The Optimal Monetary Policy Response to Exchange Rate Misalignments*

Campbell Leith
University of Glasgow

Simon Wren-Lewis
University of Exeter

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Abstract: A common feature of exchange rate misalignments is that they produce a divergence between traded and non-traded goods sectors, which appears to pose a dilemma for policy makers. In this paper we develop a small open economy model which features traded and non-traded goods sectors with which to assess the extent to which monetary policy should respond to exchange rate misalignments. To do so we initially contrast the efficient outcome of the model with that under flexible prices and find that the flex price equilibrium exhibits an excessive exchange rate appreciation in the face of a positive UIP shock. By introducing sticky prices in both sectors we provide a role for policy in the face of UIP shocks. We then derive a quadratic approximation to welfare which comprises quadratic terms in the output gaps in both sectors as well as sectoral rates of inflation. These can be rewritten in terms of the usual aggregate variables, but only after including terms in relative sectoral prices and/or the terms of trade to capture the sectoral composition of aggregates. We derive optimal policy analytically before giving numerical examples of the optimal response to UIP shocks. Finally, we contrast the optimal policy with a number of alternative policy stances and assess the robustness of results to changes in model parameters.

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Address for correspondence: Campbell Leith, Department of Economics, University of Glasgow, Adam Smith Building, Glasgow G12 8RT. E-mail c.b.leith@lbss.gla.ac.uk.

1 Overview

One of the major arguments in favour of fixed exchange rate regimes or monetary unions is that under floating rates exchange rate misalignments are frequent, large and damaging to the economy (see Buiter and Grafe, 2003, for example). In contrast, the New Open Economy Macroeconomics (NOEM) literature (see Lane, 2001 for a survey) has mainly focused on technology, preference or cost-push shocks. This is especially true when deriving fully optimal policy (see for example Clarida et al (2001), Corsetti and Pesenti (2005)). Shocks to Uncovered Interest Parity (UIP) or International Risk Sharing (IRS) have generally been introduced to explain the exchange rate fluctuations found in the data (see for example Wang(2005), Bergin (2006), Kollmann(2002)) but in models in which all firms produce traded output. However, one of the notable features of economies that experience large misalignments is the divergence between traded and non-traded sectors. For example, during the appreciation in Sterling between 1997 and 2002, declines in manufacturing output were accompanied by strong growth in the service sector.[1] This appears to accord with the stylised facts on the impact of large misalignments: see Marston (1988) for example.

This suggests that any analysis of the welfare implications of misalignments needs to consider a model which includes non-traded as well as traded goods. In this paper we extend the model of a small open economy in Gali and Monacelli (2005) (henceforth GM) to include a non-traded sector. We compare allocations chosen by a benevolant social planner to the outcome of a market equilibrium assuming flexible prices. We compute the steady state subsidies that would be required for the flexible price equilibrium to reproduce the efficient equilibrium. In the efficient allocation, IRS/UIP shocks have no impact on production in either sector. However, under flexible prices (and assuming constant subsidies), an IRS/UIP shock that causes an appreciation will lead to a reduction in the output of traded goods, but an increase in the output of non-traded goods, consistent with the stylised facts, and this will generate welfare losses.

If we add nominal inertia into the model using Calvo contracts, then policy has the opportunity to respond to these shocks. We compute, for the first time, a quadratic approximation to social welfare based on the utility of the representative agent in this economy.[2] We show that this expression for welfare cannot be expressed in quadratic terms of aggregate output and inflation alone. Either welfare needs to include terms related to individual sectors, or it can proxy sectoral differences by including terms in relative sector prices and/or the terms of trade. We analytically derive optimal policy responses to IRS/UIP shocks under commitment. We then, in a series of numerical simulations, contrast this

[1] See Cobham (2006) for details of this episode and the lack of consensus in the policy debate surrounding it.

[2] There have been papers examining optimal simple rules for such an economy either by examining unconditional expectations of utility (see for example, Ortega and Rebei (2004)) or by looking at conditional expectations of utility using higher order solution methods (e.g. Doyle et al (2006)). However, we believe we are the first to analytically derive a quadratic approximation to utility which enables us to formulate a linear-quadratic policy problem with which to characterise the optimal policy response to shocks in a two-sector open economy.

optimal commitment policy to discretionary policy and other possible policy stances. In general monetary policy attempts to offset these shocks, but cannot do so completely for two reasons: they generate inflation, and they move consumption in the two sectors in different directions.[3] To the extent that prices are more sticky in the non-traded goods sectors this tends to reduce the desire to offset the shock. We examine how these costs vary with the proportion of non-traded goods in total output, and the extent of home-bias in the consumption of traded goods.

The structure of the paper is as follows. Section 2 outlines most of the model, including the household's allocation problem. Section 3 computes the social planner's problem. Section 4 looks at price setting by monopolistically competitive firms under flexible prices, and computes the flexible price equilibrium and the subsidies required to reproduce the efficient allocation. Section 5 adds nominal inertia in the form of Calvo contracts, and Section 6 computes a second order approximation to social welfare in the presence of nominal inertia. Section 7 derives optimal policy under commitment for our two-sector small open economy, and Section 8 evaluates the welfare benefits of optimal and alternative policies for a calibrated version of the model in the face of IRS/UIP shocks. Section 9 concludes and suggests further areas for reseach.

2 The Model

2.1 Households

There are a continuum of households of size one, who differ in that they provide differentiated labour services to firms in their economy. However, we shall assume full asset markets, such that, through risk sharing, they will face the same budget constraint and make the same consumption plans. As a result the representative household will seek to maximise the following objective function,

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t) \tag{1}$$

where C and N are a consumption aggregate and labour supply respectively.

There are three types of good: non-traded goods (subscript N), which are consumed and produced in the home economy, and traded goods (subscript T) which are then broken down into two sub-types depending on whether or not they are produced at home or abroad. (Subscript T,H denotes goods tradeable goods produced at home, while subscript T,F are traded goods produced abroad). Preferences between traded and non-traded goods are given by,[4]

[3] If we allow taxes to be varied optimally along with monetary policy, the inflationary effects of IRS shocks can be eliminated, but the costs associated with different movements in output in each sector remain. To offset the shock completely we would require an additional tax instrument which affected the two sectors differently.

[4] Imposing a unitary elasticity of substitution between home and foreign goods makes our model derivation tractable and does not affect our basic argument. Additionally, Bergin (2006)

$$C = \frac{C_T^{\gamma} C_N^{1-\gamma}}{(1-\gamma)^{(1-\gamma)}\gamma^{\gamma}} \tag{2}$$

The equivalent price index is

$$P = P_T^{\gamma} P_N^{1-\gamma} \tag{3}$$

Optimisation implies the demand curves,

$$C_N = (1-\gamma)(\frac{P_N}{P})^{-1} C \tag{4}$$

$$C_T = \gamma(\frac{P_T}{P})^{-1} C \tag{5}$$

The aggregate C_N takes the form

$$C_N = \left(\int_0^1 C_N(j)^{\epsilon - 1/\epsilon} dj \right)^{\epsilon/(\epsilon-1)} \tag{6}$$

implying the individual demand curve,

$$C_N(j) = (\frac{P_N(j)}{P})^{-\epsilon} C_N \tag{7}$$

The definition of the tradeable goods aggregate is given by,

$$C_T = \frac{C_{T,H}^{1-\alpha} C_{T,F}^{\alpha}}{(1-\alpha)^{1-\alpha}\alpha^{\alpha}} \tag{8}$$

with a corresponding price index,

$$P_T = P_{T,H}^{1-\alpha} P_{T,F}^{\alpha} \tag{9}$$

$C_{T,H}$ are tradeable goods produced at home, and $C_{T,F}$ are tradeable goods produced abroad. Since we are imagining our economy to be small any value of α less than 1 implies a home-bias in preferences over tradeable goods. The introduction of such a home-bias has often seen as being equivalent to introducing a non-tradeables sector to a NOEM model since both break the assumption of PPP.[5] However, we choose to retain both for several reasons. Firstly, the presence of both non-traded goods and home-bias allows us to capture the observed structure of production and key aspects of the international business cycle (see Benigno and Theossien (2006) or Corsetti et al (2003)). Secondly, we shall show that it is only in the presence of both non-traded goods and home-bias

finds that such an assumption is a valid parameter restriction in an econometrically estimated NOEM model.

[5] See, for example, Bowman and Boyle (2003).

that our welfare measure needs to discriminate between the two sectors. Finally, by retaining home bias our model reduces to that of GM in the absence of non-tradeables, which is a useful benchmark for assessing the contribution of non-tradeables to the description of optimal policy.

