Testing a Simple Structural Model of Endogenous Growth*

Patrick Minford
Cardiff University and CEPR

David Meenagh
Cardiff University

Jiang Wang
Cardiff University

*We are grateful to Max Gillman for discussions and useful advice; and for other comments from Laurence Copeland, Gerry Makepeace, Gerald Harbour and others in the Cardiff University economics workshop. .

1 Introduction:

Our aim in this paper is to take the simplest possible Real Business Cycle model and adapt it to endogenous growth in such a way that we can use dynamic testing techniques on its aggregate predictions, using the Penn Table post-war data. The empirical testing of endogenous growth has largely to date consisted of running 'reduced form' equations on panel data for large numbers of countries over the post-war period.[1] However there are difficulties in this approach. If one writes down a model of endogenous growth (as we will shortly do) one finds that it is complex and non-linear so that it does not have a linear reduced form; thus the 'reduced forms' written down for testing are no more than guesses at the variables, either exogenous or predetermined, that might be included among the determinants of growth. Even if their inclusion is correct, the omitted variables will in general include powers or other combinations of these included variables; hence the error terms will be correlated with the regressors and there will be bias whose size and direction cannot be estimated reliably.

A further problem is one of identification. We do not know what model is generating these 'reduced forms'; many different models could give rise to some relationships between the chosen regressors and growth. For example if the regressors are correlated (due to transmission within the model) with the true causal mechanisms whatever they are one could obtain significant regression coefficients on the chosen regressors which in fact come from a quite different set of causes

For such reasons the large literature above may not be regarded as entirely persuasive evidence. Those for example who think R&D is the major factor determining growth will not be impressed by regressions showing that tax rates are correlated with growth. Vice versa with regressions highlighting R&D those favouring the tax explanation are unwilling to be persuaded.

In this paper we take a new approach to testing, one that we have used in other areas of macro with we believe some promising results (see Minford, 2006, for an informal account of these; a recent example applying the method to models of inflation persistence is Minford, Sofat, Nowell and Srinivasan, 2006). This approach is Popperian; we start by insisting on a clearcut 'null hypothesis' by which we mean a hypothesis treated as true for purposes of testing (by 'null' is strictly meant the 'zero' hypothesis of no relationships at all because this is the one that is taken to be rejected in much statistical testing; however we adopt the definition of 'working' or 'initially believed' hypothesis here because in our approach it will be this, not the zero hypothesis that is to be rejected.). This null hypothesis is the micro-founded (structural) theory of endogenous growth in this case that we are going to test.

This theory being specified is then calibrated or estimated on the data so that its structural equations have an implied set of errors. Under the null these errors are the true errors, warts of approximation and all. Thus these errors contain vital information about the model's implications; different models 'slice reality (data)' between model and error differently. We exploit this fact in what follows; this exploitation we may note in passing crucially distinguishes what we do from methods of simulation in widespread use (such as the Simulated Method of Moments- see Minford, Theodoridis and Meenagh, 2005) where errors are typically imposed. Our concern is to use the information in the data to the maximum in the testing process.

We use approximative techniques to derive the structural equations to the full non-linear model. The implication is that the errors contain the terms omitted via the approximation- these will include second and higher order terms of Taylor series expansion when only first order approximation is done as is our practice here. In addition there may be other terms in the error that have deliberately been omitted even after first-order approximation; an example would be where a variable cannot be explicitly solved for (such as below the ratio of consumption to GDP). Thus the error terms contain all the information about the effect of variables in the model but omitted from an equation as well as of pure error terms such as productivity or preference shocks. (Notice that they will in general all be simultaneously correlated for this reason.) When drawing policy conclusions this must be carefully taken into account- Lucas' critique-; provided the policy change does not alter the stochastic processes of the model it can be evaluated.

However our first purpose is to test the null hypothesis against the data. We may use the data to estimate the structural equations and we then use the data to imply the model errors. The model may still not fit the 'facts of correlation between variables'. This could be so even with a model estimated carefully on the data at a 'structural equation by equation' level; the reason for example could be that

[1] Leach (2003) and OECD (Leibfritz et al, 1997) provide useful surveys of this literature. Studies include Barro (1991), Koester and Kormendi (1989), Hansson and Henrekson (1994), Cashin (1995), Engen & Skinner (1996), Leibfritz et al (op. cit.), Alesina et al. (2002), Bleaney, Gemmell & Kneller (2000), Folster & Henrekson (2000), Bassanini & Scarpetta (2001), Benson and Johnson (1986), Chao and Grubel (1998), Easterly and Rebelo (1993), Grier and Tullock (1989), King and Rebelo (1990), Levine and Renelt (1992), Peden and Bradley (1989), Plosser (1992), Scully (1989, 1991, 1995), Slemrod (1995), Smith (2001), Vedder and Gallaway (1998).

it is mis-specified in some way. As we know a mis-specified model may still fit at the equation level with the error term picking up the mis-specification.

We can think of the facts against which the model is being measured as being like the 'reduced forms' discussed above. These are not true reduced forms of any relevant model; but they are descriptions of relationships between variables. In business cycle studies the facts are often represented by VARs. However in growth studies the facts of interest are naturally represented by panel relationships between growth and various regressors. Many such descriptions are possible; the empirical studies referred to earlier examplify a wide variety. One thing we can say is that the true model will 'predict these facts' in a certain sense: these facts must lie within the statistical distribution implied by the model.That is, if we can derive the distribution of possible results for such relationships implied by the model, then we can ask whether the relationship found in the actual facts lies within some agreed (say 95%) set of confidence limits.

We can derive these confidence limits by bootstrapping the errors in the model. For these errors represent the true source of sampling variability under the null hypothesis. Thus by replicating these errors in repeated random draws from them and inputting these draws into the model we may replicate the sampling variability of the model.and hence the statistical distributions implied for our 'relationship facts'. It is this procedure that underlies our testing of the model against the facts. We derive a pure test of the model in this way. Of course matters do not end there. If the model is rejected we have to decide on an alternative and test that in turn; if it is accepted we still must discover whether an alternative would also be accepted or whether it is rejected. These questions we attempt to answer in this paper for a limited 'alternative' to our basic model.

In what follows we go into the exact methods in a lot of detail but this account has, we hope, outlined our method and the reasons for it.

