opposite holds.

Elimination of one of the income channels at the time allowed us to understand the importance and the workings of each of them. From the welfare point of view, both income channels are important. Labor income source seems however to be essential as its elimination causes huge welfare losses to capital owners. Capital income channel does not seem to affect considerably workers' welfare while labor channel does. When eliminated, labor income channel generates additional welfare gains for workers.

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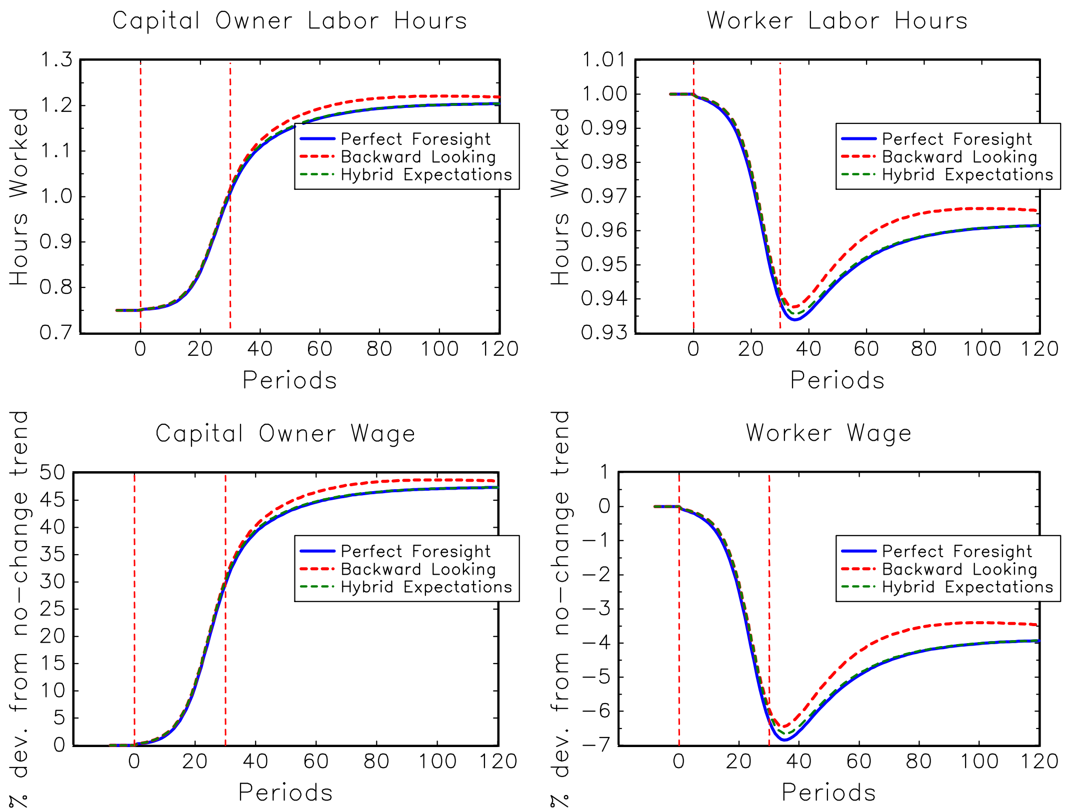
Appendix

Figure 12: Evolution of wages and hours of capital owners and workers in the simulated benchmark model



The figure plots hours and wages of capital owners and workers. The paths of wages are computed as deviations from the no technological-change trend.

Table 2: Baseline Parameter Values

Parameter	Value	Description/Target
n	9	Capital owners = top income decile
α	2.5	Risk aversion
β	0.98	Mean r_t= 0.08 (equity return)
σ_k	0.25	Empirical estimates
σ_l	1.25	Empirical estimates
γ^c	2	Empirical estimates
γ^w	2	Empirical estimates
B	1.22	Mean $k_t/y_t = 0.8 \times 2.6$
A	0.80	$\bar{k}_{n,1}^{CD}$
λ	0.09	Mean $i_t/y_t = 0.8 \times 0.22$
μ	2.03%	Mean consumption growth
θ_1	0.30	Initial capital income share: $s_1^k = 0.280$
θ_2	0.43	Final capital income share: $s_2^k = 0.312$
ρ_1	1×10^{-5}	$s_1^k = 0.280$, $s_1^c = 0.36$
ρ_2	7×10^{-5}	$s_2^k = 0.312$, $s_2^c = 0.5$
κ	0.22	Empirical estimates
δ_θ	0.32	θ_1= 0.33, θ_2= 0.44
δ_ρ	1.43	$\rho_1 \simeq 1 \times 10^{-5}$, $\rho_2 = 7 \times 10^{-5}$
δ_τ	0.47	τ_1= 10%, τ_2 = 16%

The table displays the parameter values set in the benchmark calibration. The first column shows the symbol, the second the value of the parameter and the third column describes how the value was obtained. n denotes the number of workers in the economy, α is the discount factor, σ_k is the elasticity of substitution between capital and workers and σ_l elasticity of substitution between capital and entrepreneurial labor. γ^c and γ^w are the elasticities of entrepreneurial and workers' labor supplies, respectively. B and A are production technology parameters. ρ_1 and ρ_2 are the entrepreneurial labor share parameters at the beginning and at the end of technological transition. λ stands for adjustment cost coefficient, μ for long run growth trend. θ_1 and θ_2 are initial and final capital intensity, respectively. β is a discount factor, κ speed of diffusion, δ_θ is growth in capital intensity, δ_ρ denotes growth in marginal product of entrepreneurial labor and δ_τ growth in personal transfer receipts.