Since we are interested in modelling the home country as a small open economy unable to influence variables in the rest of the world. The rest of the world can either be viewed as being an economy similar in structure to the domestic economy, but where the weight of home-country goods in their imports is negligible (ie. $\alpha^* = 1$, as in GM), or as a continuum of economies similar in structure to the home economy, but where each country is small relative to the whole (see Gali and Monacelli (2006) or Leith and Wren-Lewis (2006a)). Accordingly we take rest of the world variables as given and assume $P_{T,F} = \varepsilon P^* = \varepsilon P_N^*$ for simplicity. The demand curves within the tradeables sector are given by,

$$C_T = \gamma(\frac{P_T)}{P})^{-1}C \tag{10}$$

and the demand for tradeable goods produced at home,

$$C_{T,H} = (1-\alpha)(\frac{P_{T,H})}{P_T})^{-1}C_T \tag{11}$$

and for tradeable goods produced abroad,

$$C_{T,F} = \alpha(\frac{P_{T,F}}{P_T})^{-1}C_T \tag{12}$$

Demand for individual tradeable goods produced at home,

$$C_{T,H}(j) = (\frac{P_{T,H}(j)}{P_{T,H}})^{-\epsilon}C_{T,H} \tag{13}$$

and goods produced abroad,

$$C_{T,F}(j) = (\frac{P_{T,F}(j)}{P_{T,H}})^{-\epsilon}C_{T,F} \tag{14}$$

assuming a similar form of basket (aggregated across a continuum of identical small countries).

2.2 Households' Intertemporal Consumption and Labour Supply Problems

The first of the representative household's intertemporal problems involves allocating consumption expenditure across time. For tractability assume that (1) takes the specific form,

$$E_0 \sum_{t=0}^{\infty} \beta^t (\ln C_t - \frac{(N_t)^{1+\varphi}}{1+\varphi}) \tag{15}$$

The budget constraint at time t is given by

$$P_tC_t + E_t\{M_{t,t+1}D_{t+1}\} = \Pi_t + D_t + W_tN_t + T_t \tag{16}$$

where D_{t+1} is the nominal payoff of the portfolio held at the end of period t, Π is the representative household's share of profits in the imperfectly competitive firms, W are wages, and T are lump sum taxes. $M_{t,t+1}$ is the stochastic discount factor for one period ahead payoffs.We can then maximise utility subject to the budget constraint (16) to obtain the optimal allocation of consumption across time,

$$\beta(\frac{C_t}{C_{t+1}})(\frac{P_t}{P_{t+1}}) = M_{t,t+1} \tag{17}$$

Taking conditional expectations on both sides and rearranging gives

$$\beta R_t E_t\{(\frac{C_t}{C_{t+1}})(\frac{P_t}{P_{t+1}})\} = 1 \tag{18}$$

where $R_t = \frac{1}{E_t\{M_{t,t+1}\}}$ is the gross return on a riskless one period bond paying off a unit of domestic currency in $t+1$. This is the familiar consumption Euler equation which implies that consumers are attempting to smooth consumption over time such that the marginal utility of consumption is equal across periods (after allowing for tilting due to interest rates differing from the households' rate of time preference).

The other optimality condition is for labour supply

$$W = PCN^{\varphi} \tag{19}$$

We assume that the representative household supplies labour to both non-traded and traded goods firms, such that $N = N_{T,H} + N_N$, and the wage rate will be the same in both sectors.

2.3 Price and Exchange Rate Identities

The effective terms of trade are given by,

$$S = \frac{P_{T,F}}{P_{T,H}} \tag{20}$$

Recall the definition of tradeables prices,

$$P_T = P_{T,H}^{1-\alpha}P_{T,F}^{\alpha} \tag{21}$$

combining with the terms of trade yields,

$$P_T = P_{T,H}S^{\alpha} \tag{22}$$

The real exchange rate is given by,

$$Q = \frac{\varepsilon P^*}{P} \tag{23}$$

and using the definition of the aggregate price level allows us to rewrite the real exchange rate as,

$$Q = S^{1-\gamma\alpha}\left(\frac{P_N}{P_{T,H}}\right)^{\gamma-1} = S^{1-\gamma\alpha}T^{\gamma-1} \tag{24}$$

where $T \equiv \frac{P_N}{P_{T,H}}$ is the ratio of non-tradeable goods prices to tradeable goods produced in the home country. If $\gamma = 1$ and there are no non-tradeables then this reduces to the same expression in GM.

Other relative price terms which are useful later include,

$$\frac{P_N}{P} = \frac{P_N}{P_T^{\gamma}P_N^{1-\gamma}} = \left(\frac{P_N}{P_T}\right)^{\gamma} \tag{25}$$

$$\frac{P_T}{P} = \frac{P_T}{P_T^{\gamma}P_N^{1-\gamma}} = \left(\frac{P_N}{P_T}\right)^{\gamma-1} \tag{26}$$

$$\frac{P_N}{P_T} = \frac{P_N}{P_{T,H}^{1-\alpha}P_{T,F}^{\alpha}} = \frac{P_N}{P_{T,H}}S^{-\alpha} \tag{27}$$

2.4 International Risk Sharing

There is a similar first-order-condition for consumption in the representative foreign economy,

$$\beta(\frac{C_t^i}{C_{t+1}^i})(\frac{P_t^i}{P_{t+1}^i}) = M_{t,t+1}^i \tag{28}$$

which we equate to the domestic foc after introducing an IRS/UIP shock which implies,

$$E_t\{M_{t,t+1}^i\frac{\varepsilon_t\varsigma_t}{\varepsilon_{t+1}\varsigma_{t+1}}\} = E_tM_{t,t+1} \tag{29}$$

where ς_t is an IRS/UIP shock. Taking expectations and log-linearising implies,

$$\widehat{\varepsilon}_t - E_t\widehat{\varepsilon}_{t+1} = \widehat{R}_t^i - \widehat{R}_t + (\rho - 1)\widehat{\varsigma}_t \tag{30}$$

where

$$\widehat{\varsigma}_t = \rho\widehat{\varsigma}_{t-1} + v_t \tag{31}$$

which makes it clear that this shock is equivalent to the form of UIP shock considered in Kollmann (2005).

Assuming symmetric initial conditions (i.e. zero net foreign assets, structurally similar economies and the *ex ante* expectation that policy regimes will be similar across economies) and equating the first order conditions (focs) for consumption between two economies also implies,

$$Q_{i,t+1}\left(\frac{C_{t+1}^i}{C_{t+1}}\right)\varsigma_{t+1} = Q_{i,t}\left(\frac{C_t^i}{C_t}\right)\varsigma_t \tag{32}$$

where the real exchange rate between home and country i is, $Q_{i,t} = \frac{\varepsilon_{it} P_t^*}{P_t}$, implying

$$C_t = z^i C_t^i Q_{i,t} \varsigma_t \tag{33}$$

where z^i is a constant which depends upon initial conditions. Loglinearising and integrating over all countries yields,

$$\widehat{C} = \widehat{C}^* + \widehat{Q} + \widehat{\varsigma} \tag{34}$$

where $\widehat{C}^* = \int_0^1 \widehat{C}^i di$ is the average level of consumption in the domestic economies trading partners.

2.5 Equilibrium

Equilibrium in the non-tradeables sector is straightforward,

$$Y_N = C_N \tag{35}$$

but in the tradeables sectors is more complex. Consider tradeable good j produced in the home economy,

$$Y_T(j) = C_{T,H}(j) + C_{T,F}^*(j) \tag{36}$$

with symmetrical preferences,

$$C_{T,H}(j) = (\frac{P_{T,H}(j)}{P_{T,H}})^{-\epsilon} C_{T,H} = (\frac{P_{T,H}(j)}{P_{T,H}})^{-\epsilon}(1-\alpha)(\frac{P_{T,H}}{P_T})^{-1} C_T \tag{37}$$

and,

$$\begin{aligned} C_{T,F}^*(j) &= (\frac{P_{T,H}(j)}{P_{T,H}})^{-\epsilon} C_{T,F}^* = (\frac{P_{T,H}(j)}{P_{T,H}})^{-\epsilon}\alpha(\frac{P_{T,F}^*}{P^*})^{-1} C_T^* \\ &= (\frac{P_{T,H}(j)}{P_{T,H}})^{-\epsilon}\alpha(\frac{P_{T,H}}{\varepsilon P^*})^{-1} C_T^* = (\frac{P_{T,H}(j)}{P_{T,H}})^{-\epsilon}\alpha(\frac{P_{T,H}}{P_{T,F}})^{-1} C_T^* \end{aligned} \tag{38}$$

Therefore the equilibrium condition can be rewritten as,

$$Y_{T,H}(j) = (\frac{P_{T,H}(j)}{P_{T,H}})^{-\epsilon}[(1-\alpha)(\frac{P_{T,H}}{P_T})^{-1} C_T + \alpha(\frac{P_{T,H}}{P_{T,F}})^{-1} C_T^*] \tag{39}$$