2 A model of endogenous growth for a small open economy:

We begin (essentially as in Gillman and Kejak, 2005) from a standard intertemporal utility function and a perfectly competitive firm sector with a Cobb-Douglas production function, from which households derive wages for their labour supply as workers and dividends for their capital; under constant returns to scale dividends and wages add up to total GDP. We assume that each household owns a corresponding firm for which it works (at competitive wage rates because it could always decide to work elsewhere) and also may undertake entrepreneurial activity to innovate its methods, so raising its productivity. However each household must buy its consumption and investment goods from other firms.Government taxes both in order to make transfer payments back to households (for redistributive purposes) and there is no government spending. The economy is open but is 'small' in the strictest sense; that is, it can borrow on world markets at the world real interest rate and its goods prices are also set on world markets. To this set-up we add two endogenous growth mechanisms that have been extensively studied in the microeconomic literature: learning by doing (so that additional labour supply raises the rate of growth of productivity- see for example Fudenberg and Tirole, 1983) and innovation/entrepreneurship activity (here see for example Klette and Kortum, 2004)

We go on to show that this economy for our purposes (examining its growth behaviour) can be summarised in three equations: the production function reduced to a function of productivity and labour supply, a labour supply function of labour/consumption taxation, and a productivity function of the accumulated tax rates on labour/consumption and entrepreneurial activity.

3 Derivation of the 3-equation model:

The representative household's utility function seen from period 0 is:

$$U_t = E_0 \sum_{t=0} \beta^t (\ln c_t + \alpha_t l \ln x_t) \tag{1}$$

where α_t is a stationary preference error process,
subject to

$$(1+\tau_t)c_t + k_t - (1-\delta)k_{t-1} + b_t = y_t + (1+r_{t-1})b_{t-1} + \Gamma_t - \pi_t z_t \tag{2}$$

where:

τ is the tax rate on consumption- this is assumed to be the sole general tax (so that dividends and wages are taxed indirectly through consumption);

π is tax levied on entrepreneurial activity,

consumption (c), capital stock (k), foreign bonds (b), leisure (x), entrepreneurial activity (z) and government transfers (Γ) are all expressed per capita;

δ is depreciation and r is the real rate of interest on foreign bonds. Goods are bought by some system of organised barter and so we ignore the role of money in this economy;

$y_t = A_t k_t^{\gamma} X_t^{\zeta}(1 - x_t - z_t)^{1-\gamma-\zeta}$ is the Cobb-Douglas production function of the household (and firm combined). X represents exogenous other production factors- such as 'land'/natural resources- assumed to be owned by households.

The household's first order conditions familiarly yield (where λ are the Lagrangean parameters):

$$\lambda_0 = \frac{1}{c_0(1+\tau_0)}; \lambda_1 = \frac{\beta}{c_1(1+\tau_1)}$$

(from the first derivatives of the Lagrangean with respect to current and future consumption)

$E_0\lambda_1[1-\delta] = \lambda_0[1 - \frac{\gamma y_0}{k_0}]$ (from the first derivative with respect to capital, k_t)

$E_0\lambda_1[1+r_0] = \lambda_0$ (from the first derivative with respect to foreign bonds, b_t)

$\lambda_0[\frac{(1-\gamma)y_0}{1-x_0-z_0}] = \frac{\alpha_0 l}{x_0}$ (from the first derivative with respect to leisure).

At this stage we treat entrepreneurial activity, z, as fixed. But we will return to it once we have introduced productivity determination below.

From these conditions (letting time zero be generalised to any t) we can derive the consumption condition:

$$c_t = \frac{1}{\beta(1+\tau_t)} E_t \frac{c_{t+1}(1+\tau_{t+1})}{1+r_{t+1}} \tag{3}$$

the condition relating the marginal product of capital (which we also denote by the shadow real dividend rate, d_t) to world real interest rates plus depreciation:

$$r_t + \delta = \frac{\gamma y_t}{k_t} = d_t \tag{4}$$

and the condition relating labour supply to the marginal product of labour (which we also denote by the shadow real wage,w_t):

$$a) w_t = \frac{(1-\gamma-\zeta)y_t}{(1-x_t-z_t)}; b) x_t = \frac{\alpha_t l c_t(1+\tau_t)}{w_t} \tag{5}$$

Using the marginal productivity of capital condition, we can replace capital in the production function by terms in the shadow dividend (determined in 5).

$$y_t = A_t^{\frac{1}{1-\gamma}} (\frac{\gamma}{d_t})^{\frac{\gamma}{1-\gamma}} X_t^{\frac{\zeta}{1-\gamma}} (1 - x_t - z_t)^{\frac{1-\gamma-\zeta}{1-\gamma}} \tag{6}$$

What this means is that the household can obtain whatever capital it needs to produce its desired output at a fixed price on world markets; thus it is only limited in the output it can produce by the supply of labour offered at the going shadow wage.

We now turn to the determination of productivity growth and the marginal condition determining z.

In this model representative households choose how much to invest and work within their available production technology. This technology is assumed here to improve through two channels.

The first is learning by doing; as households work harder and therefore produce more they learn better how to run their processes and this raises A_t.across the economy. This greater productivity is an externality here. in the sense that it comes about because the whole economy produces more and not through the product of a single household on its own. Therefore the marginal product to the household is unaffected because that household's work efforts alone will not add to its productivity.

The second is innovation by households in finding out about better processes. We assume that there is some innovative or entrepreneurial activity a household can undertake which involves spending the time denoted as z above. In endogenous growth models one key channel of growth is via labour being withdrawn from 'normal' work and being used for an activity that raises productivity. Here we think of it as 'innovation', as in Klette and Kortum (2004); in Lucas' models (Lucas, 1988) it would be 'education'; in models stressing R&D, as in Aghion and Howitt (1998), it would be research activity. Notice that in all three ways that productivity growth might be enhanced, the maximisation issue is exactly the same: the household must divert an appropriate amount of time away from standard work into this growth-enhancing activity. It decides how much time to devote to z by maximising its expected welfare as above.

We write the growth of productivity as:

$$\frac{A_{t+1}}{A_t} = a_0 + a_1 z_t + a_2(1 - x_t) + u_t \tag{7}$$

where u_t is an error process, and the parameter a_1 nets out any effect of more entrepreneurial time on learning by doing (hence to be precise $a_1 = a_1' - a_2$, where a_1' is the direct effect of the entrepreneurial activity on productivity growth).

Going back therefore to the household's optimising decision, its first order condition for z_t at time 0 is given by[2]

$$0 = E_0 \sum_{t=1}^{\infty} a_1 \frac{A_0}{A_1} \beta^t \frac{y_t}{(1+\tau_t)c_t} - \lambda_0(w_0 + \pi_0)$$

from which we can obtain

$$z_0 = \frac{\left\{E_0 \sum_{t=1}^{\infty} \beta^t (\frac{1}{(1+\tau_t)}) \frac{y_t}{c_t}\right\}}{\lambda_0(w_0+\pi_0)} - \frac{(a_0 + a_2(1-x_0)) + u_0}{a_1}$$

We now compare $a_1 z_0$ with (7) and find that

$$\frac{A_1}{A_0} = \frac{a_1 \left\{E_0 \sum_{t=1}^{\infty} \beta^t (\frac{1}{(1+\tau_t)}) \frac{y_t}{c_t}\right\}}{\lambda_0(w_0+\pi_0)} \tag{7a}$$

What this is telling us is that entrepreneurs make allowance for the productivity growth already coming from learning by doing when they decide on optimal effort; they exactly offset this effect in their decision, so that it is purely entrepreneurs that determine productivity growth. To evaluate this equation we note that our tax rates are a random walk and that (see appendix) $\frac{c_t}{y_t}$ is non-stationary. We approximate the latter as a random walk. Omitting second order (variance and covariance) terms then the numerator of (7a) is given by $\frac{\beta}{1-\beta}(\frac{1}{(1+\tau_0)})\frac{y_0}{c_0}$.; then using (5) for w_0 and substituting for λ_0, we obtain

$$\frac{A_1}{A_0} = \left\{\frac{a_1\beta}{1-\beta}\frac{1}{(1+\tau_0)}\frac{y_0}{c_0}\right\} / \left\{\frac{1}{c_0(1+\tau_0)}(\frac{\alpha_0 l c_0(1+\tau_0)}{x_0} + \pi_0)\right\} = \left\{\frac{a_1\beta}{1-\beta}\frac{y_0}{c_0}\frac{x_0}{\alpha_0 l}\right\} / \left\{(1+\tau_0) + \pi_0\frac{x_0}{\alpha_0 l c_0}\right\} \tag{7b}$$

We now linearise this as

$$\frac{A_1}{A_0} = \phi_0 - \phi_1(\tau_0 + \pi_0') + error_0$$

where $\pi_0' = \pi_0 \frac{x_0}{\alpha_0 l c_0}$ is the tax on entrepreneurs normalised by the ratio of preference-adjusted leisure to consumption; and since $\frac{A_{t+1}}{A_t} = \Delta \ln A_{t+1} + 1$, gathering constants as ϕ_0' and letting $u_t' = error_t$ we obtain

$$\Delta \ln A_{t+1} = \phi_0' - \phi_1(\tau_t + \pi_t') + u_t'$$

What we see is that the 'tax rate on entrepreneurs' consists of both the general tax rate and the particular imposts levied on business activity as such. These would include corporation tax for example if it is not rebated to the shareholder as an imputed tax already paid on dividends. Here we pay especial attention to the levies on entry and exit from business as measured by international bodies.[3]

[2] This is obtained by differentiating the Lagrangean above with respect to z_0 remembering that (7) determines A_t. Thus we obtain
$0 = E_0 \sum_{t=1}^{\infty} a_1 \frac{A_0}{A_1} \lambda_t y_t - \lambda_0(w_0 + \pi_0)$
Note that $\frac{\partial y_t}{\partial z_0} = k_t^{\gamma} X_t^{\zeta} (1 - x_t - z_t)^{1-\gamma-\zeta} \frac{\partial A_t}{\partial z_0} = \frac{y_t}{A_t}\frac{\partial A_t}{\partial z_0} (t \geq 1)$; while since $A_t = \frac{A_t}{A_1}\frac{A_1}{A_0} A_0$ and $\frac{A_t}{A_1}$ is independent of z_0 it follows that $\frac{\partial A_t}{\partial z_0} = \frac{A_t}{A_1} A_0 \left\{\partial \frac{A_1}{A_0} / \partial z_0\right\} = \frac{A_t}{A_1} A_0 a_1$. Hence finally $\frac{\partial y_t}{\partial z_0} = y_t \frac{A_0}{A_1} a_1 = y_t \frac{a_1}{a_0 + a_1 z_0 + a_2(1-x_0) + u_0}$

[3] Our assumption in the above is that learning by doing is external to the household, that is it only kicks in in response

3.1 Completing the model:

To complete the model, we require:

(1) the government budget constraint which brings together the revenues it raises from households and the transfer it pays over; the government too can borrow from abroad via foreign bonds but for simplicity we assume it does not as it has no impact on the model's workings.

$$\tau_s c_t + \pi_t z_t = \Gamma_t$$

(2) goods market clearing in which households buy consumption and investment goods (gross investment = $k_{t+1} - (1-\delta)k_t$) from firms who may supply them either from their own output or from net imports (m) purchaseable on the world market at going (exogenous) world prices. If firms have excess output they export it onto the world market at these prices. We set world prices at unity, ignoring terms of trade changes as an exogenous variable with no impact on the model's workings.

$$y_t + m_t = c_t + k_{t+1} - (1-\delta)k_t$$

It can easily be verified that the balance of payments constraint is implied (via Walras' Law) by the household and government budget constraints, the constraint that firms have no surplus profits (all earnings are distributed via wages and dividends) and goods market clearing.[4]

3.2 Solution of the model:

The model is most conveniently analysed in loglinear form.We have from (3):

$$\ln c_t = -\ln(1+\tau_t) + E_t \ln c_{t+1} + E_t \ln(1+\tau_{t+1}) - E_t \ln(1+r_{t+1}) + constant \quad (8)$$

Here we have made use of the fact that when x is lognormally distributed $\ln Ex = E\ln x + 0.5 var \ln x$. We assume throughout that our errors are lognormal and have a constant variance, so that the variance and covariance terms are included in the constant term above. To loglinearise x_t we proceed by linearising (5b) as:

$\frac{\Delta x_t}{\bar{x}} = \overline{(\frac{\alpha lc}{wx})}\Delta\tau_t + (\frac{1+\tau_t}{x_t})\Delta(\frac{\alpha_t lc_t}{w_t})$ which we can approximate, adding a constant of integration, as:

$$\ln x_t = \overline{(\frac{\alpha lc}{wx})}\tau_t + \ln c_t - \ln w_t + \ln \alpha_t + \text{constant} \quad (9)$$

Using (5a) above to substitute out wages (and assuming $\ln x_t \approx \ln(1-x_t-z_t)$ because leisure and working time are approximately equally divided and assuming entrepreneurial time is very small relative to the other two) yields:

$$\ln x_t = l^*\tau_t + 0.5\{\ln c_t - \ln y_t\} + 0.5\ln\alpha_t + \text{ constant} \quad (10)$$

where $l^* = 0.5\left[\overline{\{\frac{\alpha lc}{wx})}\right]$.