Defining aggregate tradeable output as,

$$Y_{T,H} = [\int_0^1 Y_{T,H}(j)^{\frac{\epsilon-1}{\epsilon}} dj]^{\frac{\epsilon-1}{\epsilon}} \tag{40}$$

we can write,

$$\begin{aligned} Y_{T,H} &= S^\alpha[(1-\alpha)C_T + \alpha(\frac{P_{T,F}}{P_{T,H}})^{-\alpha}(\frac{P_{T,H}}{P_{T,F}})^{-1} C_T^*] \\ &= S^\alpha[(1-\alpha)C_T + \alpha\gamma(\frac{P_{T,F}}{P_{T,H}})^{-\alpha}(\frac{P_{T,H}}{P_{T,F}})^{-1} C^*] \end{aligned} \tag{41}$$

From the IRS condition we can replace the term in foreign consumption,

$$Y_{T,H} = S^{\alpha}[(1-\alpha)C_T + \alpha\gamma(\frac{P_{T,F}}{P_{T,H}})^{1-\alpha}CQ^{-1}\varsigma_t^{-1}] \tag{42}$$

and using the definition of the real exchange rate, aggregate consumption and the price indices this can be re-written as,

$$Y_{T,H} = S^{\gamma\alpha}T^{1-\gamma}\gamma C[(1-\alpha) + \alpha\varsigma_t^{-1}] \tag{43}$$

Using the definition of the real exchange rate, (24), IRS implies,

$$C = S^{1-\gamma\alpha}T^{\gamma-1}C^*\varsigma_t \tag{44}$$

Combining the demands for traded and non-traded goods we obtain an expression for the relative price term,

$$T = S^{\alpha}\left(\frac{1-\gamma}{\gamma}\right)\left(\frac{C_N}{C_T}\right)^{-1} \tag{45}$$

which can be used to eliminate the relative price term in the IRS condition,

$$\begin{aligned} C &= S^{1-\gamma\alpha}\left(S^{\alpha}\left(\frac{1-\gamma}{\gamma}\right)\left(\frac{C_N}{C_T}\right)^{-1}\right)^{\gamma-1}C^*\varsigma_t \\ &= S^{1-\alpha}\left(\frac{1-\gamma}{\gamma}\right)^{\gamma-1}\left(\frac{C_N}{C_T}\right)^{1-\gamma}C^*\varsigma_t \end{aligned} \tag{46}$$

Combining this with (43) gives us the combined IRS-resource constraint,

$$\begin{aligned} Y_{T,H} &= \gamma SC^*\varsigma_t[(1-\alpha) + \alpha\varsigma_t^{-1}] \\ &= \gamma SC^*[(1-\alpha)\varsigma_t + \alpha] \end{aligned} \tag{47}$$

Alternatively we can combine (47) and (46) to eliminate the terms of trade from our combined IRS-Resource constraint,

$$\begin{aligned} C &= S^{1-\alpha}\left(\frac{1-\gamma}{\gamma}\right)^{\gamma-1}\left(\frac{C_N}{C_T}\right)^{1-\gamma}C^*\varsigma_t \\ &= Y_{T,H}^{1-\alpha}\gamma^{\alpha-1}\left(\frac{1-\gamma}{\gamma}\right)^{\gamma-1}\left(\frac{C_N}{C_T}\right)^{1-\gamma}(C^*)^{\alpha}\varsigma_t[(1-\alpha)\varsigma_t + \alpha]^{\alpha-1} \\ &= Y_{T,H}^{1-\alpha}\gamma^{\alpha-1}\left(\frac{1-\gamma}{\gamma}\right)^{\gamma-1}\left(\frac{C_N}{C_T}\right)^{1-\gamma}(C^*)^{\alpha}\varsigma_t[(1-\alpha)\varsigma_t + \alpha]^{\alpha-1} \end{aligned} \tag{48}$$

In the special case where there are no non-tradeables, $\gamma = 1$, this reduces to,

$$C = Y^{1-\alpha}(C^*)^{\alpha}\varsigma_t[(1-\alpha)\varsigma_t + \alpha]^{\alpha-1} \tag{49}$$

which is the same expression as in GM.

2.6 Production

We assume that both traded and non-traded goods are produced using a linear production technology,

$$Y_{T,H} = A_T N_{T,H} \tag{50}$$

and,

$$Y_N = A_N N_N \tag{51}$$

where we allow for sector specific differences in technology.

3 Social Planner's Problem.

The social planner seeks to maximise the representative households' utility,

$$\ln C - \frac{(N_{T,H} + N_N)^{1+\varphi}}{1+\varphi} \tag{52}$$

subject to the technologies, (50) and (51), the resouce constraint in the non-traded goods sector,

$$Y_N = C_N \tag{53}$$

the definition of aggregate consumption,(2) and our combined IRS-resource constraint (48).

To formulate this problem in a convenient form, take logs of (48,

$$\ln C = cnst + (1-\alpha)\ln Y_{T,H} + (1-\gamma)(\ln C_N - \ln C_T) + \ln\varsigma - (1-\alpha)\ln\phi \tag{54}$$

where $\phi = [(1-\alpha)\varsigma + \alpha]$ and $cnst$ is a constant made up of model parameters.[6] Combining this with the production function for home produced goods for trade the definition of the aggregate consumption bundle, (2), we obtain,

$$\begin{aligned} \ln C &= cnst + (1-\alpha)\gamma(\ln A_T + \ln N_{T,H}) + (1-\gamma)(\ln A_N + \ln N_N) \\ &\quad +\gamma\ln\varsigma - (1-\alpha)\gamma\ln\phi \end{aligned} \tag{55}$$

which allows us to write the social planner's problem as an unconstrained problem in $N_{T,H}$ and N_N. The two focs this generates are,

$$(1-\alpha)\gamma(N_{T,H})^{-1} - (N_{T,H} + N_N)^{\varphi} = 0 \tag{56}$$

and,

$$(1-\gamma)(N_N)^{-1} - (N_{T,H} + N_N)^{\varphi} = 0 \tag{57}$$

Notice that because utility and the combined IRS/Resource constraint are log-linear the IRS/UIP shock does not affect the focs implemented by the social

[6] As substitutions are made the composition of this $cnst$ term will change. However, we shall see that this does not affect the allocation of goods and services made by the social planner.

planner. Combining these gives the optimal balance between resources devoted to traded and non-traded goods production,

$$(1-\alpha)\gamma N_N = (1-\gamma)N_{T,H} \tag{58}$$

When there is no home-bias in traded goods consumption, $\alpha = 1$ no resources will be devoted to traded goods production. While if there is no weight given to non-traded goods in utility, $\gamma = 1$, then there are no resources devoted to their production. This can be substituted into the first foc to obtain the social planner's optimal value of $N_{T,H}$,

$$(1-\alpha)\gamma(N_{T,H})^{-1} = (N_{T,H})^{\varphi}(\frac{1-\alpha\gamma}{\gamma(1-\alpha)})^{\varphi} \tag{59}$$

Which can be solved as,[7]

$$((1-\alpha)\gamma)^{1+\varphi}(1-\alpha\gamma)^{-\varphi} = (N_{T,H})^{1+\varphi} \tag{60}$$

This can then be used to obtain the measure of N_N.Using the definition of aggregate labour input, $N = N_{T,H} + N_N$, we can obtain the relationship between these variables as,

$$N = N_N\frac{1-\alpha\gamma}{1-\gamma} = N_{T,H}\frac{1-\alpha\gamma}{(1-\alpha)\gamma} \tag{61}$$

Which implies, using the focs,

$$N = (1-\alpha\gamma)^{1+\varphi} \tag{62}$$

which is invariant to shocks. Therefore we can see that the social planner would not devote any extra labour to production in the face of productivity or IRS/UIP shocks. Using the production functions, the definition of aggregate consumption and the combined resource-IRS constraint we can then derive the efficient value of variables. However, the social planner's response to shocks is most easily seen by consider the efficient deviation of a variable from its steady-state value.

From the optimality conditions for labour supply we can see that,

$$\widehat{N}^e = \widehat{N}^e_{T,H} = \widehat{N}^e_N = 0 \tag{63}$$

where a hat denotes the log-linearised value of a variable. These in turn imply,

$$\widehat{Y}^e_N = a_N = \widehat{C}^e_N \tag{64}$$

and,

$$\widehat{Y}^e_{T,H} = a_T \tag{65}$$

[7] If $\gamma = 1$ this reduces to,$(1-\alpha)^{\frac{1}{1+\varphi}} = N_{T,H}$, which is the optimal labour supply found in GM.

The efficient level of aggregate consumption is given by,

$$\widehat{C}^e = \gamma(1-\alpha)a_T + \frac{\alpha(2-\alpha)\gamma}{1-\alpha}\widehat{\phi} + (1-\gamma)a_N \tag{66}$$

and the consumption of traded goods,

$$\widehat{C}^e_T = (1-\alpha)a_T + \frac{\alpha(2-\alpha)}{1-\alpha}\widehat{\phi} \tag{67}$$

Therefore the social planner does not alter the labour input in response to any shocks, but patterns of consumption do change in response to IRS/UIP shocks. From a positive IRS/UIP shock, consumption of imported tradeable goods increase without any corresponding increase in production or consumption of goods produced at home.

Additionally, even although the social planner ignores the price mechanism in making his allocation decisions, we can consider what the implied real exchange rate would be given the IRS/UIP condition,

$$\widehat{C}^e = \widehat{Q}^e + \widehat{\varsigma} \tag{68}$$

Using the definition of the efficient level of consumption our efficient real exchange rate is given by,

$$\widehat{Q}^e = \gamma(1-\alpha)a_T - (1-\alpha(2-\alpha)\gamma)\widehat{\varsigma} + (1-\gamma)a_N \tag{69}$$

Note that the coefficient on the IRS/UIP shock is negative such that a positive IRS/UIP shock implies an appreciation of the real exchange rate. Therefore, even although the IRS/UIP shock implies an exchange rate misalignment the social planner does not choose to implement a resource allocation which completely offsets that misalignment. Essentially, the social planner implements an allocation of resources which maximises the utility of the representative domestic household and this involves the optimal exploitation of the IRS condition.

4 Flexible Price Equilibrium

The representative household supplies labour to a continuum of firms operating with the trade and non-traded goods sectors, respectively. The aggregate labour supply condition was given by,

$$W = PC(N_N + N_{T,H})^{\varphi} \tag{70}$$

which given the demand curve facing a typical firm in each sector implies the two equilibrium pricing decisions within the two sectors,

$$P_N = \frac{\epsilon}{\epsilon-1}\frac{W(1-\chi_N)}{A_N} \tag{71}$$

and,

$$P_{T,H} = \frac{\epsilon}{\epsilon - 1}\frac{W(1-\chi_T)}{A_T} \tag{72}$$

where $\frac{\epsilon}{\epsilon-1}$ is the desired mark-up reflecting the market power possessed by firms due to the existence of differentiated products and χ_T and χ_N are subsidies used to ensure the model's steady-state is efficient (see below for their derivation). Note that this implies relative prices will only differ across the two sectors in response to idiosyncratic technology shocks.

Equating the demand and supply of labour implies the follow two equilibrium conditions,

$$W = \frac{A_N P_N}{(1-\chi_N)}\frac{\epsilon - 1}{\epsilon} = PCN^{\varphi} \tag{73}$$

and,

$$W = \frac{A_T P_{T,H}}{(1-\chi_T)}\frac{\epsilon - 1}{\epsilon} = PCN^{\varphi} \tag{74}$$

We need to rewrite these focs in terms of N_N and $N_{T,H}$ to facilitate comparison with the optimality conditions of social planner. Essentially the constraints faced by the social planner allow us to replace the aggregate consumption term with terms in N_N and $N_{T,H}$.

The other element of the two focs that needs to be rewritten in terms of N_N and $N_{T,H}$ is the relative price terms. Firstly note that,

$$\frac{P_N}{P_{T,H}} = S^{\alpha}\left(\frac{1-\gamma}{\gamma}\right)\left(\frac{C_N}{C_T}\right)^{-1} \tag{75}$$

where we can replace the terms of trade with the condition

$$Y_{T,H} = \gamma S C^{*}[(1-\alpha)\varsigma_t + \alpha] \tag{76}$$

and then same constraints to rewrite this in terms of N_N and $N_{T,H}$.

Finally note that

$$\frac{P_{T,H}}{P} = \left(\frac{P_N}{P_{T,H}}\right)^{\gamma-1} S^{-\alpha\gamma} \tag{77}$$

and,

$$\frac{P_N}{P} = \left(\frac{P_N}{P_{T,H}}\right)^{\gamma} S^{-\alpha\gamma} \tag{78}$$

These can then be used to rewrite the two focs in terms of N_N and $N_{T,H}$.

$$-\varphi\ln(N) - \ln(1-\chi_N) - \ln(N_N) - \ln(\frac{\epsilon}{\epsilon-1}) + \ln(1-\gamma) = 0 \tag{79}$$

and,

$$-\varphi\ln(N) - \frac{\alpha}{1-\alpha}\ln\phi - \ln(1-\chi_T) - \ln(N_{T,H}) - \ln(\frac{\epsilon}{\epsilon-1}) + \ln(\gamma) = 0 \tag{80}$$

These can then be contrasted with the focs from the social planner's problem which are replicated here for convenience,

$$(1-\alpha)\gamma(N_{T,H})^{-1} - (N_{T,H}+N_N)^{\varphi} = 0 \tag{81}$$

and,

$$(1-\gamma)(N_N)^{-1} - (N_{T,H}+N_N)^{\varphi} = 0 \tag{82}$$

The optimal values of χ_T and χ_N are then the ones that ensure that these two focs allocate the same levels of labour to production of the two types of goods as would be chosen by the social planner and can be shown to be,

$$\ln(1-\chi_T) = -\ln(\frac{\epsilon}{\epsilon-1}) + \ln(1-\alpha) - \frac{\alpha}{1-\alpha}\ln\phi \tag{83}$$

(Note that when there are no IRS/UIP shocks this reduces to $\ln(1-\chi_T) = -\ln(\frac{\epsilon}{\epsilon-1}) + \ln(1-\alpha)$ as is the case in GM) and,

$$\ln(1-\chi_N) = -\ln(\frac{\epsilon}{\epsilon-1}) \tag{84}$$

which simply offsets the inefficiencies due to imperfect competition in the non-tradeables sector.

The subsidy required to ensure the flex price equilibrium is efficient requires the subsidy to vary with the IRS/UIP shock. However, in line with the literature we assume a constant subsidy,

$$\ln(1-\overline{\chi}_T) = -\ln(\frac{\epsilon}{\epsilon-1}) + \ln(1-\alpha) \tag{85}$$

which will imply that the flex price equilibrium is not efficient. We shall explore the nature of this inefficiency below.

This implies with these constant subsidies in place that the log-linearised flex price equilibrium is given by,

$$\widehat{N}_N^f = \frac{\alpha\gamma\varphi}{(1-\alpha\gamma)(1+\varphi)}\widehat{\phi} \tag{86}$$

and,

$$\widehat{N}_{T,H}^f = -\frac{(\varphi(1-\gamma)+(1-\alpha\gamma))\alpha}{(1-\alpha)(1-\alpha\gamma)(1+\varphi)}\widehat{\phi} \tag{87}$$

and aggregate employment is given by,

$$\widehat{N}^f = -\frac{\alpha\gamma}{(1-\alpha\gamma)(1+\varphi)}\widehat{\phi}$$

This implies the following levels of non-traded goods production/consumption,

$$\widehat{Y}_N^f = a_N + \widehat{N}_N^f = \widehat{C}_N^f \tag{88}$$

and production of traded goods,

$$\widehat{Y}_{T,H}^f = a_T + \widehat{N}_{T,H}^f \tag{89}$$

The combined IRS-resource constraint and the definition of consumption can be solved to yield,

$$\widehat{C}_N^f = a_N + \frac{\alpha\gamma\varphi}{(1-\alpha\gamma)(1+\varphi)}\widehat{\phi} \tag{90}$$

and,

$$\widehat{C}_T^f = (1-\alpha)a_T - \frac{(\varphi(1-\gamma)+(1-\alpha\gamma))\alpha}{(1-\alpha)(1-\alpha\gamma)(1+\varphi)}\widehat{\phi}_t + \frac{\alpha(2-\alpha)}{1-\alpha}\widehat{\phi} \tag{91}$$

We are now in a position to describe the flex price response to an IRS/UIP shock. Following a positive IRS/UIP shock, which implies an appreciation of the exchange rate, imported traded goods are cheaper and home consumers substitute away from domestically produced traded goods towards foreign-produced goods. This reduces the price of home-produced traded goods. The reduced production of home-produced good reduces the demand for labour, which pushes down costs in the non-traded goods sector. This prompts non-traded goods producers to cut their prices and produce more goods, implying increased consumption of non-traded goods.