It can be shown that $\{\ln c_t - \ln y_t\}$ is a non-stationary process (for the formal derivation see the appendix); the reason lies in the permanent income hypothesis, that consumption equals permanent income from home output plus interest on foreign assets. The stock of foreign assets then follows a random walk because consumers use foreign assets as a way of smoothing any fluctuations of home income around permanent income..It follows that we can replace this term (plus the stationary preference

to all households' work efforts..If one were to assume that learning by doing is internal to the household- that is, when one works harder, one's own productivity increases independent of whether others are also working harder- then the maximisation is more complex. In this case, the decision to work in normal employment is influenced also by the expected productivity return. However, it turns out that productivity growth still depends negatively both on the general tax rate and on the special business imposts, though the coefficients are not the same as above.

[4] Thus taking the household budget constraint
$(1+\tau_t)c_t + k_{t+1} - (1-\delta)k_t + b_{t+1} = y_t + (1+r_t)b_t + \Gamma_t - \pi_t z_t$
we note that the tax terms cancel with the government transfer via the government's budget constraint so that
$c_t + k_{t+1} - (1-\delta)k_t + b_{t+1} = y_t + (1+r_t)b_t$
Now we use market clearing to substitute out for y_t so that
$c_t + k_{t+1} - (1-\delta)k_t + b_{t+1} = c_t + k_{t+1} - (1-\delta)k_t - m_t + (1+r_t)b_t$.
Cancelling terms yields the balance of payments
$b_{t+1} - b_t = r_t b_t. - m_t$
where net lending abroad (the capital account deficit) equals net interest from abroad minus net imports (the current account surplus).

error) with a non-stationary error process, which will plainly be correlated with the other errors in the model and may also be autocorrelated..

In order to solve the model and eliminate expected future terms it is necessary to make assumptions about the behaviour of the exogenous variables. We assume that world real interest rates, r, are stationary and autoregressive of order 1: $r_t = (1-\lambda)r^* + \lambda r_{t-1} + \epsilon_t$. We assume that all the policy variables, essentially the tax rates, are random walks, which is frequently found empirically since tax changes are generally the result of policy change which is by construction unexpected.

A full explicit solution in terms of the forcing processes requires dynamic programming. However as noted earlier we treat the log of the consumption/income ratio in (10) as a random walk error process and include it with the error due to work preferences, also likely to be a random walk like productivity Thus our model for estimation becomes:

(7) $\Delta \ln A_{t+1} = \phi_0' - \phi_1(\tau_t + \pi_t') + u_t'$;we integrate this into levels to become $\ln A_t$=$-\phi_1 \sum_{i=1960}^{t}(\tau_i + \pi_i') + \phi_0' t + \zeta_t$ where $\zeta_t = \frac{u_t'}{1-L}$ We then susbtitute this for $\ln A_t$ in equation 6,which becomes our first equation to be estimated

(10) $\ln(1-x_t) = -l^*\tau_t + v_t$;

where as noted above we have treated $\ln(1-x) \approx -\ln x$

(6) in loglinearised form is now:

$$\ln y_t = (\frac{-\phi_1}{1-\gamma}) \sum_{i=1960}^{t} (\tau_i + \pi_i') + \psi \ln(1-x_t) + (1-\psi)\ln X_t + c + \phi_0' t + \xi_t + \zeta_t \qquad (6)$$

(where we have neglected the direct effect of z_t on output for convenience as very small; where $\psi = \frac{1-\gamma-\zeta}{1-\gamma}$ and where $-\frac{\gamma}{1-\gamma}\ln r_t = \xi_t$ is the effect of world real interest rates- this is assumed to be picked up by the time effects in the panel estimation process (as is $\phi_0' t$) while $(1-\psi)\ln X_t$ is assumed to be picked up by the country and time effects..

Thus (10), and (6) are our two equations of the model to be taken to the data.

These equations have been chosen for tractability in the context of our panel set-up and data set. Forward-looking terms for example must be substituted out because we are in practice unable to solve each country model separately over the sample period. Other variables, such as wages, we have no data for. We could have had an additional equation for consumption and may do so in a more elaborate version later: but it too must be a solution equation and so little appears to be gained at the cost of an extra error term to be bootstrapped. One of our two equations (6) contains the production function which is essentially structural, an 'engineering relationship' with capital solved out in terms of its first-order condition. The labour supply and productivity equations (the latter substituted into the production function) are solution equations derived from first order conditions and a solution for their components using approximation techniques that exploit the unit root properties of the exogenous productivity, preference and tax processes and of the consumption-GDP ratio. The resulting two error terms include what the data and model imply are the omitted effects of the exogenous errors. These effects do not include the direct effects of interest in the model, of tax rates on productivity growth and on labour supply; these are explicitly included in the model.

Thus the two equations constitute a 'structural model' in the sense that they jointly exactly replicate the data country by country in a way entirely constrained by the model and its solution method.

We present in what follows two versions of the model for testing on the panel data set:

(1) The model as estimated at the structural equation level in conformity with the theory.

(2) The model with the tax coefficients set to zero: a 'no-tax-effect model', with the other coefficients re-estimated with this constraint.

Empirical work:

The procedure we follow to test the model we have set out is that of bootstrapping. The idea is that we treat the model - in the form of the two equations set out above- as the true or 'null' hypothesis.We estimate this model on the available post-war annual data, for 76 countries from 1970-2000. The resulting 2 structural errors for each country-period are thus the implied 'true errors' under the model. These errors and the tax rates have time-series properties which we assume differ country by country; we estimate a time-series process for each country error and tax process, which in turn implies a set of 4 random errors (structural and tax) for each country over the period 1970-2000. Our bootstrapping procedure is them to draw the whole vector of random errors as a 76-country bloc repeatedly with replacement for a 30-year sample period (we draw them as a vector to retain any patterns of simultaneous correlation); input them into the country time-series processes to generate a resulting set of 30-year errors; input these in turn into the model to generate a 30-year sample of data for the endogenous variables. Such a 30-year sample of data is one pseudo-sample. We generate 1000 of these pseudo-samples. The idea is that these 1000 pseudo-samples represent the sampling variation that would occur according to the model.

We then investigate whether data descriptions that would emerge from the model are rejected by the data.. We do this by estimating the descriptive form on the actual data and also on the pseudo-samples; if the estimate generated on the actual data lie within the 95% confidence limits given by the pseudo-samples, then we say that the model is not rejected by the data and vice versa.

The results for the model equations are as follows (with fixed country and time effects on each equation). We estimate equation (6) as:

$$\text{(1)}\quad \ln y_t = c_1 + \underset{(0.083)}{0.715}\ln(1-x_t) - \underset{(0.003)}{0.014}\sum_{i=1960}^{t}(\tau_i + \pi_i')$$

Number of obs	= 2280
F(105, 2174)	= 864.00
R^2	= 0.9756
$\bar{R}^2$	= 0.9745
Root MSE	= 0.1649

We estimate the structural labour supply equation, (10) as:

$$\text{(3)}\quad \ln(1-x_t) = c_2 + \underset{(0.01)}{0.0128}\ln(1-\tau_t)$$

Number of obs	= 2280
F(103, 2024)	= 278.53
R^2	= 0.9308
$\bar{R}^2$	= 0.9275
Root MSE	= 0.0442

The error term from this equation is a combination of labour supply preferences and the log of the consumption/income ratio.

Bootstrapping this model as described above generates 1000 pseudo-samples. With it we then investigate a data description for growth. In it growth depends on the (general plus entrepreneurial) tax rate and the rate of change of the general tax rate (the latter because growth in output not caused by productivity depends of the growth in labour supply which in turn depends on the rate of change of the general tax rate). Note that we have taken $\ln(1 - taxrate) \approx -taxrate$.

Growth rate and taxation- descriptions of data with model-generated 95% confidence bands

We now turn to our test of this above model against the data. We proceed as follows. First we regress the data for growth on a set of potential regressors with a view to capturing the best ('reduced form') description of the data. We consider four sets of regressors: the level of business tax, $(\tau_t + \pi_t')$; the rate of change of personal tax, $\Delta\tau_t$; country dummies; and time dummies. (A Hausman test for this equation rejected the random effects estimator as inconsistent so we do not report those estimates.)

With fixed country and time effects				
$\Delta \ln y_t = \alpha_1(\tau_t + \pi_t') + \alpha_2\Delta\tau_t$			Number of obs	= 1748
	Actual	'Reduced form' standard errors	F(100,1648)	= 3.87
			R^2	= 0.1903
α_1	-0.043	0.027	$\bar{R}^2$	= 0.1411
α_2	-0.039	0.043	Root MSE	= 0.0506

Table 1: Regression of Growth on Business Tax and the Rate of Change of Personal Tax With Fixed Time and Country Effects

In these equations $\Delta\tau_t$ was insignificant though of the right sign. This term picks up the temporary effect on growth of the change in the personal tax rate (which affects labour supply); this effect however is very poorly determined, which is perhaps not surprising as it works through labour supply and we know from other work that labour supply effects depend on expected tax and other variables. Here we are unable to pick up expectations effects (which could introduce a lead or a lag in the tax variable). We therefore decided to look also at an equation with solely the business tax effect whose level should directly determine growth on a permanent basis; we would expect this effect to come through powerfully in the data description and indeed it seems to do so.

The resulting equation is:

With fixed country and time effects			Number of obs	= 1748
$\Delta \ln y_t = \alpha_1(\tau_t + \pi'_t)$			F(99,1649)	= 3.90
	Actual	'Reduced form' standard errors	R^2	= 0.1899
			$\bar{R}^2$	= 0.1412
α_1	-0.050	0.027	Root MSE	= 0.0506

Table 2: Regression of Growth on Business Tax With Fixed Time and Country Effects

Using panel data with fixed effects may not be the most efficient model to run. Estimating the model with random effects will give a more efficient estimator (the reason for this is that the estimator saves degrees of freedom by not using the fixed country dummies but instead using the regression with fixed country dummies with a weight, to coeerct the regression with time dummies only). The results for the random effects estimator are shown in Table 3.

With random effects			Number of obs	= 1748
$\Delta \ln y_t = \alpha_1(\tau_t + \pi'_t)$			Wald $\chi^2(1)$	= 9.91
	Actual	'Reduced form'	R^2 within	= 0.0035
		standard errors	R^2 between	= 0.0615
α_1	-0.043	0.014	R^2 overall	= 0.0088

Table 3: Regression of Growth on Business Tax with Random Effects

To test whether we should use the fixed or random effects model we run a Hausman test, the results from this test were

$\Delta \ln y_t = \alpha_1(\tau_t + \pi'_t)$						
	Fixed	Random	Difference	Standard error	$\chi^2(1)$	P-value
α_1	-0.050	-0.043	-0.007	0.022	0.10	0.751

Table 4: Hausman Test

From Table 4 we find that we can use either fixed or random effects in the actual data sample without serious risk of inconsistency. This is of interest in that it tells us that the effect of business tax in the descriptive regression is well-determined and highly significant.

We now turn to the bootsrapping exercise where we wish to establish the sampling distributions of the descriptive regression coefficients according to our model. For this exercise it is essential that the estimator used is consistent in all the potential data samples; otherwise the distribution of 'potentially estimated' coefficients will be wrongly measured. Hence in what follows we use the fixed effects estimator throughout the bootstrapping process, since it is known definitely to be consistent in all samples; thus each sample estimate will give us a 'central' value for the coefficients.

We now report how our chosen descriptive equation- with the business tax rate only- compares with our basic model. We take the descriptive regression and run it on our bootstrap data for each model. As noted earlier, this allows us to find the 95% confidence interval implied by the model. In addition it gives the overall 'M-metric', that is the percentile in the bootstrap distribution of all parameters[5] jointly where the actual data regression lies; the higher the percentile, the further into the tail the actual

[5] In assessing whether the model is rejected or not we need to use the joint distribution of all the parameters in the description. The 95% confidence intervals shown by each parameter apply to that parameter taken on its own, that is holding the other parameters as given by their estimated values. For the model as a whole the question is whether the joint values of the estimated parameters lie within the '95% contour' of the joint distribution. The idea here is that the model generates a joint distribution of the descriptive ('reduced form') parameters; ranging from the most likely joint values (the mean of the bootstrap distribution) out to the least likely. The joint value likelihood can be computed by assuming a multi-variate normal distribution; however this is only used up to the ordering of the joint variates, the likelihood itself is discarded to give a metric (the Mahalanobis metric). Instead the probability of each metric then being given by the likelihood, it is determined from the bootstraps; specifically the 95% value of the metric is given by the value of the metric when 95th percentile bootstrap. The model as a whole is then rejected if the actual metric estimated on the data exceeds this 95th percentile. One can think of this 95th percentile as a contour line on a joint distribution. Clearly such a rejection is related somehow to the rejection on the parameters individually; however, this relationship depends on the covariance matrix of these parameters which is a crucial ingredient of the joint distribution. Thus there is no simple link from the individual rejections to the overall rejection of the model.

regression lies. As it happens, in this case with only one parameter of interest the M-metric is directly related to the distribution of this one parameter.