We can contrast the flex price equilibrium with the efficient allocation that would be chosen by the social planner,

$$\widehat{N}_N^f - \widehat{N}_N^e = \frac{\alpha\gamma\varphi}{(1-\alpha\gamma)(1+\varphi)}\widehat{\phi} \tag{92}$$

$$\widehat{N}_{T,H}^f - \widehat{N}_{T,H}^e = -\frac{(\varphi(1-\gamma)+(1-\alpha\gamma))\alpha}{(1-\alpha)(1-\alpha\gamma)(1+\varphi)}\widehat{\phi} \tag{93}$$

$$\widehat{Y}_N^f - \widehat{Y}_N^e = \frac{\alpha\gamma\varphi}{(1-\alpha\gamma)(1+\varphi)}\widehat{\phi} = \widehat{C}_N^f - \widehat{C}_N^e \tag{94}$$

$$\widehat{Y}_{T,H}^f - \widehat{Y}_{T,H}^e = -\frac{(\varphi(1-\gamma)+(1-\alpha\gamma))\alpha}{(1-\alpha)(1-\alpha\gamma)(1+\varphi)}\widehat{\phi} \tag{95}$$

$$\widehat{C}_T^f - \widehat{C}_T^e = -\frac{(\varphi(1-\gamma)+(1-\alpha\gamma))\alpha}{(1-\alpha\gamma)(1+\varphi)}\widehat{\phi} \tag{96}$$

Combining the two expressions for consumption of tradeable and non-tradeable goods with the definition of aggregate consumption yields,

$$\widehat{C}^f - \widehat{C}^e = -\frac{\alpha\gamma}{(1+\varphi)}\widehat{\phi} = \widehat{Q}^f - \widehat{Q}^e \tag{97}$$

and we see that the net impact of relatively higher non-tradeables consumption and lower tradeables consumption under flexible prices relative to the efficient outcome is a relative decline in aggregate consumption and total employment. Given the IRS/UIP condition this implies that there has been a relative appreciation of the real exchange rate under flexible price. The social planner simply allows consumers to enjoy the extra consumption of foreign-produced traded goods made available by the exchange rate appreciation under an IRS/UIP shock. Under flexible prices this is not a sustainable allocation as the shifts in relative demands across goods this implies will induce price changes which will then prompt production changes. This is what the social planner avoids by ignoring the shock in deciding on domestic production levels. It is also interesting to note that the extent of the excessive appreciation under flexible prices is lower as the degree of home bias $(1-\alpha)$ increases and the proportion of non-tradeables $(1-\gamma)$ increases. The intuition is that the IRS/UIP shock is essentially a shock to tradeable goods, and the less consumers care about such goods the less there is a price response to such shocks.

5 Sticky Prices

In this section we introduce sticky prices through the device of Calvo contracts whereby firms are only able to change their prices after a random interval of time. This implies that monetary policy can affect real variables, such that it may be able to move an economy closer to the efficient equilibrium in the face of IRS/UIP shocks. It also allows us to capture a key stylised fact which may affect the transmission of both IRS/UIP shocks and monetary policy through the economy - namely that non-traded goods prices are typically thought to be sticky relative to prices in the traded goods sector.[8] We examine the pricing decisions in both sectors, before considering a second order approximation to welfare in the presence of price stickiness in the next Section.

5.1 Non-Tradeable Goods Pricing

The production function is linear, so for firm j

$$Y_N(j) = A_N N_N(j) \tag{98}$$

where $a_N = \ln(A_N)$ is time varying and stochastic. While the demand curve they face is given by,

$$Y_N(j) = (\frac{P_N(j)}{P_N})^{-\epsilon} Y_N \tag{99}$$

where $Y_N = \left[\int_0^1 Y_N(j)^{\frac{\epsilon-1}{\epsilon}} dj\right]^{\frac{\epsilon}{\epsilon-1}}$. The objective function of the firm is given by,

$$\sum_{s=0}^{\infty}(\theta_N)^s M_{t,t+s}\left[\frac{P_N(j)_t}{P_{t+s}} Y_N(j)_{t+s} - \frac{W_{t+s}}{P_{t+s}}\frac{Y_N(j)_{t+s}(1-\varkappa_N)}{A_{N,t+s}}\right] \tag{100}$$

where $\varkappa_N$ is an employment subsidy which can be used to eliminate the steady-state distortion associated with monopolistic competition (assuming there is a lump-sum tax available to finance such a subsidy). Using the demand curve for the firm's product,

$$\sum_{s=0}^{\infty}(\theta_N)^s M_{t,t+s}\left[\begin{array}{c} \frac{P_N(j)_t}{P_{t+s}}(\frac{P_N(j)_t}{P_{N,t+s}})^{-\epsilon} Y_{N,t+s} \\ -\frac{W_{t+s}}{P_{t+s}}(\frac{P_N(j)_t}{P_{N,t+s}})^{-\epsilon}\frac{Y_{Nt+s}(1-\varkappa_N)}{A_{N,t+s}}\end{array}\right] \tag{101}$$

[8] The section on calibration of the model discusses the emprical evidence in support of this assumption.

The solution to this problem is given by,

$$\sum_{s=0}^{\infty}(\theta_N)^s M_{t,t+s}\left[\begin{array}{c}(1-\epsilon)P_{t+s}^{-1}(\frac{P_N(j)_t}{P_{N,t+s}})^{-\epsilon}Y_{Nt+s}\\ +\epsilon\frac{W_{t+s}}{P_{t+s}}P_N(j)_t^{-\epsilon-1}P_{N,t+s}^{\epsilon}\frac{Y_{N,t+s}(1-\varkappa_N)}{A_{N,t+s}}\end{array}\right]=0 \tag{102}$$

Solving for the optimal reset price, which is common across all firms able to reset prices in period t,

$$\overline{P}_{N,t}=\frac{\sum_{s=0}^{\infty}(\theta_N)^s M_{t,t+s}\left[\epsilon\frac{W_{t+s}}{P_{t+s}}P_{N,t+s}^{\epsilon}\frac{Y_{Nt+s}}{A_{Nt+s}}\right]}{\sum_{s=0}^{\infty}(\theta_N)^s M_{t,t+s}\left[(\epsilon-1)P_{t+s}^{-1}P_{N,t+s}^{\epsilon}Y_{N,t+s}(1-\varkappa_N)\right]} \tag{103}$$

While non-tradeable goods prices evolve according to,

$$P_{N,t}=\left[(1-\theta_N)\overline{P}_{N,t}^{(1-\epsilon)}+\theta_N P_{N,t-1}^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}} \tag{104}$$

Leith and Wren-Lewis (2006) demonstrate that this implies a New Keynesian Phillips curve for non-tradeables' price inflation which is given by,

$$\begin{aligned}\pi_{N,t} &= \beta E_t\pi_{N,t+1} \\ &\quad +\frac{(1-\theta_N\beta)(1-\theta_N)}{\theta_N}(-a_t+w_t-p_{N,t}-v+\ln(\mu))\end{aligned} \tag{105}$$

where $mc=-a+w-p_N-v$ are the real log-linearised marginal costs of production, and $v=-\ln(1-\varkappa_N)$. Which can be rewritten using similar subsititutions as in the derivation of the flex price equilibrium conditions,

$$\pi_{N,t}=\beta E_t\pi_{N,t+1}+\frac{(1-\theta_N\beta)(1-\theta_N)}{\theta_N}(\varphi\widehat{N}_t+\widehat{N}_{N,t}) \tag{106}$$

or in gap form,

$$\pi_{N,t}=\beta E_t\pi_{N,t+1}+\frac{(1-\theta_N\beta)(1-\theta_N)}{\theta_N}(\varphi\widehat{N}_t^g+\widehat{N}_{N,t}^g) \tag{107}$$

where $\widehat{N}_t^g=\widehat{N}_t-\widehat{N}_t^e=\widehat{N}_t$ since $\widehat{N}_t^e=0$.

5.2 Tradeable Goods Pricing

There is a similar problem facing firms in the tradeable goods sector which results in a NKPC of the form,

$$\begin{aligned}\pi_{T,H,t} &= \beta E_t\pi_{T,H,t+1} \\ &\quad +\frac{(1-\theta_{T,H}\beta)(1-\theta_{T,H})}{\theta_{T,H}}(-a_t+w_t-p_{T,H,t}-v_t+\ln(\mu))\end{aligned} \tag{108}$$

where$1-\theta_{T,H}$ is the proportion of firms changing their price within a given period, $mc=-a+w-p_{T,H}-v$ are the real log-linearised marginal costs of production, and $v = -\ln(1-\overline{\varkappa}_T)$. This can be rewritten (using the flex price focs) as,

$$\pi_{T,H,t} = \beta E_t \pi_{T,H,t+1} + \frac{(1-\theta_{T,H}\beta)(1-\theta_{T,H})}{\theta_{T,H}}(\varphi\widehat{N}_t + \alpha\widehat{\varsigma}_t + \widehat{N}_{T,H,t}) \quad (109)$$

or in (efficiency) gap form,

$$\begin{aligned}\pi_{T,H,t} &= \beta E_t \pi_{T,H,t+1} \\ &+ \frac{(1-\theta_{T,H}\beta)(1-\theta_{T,H})}{\theta_{T,H}}(\varphi\widehat{N}^g_t + \widehat{N}^g_{T,H,t} + \alpha\widehat{\varsigma}_t)\end{aligned} \quad (110)$$

6 Welfare

Social welfare is given by the representative household's utility,

$$\Upsilon_t = E_0 \sum_{t=0}^{\infty} \beta^t [\ln C_t - \frac{(N_{T,H,t} + N_{N,t})^{1+\varphi}}{1+\varphi}] \quad (111)$$

Appendix I then derives a second order approximation to the representative household's utility which is shown to be,

$$\begin{aligned}\Upsilon_t &= E_t \sum_{s=0}^{\infty} \beta^s [(\Psi)(1+\varphi(\Psi))(\widehat{Y}^g_{N,t+s})^2 + (1-\Psi)(1+\varphi(1-\Psi))(\widehat{Y}^g_{T,H,t+s})^2 \\ &+ 2\varphi(\Psi)(1-\Psi)(\widehat{Y}^g_{N,t+s})(\widehat{Y}^g_{T,H,t+s}) + (\Psi)\frac{\epsilon}{\lambda_N}\pi^2_{N,t+s} + (1-\Psi)\frac{\epsilon}{\lambda_{T,H}}\pi^2_{T,H,t+s} + tip\end{aligned} \quad (112)$$

where $\Psi = \frac{1-\gamma}{1-\alpha\gamma}$. Here it is the case that welfare depends not only on the total labour input, but on its composition between non-traded and traded goods production as well as inflation in each sector. The inutition is simple. Welfare depends upon the costs of producing goods in either sector. These costs are captured by the output gap within each sector as well as the terms in inflation which reflect price dispersion within each sector. Any price dispersion will increase the costs of producing a given level of output. Therefore IRS/UIP shocks and producitivity shocks when prices are sticky affect traded and non-traded goods differently and will require us to examine the composition of output.