$\Delta \text{lny}_t = \alpha_1(\tau_t + \pi'_t)-$With fixed country and time effects				
95% interval for basic model	Actual	Lower	Upper	M-metric
a_1	-0.050	-0.054	0.017	90.8%

Table 5: Bootstrap Results for Model with Estimated Tax Effects

What we see is that the model is accepted at the 95% level. This is itself of some interest. However, we do not know whether the data will also accept other models that contradict our model. To assess this we create an alternative model of this sort: in this we set the tax coefficients to zero, both on business tax and on personal tax. Thus the alternative model asserts that taxes have no effect; the only identified effects are of labour in the production function, the rest is the effect of country and time dummies. The model is reestimated in this way and new error terms extracted and bootstrapped in just the same way as for the principal model. We obtain new bootstrap distributions for the data descriptive equation as follows:

Alternative (no-tax-effect) model:

$\Delta \text{lny}_t = \alpha_1(\tau_t + \pi'_t)-$With fixed country and time effects				
95% interval for 0-tax-effect model	Actual	Lower	Upper	M-metric
a_1	-0.050	-0.040	0.030	98.1%

Table 6: Bootstrap Results for Model with Zero Tax Effect

We see that this alternative model is rejected, with an M-metric of 98.1%. Thus our model is accepted by the data at the 95% level, whereas the alternative model with no tax effect is rejected..

So far we have tested the basic model from the zero side, so to speak- to see whether it dominates a no-tax-effect model. It is also of interest to test it from the other side: to see whether a business tax effect higher than freely estimated would satisfy the data description..So we also reestimated the model imposing an increased coefficient on business tax.and retrieving the implied new errors. We used two cases, one in which we set the coefficient to -0.02 and another in which we set the coefficient to -0.04. The results for the -0.02 case are shown in Table 7 and the -0.04 case in Table 8.

Coefficient on Business Tax Set to -0.02				
$\Delta \text{lny}_t = \alpha_1(\tau_t + \pi'_t)-$With fixed country and time effects				
	Actual	Lower	Upper	M-metric
a_1	-0.050	-0.060	0.012	82.1%

Table 7: Bootstrap Results for Model with Tax Effects and Coefficient on Business Tax set to -0.02

What is interesting about this is that there is an improvement in the model's performance vis-a-vis the data description.as the model's business tax effect is raised. Thus if it is raised in absolute size by two standard errors to -0.02 (from the estimated -0.014) the M-metric falls from 90.8% to 82.1%. However the improvement stops from here on. If it is raised further to -0.04 it improves barely at all; this must be because it induces errors in the model whose variation is correlated with the tax and offset its effect on the distribution of a_1. Hence the data estimation of the model itself combined with the data description tell us that a business tax parameter of between -0.014 and -0.02 is the most compatible with the data.[6]

[6] If we use the general data description with both variables entered, the distribution is not so tightly defined. We obtain:

$\Delta \text{lny}_t = \alpha_1(\tau_t + \pi'_t) + \alpha_2 \Delta\tau_t-$With fixed country and time effects				
95% interval for basic model	Actual	Lower	Upper	M-metric
a_1	-0.043	-0.044	0.027	
a_2	-0.039	-0.192	0.027	62.5%

and

$\Delta \text{lny}_t = \alpha_1(\tau_t + \pi'_t) + \alpha_2 \Delta\tau_t-$With fixed country and time effects				
95% interval for 0-tax-effect model	Actual	Lower	Upper	M-metric
a_1	-0.043	-0.029	0.041	
a_2	-0.039	-0.197	0.023	73.5%

Here we see that neither model is rejected. But if we compare the no-tax-effect model with the basic model we see that its M-metric at 73.5% lies well above the 62.5% of the basic model. We can interpret this as a measure of relative likelihood

Coefficient on Business Tax Set to -0.04				
$\Delta \text{lny}_t = \alpha_1(\tau_t + \pi'_t)$−With fixed country and time effects				
	Actual	Lower	Upper	M-metric
a_1	-0.050	-0.062	0.011	81.7%

Table 8: Bootstrap Results for Model with Tax Effects and Coefficient on Business Tax set to -0.04

3.3 A discussion of the empirical results

We may start by discussing the 'conventional' way of testing the model using the standard reduced form approach. Thus we note that the model implication- viz that the level of business tax and the rate of change of generat tax both affect growth- meets a mixed reception. The business tax effect alone is fairly significant against the usual zero alternative; the general tax effect is not. We concluded from this that the data description should not include the general tax effect as it does not contribute to explaining growth. We might also have concluded that there was evidence of a business tax effect. However as we have argued above this is not a persuasive test for two reasons. First, the error terms in the reduced form will include omitted nonlinear effects of tax on growth that can bias the 'reduced form' coefficient. Second, other models in which tax plays no part could also generate this 'reduced form' result.

So we reviewed next the evidence from the bootstrapping method, where instead of the confidence intervals generated by the 'reduced form' we look at those produced by bootstrapping the structural model. We found here that the model was accepted by the data description and furthermore that an alternative model with no tax effects was rejected by it and thus also dominated in likelihood by our model. In fact a model with a higher business tax coefficient of -0.02 is more likely viewed from its fit with the data description equation (though less likely viewed as a dierct estimate from teh production function).

What is also striking is the insight afforded by the bootstrapping procedure into the biases in the 'reduced form' coefficients under the null hypothesis. Thus we know from simulating the model for a shock to the business tax rate that growth (in steady state) increases by 0.14%-0.02% for every 0.1 (ie 10 percentage point) fall in the business tax rate under the model. However the 'reduced form' coefficients give a value for this business tax effect that is up to three and a half times as big.. This indicates a huge amount of bias in the 'reduced form' coefficients; these values bear little relation to what the model would produce as the simulated effect. The model when bootstrapped reveals that the correlation of the tax shocks with the errors creates massive bias in the 'reduced form' estimates..To put it in concrete terms, for example when the business tax rate is cut this causes a rise in consumption and labour supply as well as in productivity growth; the former two create an independent source of output increase over and above the steady state increase; this association raises the estimated effect of a business tax cut on growth.

A last point of interest is that we were unable to change the structural coefficients on tax upwards beyond a certain point, even though the 'reduced form' results would have been better fitted by a large business tax coefficient, cetris paribus. What we found was that the data forced the structural model errors to offset the effect of raising the business tax coefficient beyond a certain point. Had we kept the freedom to 'make up' the structural errors we would have been able to fit the 'reduced form' results easily. But because we forced the structural model to fit the data through the implied errors used in the bootstrapping, the fitting of the 'reduced form' was constrained. It is as if our results can only emerge satisfactorily if they can go through two mincers, each of a different shape; a structural mincer and a 'reduced form' mincer; only if the model can force its way through both are its results to be believed.

of each model, conditional on the data. That is, the data regression is closer to the most likely parameter combination according to our model than according to the alternative model. If we could assume a particular likelihood distribution- eg multi-variate normal- then we could translate the M-metrics into exact likelihoods.

We also find that the model with the higher business tax effect (of -0.02) performs better than the one with the estimated tax effect, just as in the case focused on in the text. Hence if we were to use this data description, we would get essentially the same results if we were to set the confidence level higher, at say 65%. We would reject the no-tax-effect model and accept the two tax-effect ones, with the likely tax effect lying somewhere between the two. If we maintain the 95% confidence level it still remains the case that this is the likely tax effect range.

$\Delta \text{lny}_t = \alpha_1(\tau_t + \pi'_t) + \alpha_2 \Delta\tau_t$−With fixed country and time effects				
95% interval for 0-tax-effect model	Actual	Lower	Upper	M-metric
a_1	-0.043	-0.051	0.021	
a_2	-0.039	-0.183	0.031	53.5%

4 Conclusions

The overall conclusion of this empirical work on estimating and simulating general equilibrium models of growing small open economies is that they are consistent with the panel post-war data. Growth does indeed depend on tax rates, particularly the business tax rate (interpreted as the tax, including regulative, burden on an individual businessman). But 'reduced form' estimates of this effect are unreliable and biased upwards..Instead one must construct the structural model, check whether its estimates cohere with the data both at the structural level and then at the 'reduced form' level; this paper is an illustration of how this bootstrap-based technique exploits the data at both these levels. Then the parameters in this structural construct, thus tested, can be used to estimate the effect of a shock to the tax rates.

References

[1] Agell, J., P. Englund and J. Södersten (1995), 'The Swedish tax reform: an introduction', Swedish Economic Policy Review, Vol. 2, pp. 219-228.

[2] Aghion, P., and P. Howitt (1998) Endogenous Growth Theory. Cambridge, MA: MIT Press.

[3] Alesina, A., S. Ardagna, R. Perotti and F. Schiantarelli. (2002). 'Fiscal Policy Profits and Investment,' American Economic Review, June, 92: 571-89.

[4] Barro, R.J. (1991), 'Economic growth in a cross-section of countries', Quarterly Journal of Economics, 106(2), pp.407 433, May.

[5] Barro, R.J. and J.W. Lee (2000), 'International Data on Educational Attainment: Updates and Implications,' manuscript, Harvard University, February.

[6] Bassanini, A. and S. Scarpettta (2001), 'Does human capital matter for growth in OECD countries? Evidence from PMG estimates', OECD Economics Department Working Papers, No. 282.

[7] Becsi, Z. (1996). 'Do State and Local Taxes Affect Relative State Growth?' Economic Review 81, 2 (March/April): 18-36.

[8] Benson, B., and R. Johnson (1986). 'The Lagged Impact of State and Local Taxes on Economic Activity and Political Behaviour.' Economic Inquiry 24 (July): 389-401.

[9] Bleaney, M., N. Gemmell and R. Kneller (2000). 'Testing the endogenous growth model: public expenditure, taxation and growth over the long run,' Canadian Journal of Economics.

[10] Cashin, P. (1995). 'Government spending, taxes and economic growth,' IMF Staff Papers, 42(2), 237-69.

[11] Chao, J.C.P., and H. Grubel. (1998). 'Optimal Levels of Spending and Taxation in Canada'. In Herbert Grubel (ed.), How to Use the Fiscal Surplus: What is the Optimal Size of Government? Vancouver: Fraser Institute.

[12] Dar, A. and S. AmirKhalkhali (2002), 'Government size, factor accumulation, and economic growth: evidence from OECD countries', Journal of Policy Modeling 24(7-8), 679-692.

[13] Easterly, W. and S. Rebelo (1993), 'Marginal income tax rates and economic growth in developing countries', European Economic Review, Vol. 37.

[14] Engen, E. M., and J. Skinner (1996). 'Taxation and Economic Growth,' NBER Working Papers 5826, National Bureau of Economic Research, Inc.

[15] Folster, S. and M. Henrekson (2000), 'Growth Effects of Government Expenditure and Taxation in Rich Countries', Stockholm School of Economics, Working Paper No. 391, June.

[16] Fudenberg, D. and J. Tirole (1983) 'Learning-by-Doing and Market Performance' The Bell Journal of Economics, 14 (2), 522-530.

[17] Gillman, M. and Michal Kejak (2005), ' Inflation and Balanced-Path Growth with Alternative Payment Mechanisms', Economic Journal, Vol 115 (January): 247-270.

[18] Grier, K., and G. Tullock (1989). 'An Empirical Analysis of Cross-national Economic Growth, 1951-80.' Journal of Monetary Economics 24: 259-76.

[19] Grossman, P. (1988). 'Government and Economic Growth: A Non-Linear Relationship.' Public Choice 56: 193-200.

[20] Hansson, P. and M. Henrekson (1994), 'A new framework for testing the effect of government spending on growth and productivity', Public Choice 81, pp. 381-401.

[21] Heston, A., R. Summers and B. Aten (2002), Penn World Table Version 6.1, Center for International Comparisons at the University of Pennsylvania (CICUP), October.

[22] King, R.G. and S. Rebelo (1990), 'Public policy and economic growth: developing neoclassical implications', Journal of Political Economy, Vol. 98.

[23] Klette, T. J. and S. Kortum (2004) 'Innovating firms and Aggregate Innovation', Journal of Political Economy, 112 (5), 986-1018.

[24] Koester, R. and R. Kormendi (1989), 'Taxation, aggregate activity and economic growth: cross-country evidence on some supply-side hypotheses', Economic Inquiry, Vol. 27, Number 3.

[25] Leach, G. (2003) The negative impact of taxation on economic growth. London: Reform. September.

[26] Leibfritz, W., J. Thornton and A. Bibbee (1997). 'Taxation and economic performance,' OECD Working paper No. 176.

[27] Levine, R. and D. Renelt (1992), 'A sensitivity analysis of cross-country growth regressions', American Economic Review, Vol. 82.

[28] Lucas, R.E. (1988) 'On the mechanics of economic development', Journal of Monetary Economics, 75(4), 321-334.