Here we can discern the relative contribution to home bias in preferences and the presence of non-tradeables in generating this sectoral version of the quadratic welfare function. With no non-traded goods then that sector is, obviously, not present in the welfare function, while in the case of no home bias ($\alpha = 1$), we find,$\Psi = 1$ and our welfare measure reduces to,

$$E_t \sum_{s=0}^{\infty} \beta^s \Gamma_{t+s} = E_t \sum_{s=0}^{\infty} \beta^s [(1+\varphi)(\widehat{Y}^g_{N,t+s})^2 + \Psi\frac{\epsilon}{\lambda_N}\pi^2_{N,t+s}]$$

With no home-bias in tradeable goods consumption, the share of domestically produced traded goods in the consumer's basket of tradeable goods would be so small (given our small open economy assumption) that the production of tradeable goods would be negligible. Therefore we need both non-traded goods and a home-bias in traded goods to ensure that both sectors remain in our welfare measure.

6.1 Relating Welfare to Aggregate Variables

Our welfare criterion reflects the costs of fluctuations in our two sectors. It differs from the criteria typically employed by central banks in that such criteria would usually focus on aggregate variables such as a single measure of price inflation (e.g. output price or consumer price inflation) and a single measure of output disequilibrium (the output gap). It is obvious that if we remove non-tradeable goods from our economy then our welfare measure would reduce to the combination of quadratic terms in output price inflation and the output gap, confirming the result of Clarida et al (2001) that the policy problem facing policy makers in the small open economy is isomorphic to that in the closed economy.

An obvious question to ask is whether or not our welfare measure in the presence of non-tradeables can be similarly reduced to terms in aggregate variables? To answer this question it is helpful to consider the relationships we have available to rewrite our objective function with. These are summarised below, in gap form:

Equation (47),

$$\widehat{Y}_{T,H}^{g} = \widehat{S}^{g} \tag{113}$$

The relative demand between the two sectors,

$$\widehat{T}^{g} = \alpha\widehat{S}^{g} - \left(\widehat{C}_{N}^{g} - \widehat{C}_{T}^{g}\right) \tag{114}$$

The definition of aggregate consumption,

$$\widehat{C}^{g} = \gamma\widehat{C}_{T}^{g} + (1-\gamma)\widehat{C}_{N}^{g} \tag{115}$$

The combined resource-IRS constraint, (55),

$$\widehat{C}^{g} = (1-\alpha)\gamma\widehat{Y}_{T,H}^{g} + (1-\gamma)\widehat{Y}_{N}^{g} \tag{116}$$

The relationship between the output and consumption of non-tradeable goods,

$$\widehat{Y}_{N}^{g} = \widehat{C}_{N}^{g} \tag{117}$$

These can be utilised to eliminate the terms in the sectoral output gaps, but two variables will always be required to capture these terms. Suppose we construct an aggregate measure of the output gap which is a weighted average of the sectoral outputs,

$$\widehat{Y}^{g} = (1-\Psi)\widehat{Y}_{T,H}^{g} + \Psi\widehat{Y}_{N}^{g} \tag{118}$$

such a measure would always need to be augmented with another gap variable, such as consumption, the relative price term, the terms of trade. Two variables are required to capture the composition of output across the two sectors which is otherwise potentially masked in the single output gap measure. For example we could rewrite terms in the individual output gaps as expressions in the aggregate output gap and the relative price term,

$$\widehat{Y}^g_{T,H} = \widehat{Y}^g + \Psi\widehat{T}^g, \text{ and } \widehat{Y}^g_N = \widehat{Y}^g - (1-\Psi)\widehat{T}^g$$

or aggregate output and the terms of trade,

$$\widehat{Y}^g_{T,H} = \widehat{S}^g, \text{ and } \widehat{Y}^g_N = \frac{1}{\Psi}(\widehat{Y}^g - (1-\Psi)\widehat{S}^g)$$

Therefore, we can see that introducing non-tradeable goods to the open economy model provides a rationale for including quadratic terms in the terms of trade or real exchange rate gaps in the welfare measure[9].

A similar reasoning applies to the inflation terms. Since we care about the dispersion of prices within each sector it is not possible to consider a single aggregate measure of inflation which aggregates inflation across the two sectors since this may mask and divergence in inflation between the two sectors. However from the definition of the relative price term, $\widehat{T}$ we know,

$$\Delta\widehat{T}_t = \pi_{N,t} - \pi_{T,H,t} \tag{119}$$

From our model we have a definition of consumer price inflation,

$$\begin{aligned} \pi_t &= \gamma\pi_{T,t} + (1-\gamma)\pi_{N,t} \\ &= \gamma((1-\alpha)\pi_{T,H,t} + \alpha\pi_{T,F,t}) + (1-\gamma)\pi_{N,t} \end{aligned} \tag{120}$$

If we exclude the element of foreign price inflation in formulating an inflation measure for the monetary policy maker then we obtain,

$$\begin{aligned} \pi^{cb}_t &= \gamma(1-\alpha)\pi_{T,H,t} + (1-\gamma)\pi_{N,t} \\ &= (1-\alpha\gamma)(((1-\Psi)\pi_{T,H,t} + \Psi\pi_{N,t}) \end{aligned} \tag{121}$$

which is an output price measure based on a weighted average of inflation in our tradeables and non-tradeables sectors. Clearly we can rewrite our terms in inflation using a combination of this aggregate measure in conjunction with ΔT. It is important to note that this version of T is not in a gapped form and that the relationship between this and the gapped version is given by,

$$\widehat{T}_t = \widehat{T}^g_t - a_{T,t} + a_{N,t} - \alpha\widehat{\varsigma}_t \tag{122}$$

Using the various price and exchange rate identities this term can then be rewritten using a combination of changes in the real exchange rate and terms of trade from,

$$\Delta\widehat{Q}_t = (1-\gamma)\Delta\widehat{T}_t + (1-\gamma\alpha)\Delta\widehat{S}_t \tag{123}$$

[9] Kirsanova et al (2006) provide further reasons for including such terms in a model without a non-tradeables sector.

Therefore, by introducing non-tradeables alongside tradeable goods we create a two sector economy where it is not possible to capture the extent of price dispersion in the economy using a single inflation measure alone. Instead any aggregate inflation measure must be augmented with terms in changes in relative prices to reflect differences in the rates of inflation across the two sectors. This, therefore, provides a rationalisation for including terms in the change in actual exchange rates (or related variables) rather than simply the gapped values of such variables.

It could be argued that these results are not surprising, in the sense that by modelling the sectoral composition of the economy and introducing a shock which changes patterns of production across these sectors in a distortionary way is bound not to be captured by the simple closed economy welfare metric based on terms in aggregate output and inflation. However, what our results also show is that we can still formulate a welfare function based on aggregate variables by introducing terms in relative prices, such as the real exchange rate, which allow us to capture the sectoral composition of the standard aggregate variables.