[29] Mackness, W. (1999). Canadian Public Spending: The Case for Smaller, More Efficient Government. Public Policy Source 13. Vancouver, BC: The Fraser Institute.

[30] Minford, P. (2006) 'What are the right models and policies for a world of low inflation?' NIESR Review, May 2006.

[31] Minford, P., P. Sofat, E. Nowell and N Srinivasan (2006) 'UK Inflation Persistence: Policy or Nature?', CEPR Discussion Paper No 5608, available on Minford Cardiff University webpage.(http://www.cf.ac.uk/carbs/econ/minfordp/index.html)

[32] Minford, P., K. Theodoridis and D. Meenagh (2005) 'A Bootstrap Test for the dynamic performance of Macroeconomic Models - an outline and some experiments', available on Minford Cardiff University webpage (see last ref).

[33] Peden, E. (1991). 'Productivity in the United States and Its Relationship to Government Activity: An Analysis of 57 Years, 1929-1986.' Public Choice 69: 153-73.

[34] Peden, E., and M. Bradley (1989). 'Government Size, Productivity, and Economic Growth: The Post-War Experience.' Public Choice 61: 229-45.

[35] Plosser, C.I. (1992), 'The search for growth', in Policies for Long-Run Economic Growth, Federal Reserve Bank of Kansas City.

[36] Scully, G. W. (1989). 'The Size of the State, Economic Growth and the Efficient Utilization of National Resources.' Public Choice 63: 149-64.

[37] — (1991). Tax Rates, Tax Revenues and Economic Growth. Policy Report 98. Dallas, TX: National Center for Policy Analysis.

[38] — (1995). 'The 'Growth Tax' in the United States.' Public Choice 85: 71-80.

[39] Slemrod, J. (1995), 'What do cross-country studies teach about government involvement, prosperity, and economic growth?', Brookings Papers on Economic Activity, 2.

[40] Smith, D. B. (2001), Public Rags or Private Riches? High Public Spending Makes Us Poor, Politeia, London.

[41] Vedder, R. K. (1993). Economic Impact of Government Spending: A 50-State Analysis. Policy Report 178. Dallas, TX: National Center for Policy Analysis.

[42] Vedder, R. K., and L. E. Gallaway (1998). Government Size and Economic Growth. Paper prepared for the Joint Economic Committee of the US Congress.

5 Appendix: the non-stationarity of $\{\ln c_t - \ln y_t\}$

Start with the household budget constraint after substituting out tax and transfer terms via the government budget constraint and wage and dividends from the firm's first order conditions; this is line 3 of footnote 2:

$$c_t + k_{t+1} - (1-\delta)k_t + b_{t+1} = y_t + (1+r_t)b_t \tag{A1}$$

In expectational form the household's consumption plan must satisfy this constraint as follows after an infinite forward recursion in the value of future bonds:

$$(1+r_t)b_t = c_t - y'_t + E_t \sum_{i=0}^{\infty} \left\{ \prod_{j=1}^{i} (1+r_{t+j} \right\}^{-1} (c_{t+i} - y'_{t+i}) \tag{A2}$$

where $y'_t = y_t - [k_{t+1} - (1-\delta)k_t]$
Now note that from the household's first order condition

$$E_t \left\{ \prod_{j=1}^{i} (1+r_{t+j} \right\}^{-1} c_{t+i} = \beta^i c_t \text{since for example } c_t = \frac{1}{\beta} E_t \frac{c_{t+1}}{1+r_{t+1}} = \frac{1}{\beta^2} E_t \frac{c_{t+2}}{(1+r_{t+1})(1+r_{t+2})} \tag{A3}$$

It follows that

$$c_t = (1-\beta) \left\{ (1+r_t)b_t + y'_t + E_t \sum_{i=0}^{\infty} \left\{ \prod_{j=1}^{i} (1+r_{t+j} \right\}^{-1} y'_{t+i}) \right\} \tag{A4}$$

The term inside the braces is the household's spendable wealth hence the whole RHS expression is permanent net income or

$$c_t = .(1-\beta)(1+r_t)b_t + \overline{y}'_t \tag{A5}$$

In steady state (at T) we have (where g is the growth rate)

$$\begin{aligned} c_T &= (1-\beta) \left\{ (1+r^*)b_T + \sum_{i=0}^{\infty} \left\{ \frac{1+g}{1+r^*} \right\}^i [1 + \frac{\gamma\delta}{r^*+c}] y_T \right\} = (1-\beta)(1+r^*) \left\{ b_T + \frac{1}{r^*-g} [1 + \frac{\gamma\delta}{r^*+c}] y_T \right\} \\ &= (1-\beta)(1+r^*)b_T + \overline{y}'_T \end{aligned} \tag{A6}$$

in which all of c_T, b_T, y_T will be growing at g.
Now consider the movement of $\frac{c_t}{y_t}$ which from A5) is:

$$\frac{c_t}{y_t} = .(1-\beta)(1+r_t)\frac{b_t}{y_t} + \frac{\overline{y}'_t}{y_t} \tag{A7}$$

Hence using the approximation that $\ln(x+y) = \frac{x}{x+y}\ln x + \frac{y}{x+y}\ln y$

$$\ln c_t - \ln y_t = .(\text{share of net income from abroad}) \ln \frac{b_t}{y_t} + \ln \frac{\overline{y}'_t}{y_t} \tag{A8}$$

From the balance of payments (footnote 2)

$$\frac{b_{t+1}}{y_{t+1}} \frac{y_{t+1}}{y_t} - \frac{b_t}{y_t} = r_t \frac{b_t}{y_t}. - \frac{m_t}{y_t} \tag{A9}$$

or

$$\frac{b_{t+1}}{y_{t+1}} - \frac{b_t}{y_t} = (r_t - g_t)\frac{b_t}{y_t}. - \frac{m_t}{y_t}$$

We know that in steady state $\frac{b_t}{y_t}$ will tend to some steady level. because of household behaviour. However until this has occurred it is driven by a difference equation of the form:

$$x_{t+1} = (1 + q_t)x_t + \xi_t \tag{A10}$$

where $q_t = r_t - g_t$ will vary from positive to negative and $\xi_t = -\frac{m_t}{y_t}$ will move randomly between steady states. Plainly $x_t = \frac{b_t}{y_t}$.will for at least some of the periods between steady states will be a randomly disturbed explosive (or unit root) difference equation and will therefore be non-stationary (in other words it will end up at a new steady state randomly different from its initial value). So therefore will $\ln c_t - \ln y_t$ which contains its log.