7 Policy Problem

In this section we utilise our welfare measure to derive fully optimal policy under commitment. Our objective function has been derived as (112), and our model consists of our two NKPCs, (107) and (110), and the evolution of their relative prices,

$$\widehat{T}_t = \widehat{T}_{t-1} + \pi_{N,t} - \pi_{T,H,t} \tag{124}$$

However, we need to relate this relative price term to output gap variables. Appendix 2 derives this link as,

$$\widehat{T} = \widehat{Y}^g_{T,H} - \widehat{Y}^g_N + \alpha\widehat{\varsigma} + a_T - a_N \tag{125}$$

Therefore the policy problem is given by the following Lagrangian,

$$\begin{aligned}
&E_0 \sum_{t=0}^{\infty} \beta^s \frac{1}{2} [[(\Psi)\,(1+\varphi\,(\Psi))\,(\widehat{Y}^g_{N,t})^2 + (1-\Psi)(1+\varphi(1-\Psi))(\widehat{Y}^g_{T,H,t})^2 \\
&+2\varphi\,(\Psi)\,(1-\Psi)(\widehat{Y}^g_{N,t})(\widehat{Y}^g_{T,H,t}) + (\Psi)\,\frac{\epsilon}{\lambda_N}\pi^2_{N,t} + (1-\Psi)\,\frac{\epsilon}{\lambda_{T,H}}\pi^2_{T,H,t}] \\
&+\lambda^{\pi_N}(\pi_{N,t} - \beta E_t \pi_{N,t+1} - \lambda_N(\varphi((\Psi)\,\widehat{Y}^g_{N,t} + (1-\Psi)\,\widehat{Y}^g_{T,H,t}) + \widehat{Y}^g_{N,t})) \\
&+\lambda^{\pi_{T,H}}(\pi_{T,H,t} - \beta E_t \pi_{T,H,t+1} - \lambda_{T,H}(\varphi((\Psi)\,\widehat{Y}^g_{N,t} + (1-\Psi)\,\widehat{Y}^g_{T,H,t}) + \widehat{Y}^g_{T,H,t} + \alpha\widehat{\varsigma}_t)) \\
&+\lambda^T(\widehat{T}_t - \widehat{T}_{t-1} - \pi_{N,t} + \pi_{T,H,t}) \\
&+\lambda^Y(\widehat{T}_t - \widehat{Y}^g_{T,H,t} + \widehat{Y}^g_{N,t} - a_{T,t} + a_{N,t} - \alpha\widehat{\varsigma}_t)]
\end{aligned}$$

The first-order conditions this implies are as follows. Firstly for non-traded goods price inflation,

$$\frac{\Psi\epsilon}{\lambda_N}\pi_{N,t} + \Delta\lambda_t^{\pi_N} - \lambda_t^T = 0 \qquad (126)$$

domestically produced traded-goods price inflation,

$$\frac{(1-\Psi)\epsilon}{\lambda_T}\pi_{T,H,t} + \Delta\lambda^{\pi T,H} + \lambda_t^T = 0 \qquad (127)$$

Output of non-tradeables,

$$\Psi(1+\varphi\Psi)\widehat{Y}^g_{N,t} + \varphi\Psi(1-\Psi)\widehat{Y}^g_{T,t} - \lambda_N(1+\Psi\varphi)\lambda_t^{\pi_N} - \lambda_T\Psi\varphi\lambda_t^{\pi T,H} + \lambda_t^Y = 0 \quad (128)$$

Output of tradeables,

$$\begin{aligned} 0 \;=\; & (1-\Psi)\varphi\Psi\widehat{Y}^g_{N,t} + (1+\varphi(1-\Psi))(1-\Psi)\widehat{Y}^g_{T,t} - \lambda_N\varphi(1-\Psi)\lambda_t^{\pi_N} \\ & -\lambda_T((1-\Psi)\varphi+1)\lambda_t^{\pi T,H} - \lambda_t^Y \end{aligned} \qquad (129)$$

Relative price of non-tradeables to tradeables,

$$\lambda_t^T - \beta\lambda_{t+1}^T + \lambda_t^Y = 0 \qquad (130)$$

Combining the focs for the two forms of inflation yields,

$$\frac{\Psi\epsilon}{\lambda_N}\pi_{N,t} + \Delta\lambda_t^{\pi_N} + \frac{(1-\Psi)\epsilon}{\lambda_T}\pi_{T,H,t} + \Delta\lambda_t^{\pi T,H} = 0 \qquad (131)$$

Doing the same for the focs for the two forms of output and taking the first-difference,

$$\Psi\Delta\widehat{Y}^g_{N,t} + (1-\Psi)\Delta\widehat{Y}^g_{T,t} - \lambda_N\Delta\lambda_t^{\pi_N} - \lambda_T\Delta\lambda_t^{\pi T,H} = 0 \qquad (132)$$

Note that when $\lambda_T = \lambda_N$ these combine to yield the simple target crtierion,

$$\Psi\epsilon\pi_{N,t} + (1-\Psi)\epsilon\pi_{T,H,t} + \Psi\Delta\widehat{Y}^g_{N,t} + (1-\Psi)\Delta\widehat{Y}^g_{T,H,t} = 0 \qquad (133)$$

Therefore with equally sticky prices in both sectors we obtain a target crtierion which is similar to that found in a simple one-sector New Keynesian model (see for example Woodford (2003), chapter 7) which exhibits the property of price level control within each sector. In other words, in order to improve the trade-off between inflation and output stabilisation the policy maker will commit to return the price level in each sector to its pre-shock level.

Returning to the case with differing degrees of stickiness across sectors we can solve these for the changes in the two lagrange multipliers,

$$\Delta\lambda_t^{\pi_N} = -\frac{\Psi\epsilon\lambda_T}{\lambda_N(\lambda_T-\lambda_N)}\pi_{N,t} - \frac{(1-\Psi)\epsilon}{(\lambda_T-\lambda_N)}\pi_{T,H,t} - \frac{\Psi}{\lambda_T-\lambda_N}\Delta\widehat{Y}^g_{N,t} - \frac{(1-\Psi)}{\lambda_T-\lambda_N}\Delta\widehat{Y}^g_{T,H,t} \qquad (134)$$

and,

$$\Delta\lambda_t^{\pi T,H} = \frac{\Psi\epsilon}{(\lambda_T - \lambda_N)}\pi_{N,t} + \frac{(1-\Psi)\epsilon\lambda_N}{\lambda_T(\lambda_T - \lambda_N)}\pi_{T,H,t} + \frac{\Psi}{\lambda_T - \lambda_N}\Delta\widehat{Y}_{N,t}^g + \frac{(1-\Psi)}{\lambda_T - \lambda_N}\Delta\widehat{Y}_{T,H,t}^g \quad (135)$$

Substituting these back into the foc for traded output we obtain,

$$-\Delta\lambda_t^Y = \frac{\Psi\epsilon\lambda_T}{(\lambda_T - \lambda_N)}\pi_{N,t} + \frac{(1-\Psi)\epsilon\lambda_N}{(\lambda_T - \lambda_N)}\pi_{T,H,t} + \frac{\Psi\lambda_T}{\lambda_T - \lambda_N}\Delta\widehat{Y}_{N,t}^g + \frac{(1-\Psi)\lambda_N}{\lambda_T - \lambda_N}\Delta\widehat{Y}_{T,H,t}^g \quad (136)$$

Similarly taking the first difference of the foc for traded-goods price inflation we obtain,

$$-\Delta\lambda_t^T = \frac{\Psi\epsilon}{(\lambda_T - \lambda_N)}\Delta\pi_{N,t} + \frac{(1-\Psi)\epsilon}{(\lambda_T - \lambda_N)}\Delta\pi_{T,H,t} + \frac{\Psi}{\lambda_T - \lambda_N}\Delta^2\widehat{Y}_{N,t}^g + \frac{(1-\Psi)}{\lambda_T - \lambda_N}\Delta^2\widehat{Y}_{T,H,t}^g \quad (137)$$

Using the final foc in first-differenced form we obtain the target criterion for optimal policy,

$$-\Delta\lambda_t^Y = \Delta\lambda_t^T - \beta\Delta\lambda_{t+1}^T \quad (138)$$

which implies,

$$\begin{aligned} & \Psi\epsilon(\lambda_T\pi_{N,t} + \Delta\pi_{N,t} - \beta E_t\Delta\pi_{N,t+1}) \\ & +(1-\Psi)\epsilon(\lambda_N\pi_{T,H,t} + \Delta\pi_{T,H,t} - \beta E_t\Delta\pi_{T,H,t+1}) \\ & +\Psi(\Delta^2\widehat{Y}_{N,t}^g - \beta E_t\Delta^2\widehat{Y}_{N,t+1}^g) + (1-\Psi)(\Delta^2\widehat{Y}_{T,H,t}^g - \beta E_t\Delta^2\widehat{Y}_{T,H,t+1}^g) \\ = & \ 0 \end{aligned} \quad (139)$$

We therefore get a backward and forward mix of dynamics incorporating the desire make policy history dependent as well as take account of the fact that policy in this period will affect the relative price of non-tradeables and tradeables in the following period.

8 Simulations

In this section we simulate the model in the face of IRS/UIP shocks. Our central parameter set is given by, $\beta = 0.99$, $\theta^T = \theta^N = 0.75$, $\varphi = 1$, $\gamma = 0.55$, $\epsilon = 6$ and $\alpha = 0.28$. The bulk of these parameters come from Leith and Wren-Lewis(2006), but the shares of tradables and non-tradeables in consumption baskets is given by, Theossein and Benigno (2006). Our IRS/UIP shock follows an autoregressive process with persistence of 0.5 and an innovation with a standard deviation of 6.6,

$$\widehat{\varsigma}_t = \rho\widehat{\varsigma}_{t-1} + \nu_t \quad (140)$$

where $\nu_t \sim N(0, 6.6)$. This ensures that our shock matches the UIP shock in the form of equation (30) estimated by Kollmann (2005) using post-Bretton Woods data.

The paths for our variables under optimal discretion and commitment policy in the face of a 1 standard deviation IRS/UIP shock is given in Figure 1. Policy acts to reduce the excessive real appreciation that would emerge under flexible prices, however it does not do so completely (the consumption gap remains negative) as to do so would fuel inflation. The welfare costs of this amount to 0.71% and 0.79% of one period's steady-state consumption under commitment and discretion, respectively.

Given the debate within the policy making circles as to the appropriate response to exchange rate misalignments (see Cobham, 2006) it is informative to contrast the optimal policy with alternative policies. Since the model would be indeterminate under a policy which ignored the shock and simply fixed nominal interest rates, we need to consider other policy stances. Here we examine two alternative policies, (1) fixed real interest rates and (2) strict output price inflation targeting[10]. Under the first policy there is no attempt to offset the misalignment of the real exchange rate, while the second is a rigid application of domestic inflation targeting. Both policies can be thought of as not affording the UIP shock any special status, in contrast to the optimal policy which deliberately seeks to offset the exchange rate misalignment caused by the UIP shock.

8.0.1 Fixed Real Interest Rates

The policy of fixed real interest rates can be thought of as a simple interest rate rule relating real interest rates to some measure of inflation,

$$\widehat{r}_t = \delta \pi_t$$

but where the coefficient $\delta \to 0$, such that the rule satisfies the Taylor principle to ensure determinacy, but the response of real interest rates to inflation is negligible. Since the log-linearised consumption Euler equation is given by,

$$\widehat{C}_t = E_t \widehat{C}_{t+1} - \widehat{r}_t$$

this policy implies that aggregate consumption does not deviate from its steady-state value. The results of such a policy stance are given in Figure 2. Essentially we do not get the fall in real interest rates that would occur under the optimal policy, and the exchange rate appreciates by more than is desirable. This drives down domestic inflation and output in both sectors. In contrast to the optimal policy the welfare costs of the shock rise from 0.71% to 20.57% of one period's steady-state consumption. It is clearly sub-optimal not to respond to the IRS/UIP shock.

8.0.2 Strict Output Price Inflation Targeting

[10] An obvious alternative to consider is consumer price inflation targetting. However, Leith and Wren-Lewis (2006b) demonstrate that although the cpi inflation measure reflects movements in the exchange rate, targetting this measure of inflation is clearly damaging to welfare.

We now consider a policy which seeks to ensure that

$$\Psi\lambda_T\pi_{N,t} + (1-\Psi)\lambda_N\pi_{T,H,t} = 0$$

which since we have equal degrees of price stickiness across the two sectors in our becnhmark case, implies $\pi_{N,t} = \pi_{T,H,t} = 0$. Here we find (see Figure 3) that interest rates fall, but not quite as much as under the optimal policy. However, the welfare implications of this are not very drastic with the costs of the shock rising from 0.7114% to 0.8135% under commitment, reflecting the importance of minimising price dispersion under the optimal policy.

8.1 Robustness

In this subsection we explore a number of robustness checks on our basic results. Firstly, in line with the empirical evidence, we assume that tradeable goods prices are more flexible than non-tradables prices and reduce the average contact life in the tradeables goods sector from 1 year to 7.5 months ($\theta_T = 0.6$).[11] As we reduce the stickiness of tradeables goods prices (see Figure 4) we start to get different rates of inflation in the two sectors. However, a given relaxation of policy to offset the appreciation will tend to raise the relative price of traded to non-traded goods when non-traded goods prices are relatively sticky. This will tend to exacerbate the sectoral imbalances implied by the IRS/UIP shock. In other words the optimal monetary policy reacts less to the excessive appreciation of the real exchange rate when non-tradeables are relatively more sticky than tradeables. This reduced ability to offset the shock results in a slight increase in the welfare costs of the shock with costs under commitment rising from 0.71% to 0.76%, and under discretion from 0.79% to 0.81%. In constrast the asymmetries in inertia across the two sectors has a negligible impact on the policy of strict output price inflation targetting, which remains as 0.81%. This reflects the fact that the policy of strict output price inflation targetting was already shown to involve a slight moderation of the policy response to the shock, which is appropriate when non-traded goods prices are relatively sticky.

In Figure 5 we plot the welfare costs of optimal discretionary and commitment policy, as well as the policy of strict output inflation targetting, against the proportion of non-tradeables goods in the consumption basket. As the proportion approaches one the welfare costs of IRS/UIP shocks are eliminated, since we essentially move to a closed economy special case. For discretionary policy and strict output price inflation targetting the welfare costs are close to being a monotonically decreasing function of the proportion of non-tradeables. However, for commitment policy there is a more obvious non-zero proportion of non-tradeable goods for which the welfare costs of the shock are higher.

In Figure 6 we undertake the same exercise, but considering variation in the extent of home bias. As with non-tradeable goods when the extent of home bias

[11] Cristadoro et al (2005), de Walque et al (2006) and Ortega and Rebei (2004) all find non-traded goods prices to be relatively more sticky than traded-goods prices. Our figure of $\theta_T = 0.75$ and $\theta_N = 0.6$ is consistent with the estimates of de Walque et al (2006).

is 100% our model essentially reduces to a closed economy model and IRS/UIP shocks have no welfare consequences. While with no home-bias we have already noted that the share of traded-goods in domestic production are insignificant such that this also reduces to a closed economy model where IRS shocks do not matter. Accordingly, as we increase the extent of home bias there is initially a sharp increase in the welfare costs of the shock under all policies, prior to there being a sharp decline. The reason is that initially the introduction of some home bias means that welfare now has a sectoral dimension which inhibits the policy response to the shock.

9 Conclusion

In this paper we have outlined a model of a small open economy with both traded and non-traded goods production, where both goods are sold in imperfectly competitive markets. The representative consumer maximises consumption and leisure over the infinite lifetime, and chooses among different types of good with some bias in favour of domestically produced traded goods. We assume than International Risk Sharing (IRS) holds, but is stochastic, and we focus on the impact of these shocks.

We show that the benevolant social planner will choose an allocation where production in each sector is constant, and independent of IRS/UIP shocks. However, the market equilibrium under flexible prices will not be independent of such shocks: a shock that generates a real appreciation will be associated with a reduction in the output of traded goods, but an increase in the output of non-traded goods, and this will generate welfare losses. This accords with the stylised facts on the impact of large misalignments.

By introducing sticky prices in both sectors we allow policy a potential role in offsetting such shocks. We show that a quadratic approximation to social welfare based on the utility of the representative agent in this sticky-price economy depends on output gaps and inflation in both sectors. This expression for welfare cannot be expressed in terms of aggregate output and inflation alone, but sectoral differences in output gaps can be replaced by gap terms in relative sector prices and/or the terms of trade/real exchange rate, and differences in inflation rates can be replaced by terms in the actual change in relative prices and the terms of trade/real exchange rate. Thus, the existence of non-traded goods can be seen as one justification for a concern by policy makers about both real exchange rate gaps, and changes in the exchange rate (or related measures).

We then derive optimal policy under commitment for our two sector economy. When the degree of price stickiness across the two economies is the same we get a target criterion for optimal policy which is similar to that in the closed economy- policy makers seek to return the price level in both sectors back to base following a shock. When we allow for differing degrees of price stickiness across the two sectors optimal policy becomes more forward looking as it must also assess the extent to which policy affects the evolution of relative prices between the two sectors. Using a calibrated version of our model, we contrast the optimal policy responses to IRS/UIP shocks under both commitment and discretion, as well as other descriptions of policy.

When prices are sticky, an IRS/UIP shock not only generates the changes in sectoral output noted above, but also generates inflation. Policy acts to reduce the excessive real appreciation that would emerge under flexible prices, but it does not do so completely (the consumption gap remains negative) as to do so would fuel inflation. As a result, monetary policy cannot completely offset these shocks for two reasons: they generate inflation, and they move consumption in the two sectors in different directions. This latter effect is exacerbated when non-traded goods prices are stickier than traded-goods prices. By implication, a policy that attempted to hold the terms of trade or real exchange rate constant would be significantly suboptimal, as is a policy which does not attempt to offset the shock at all. However, a policy of strict output inflation targeting is not too damaging. We also examined how these costs vary with the proportion of non-traded goods in total output, and the extent of home-bias in the consumption of traded goods.

As our model and welfare criteria capture the general policy problem facing policy makers in economies with non-traded and traded goods sectors, there is significant scope for further work utilising this model in the face of alternative shocks or alternative descriptions of policy. For example, an examination of sectoral productivity shocks would be interesting. As would an assessment of the ability of policy rules specified in terms of aggregate variables to capture the desired policy response to shocks in a sectoral economy. One can conjecture that the derivation of the quadratic loss function gives a good guide as to the appropriate form additional exchange rate terms should enter an extended Taylor-type policy rule. We leave such analysis for future research.

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Appendix I - Derivation of Welfare Measure

Social welfare is,

$$\Upsilon_t = E_0 \sum_{t=0}^{\infty} \beta^t [\ln C_t - \frac{(N_{T,H,t} + N_{N,t})^{1+\varphi}}{1+\varphi}] = E_0 \sum_{t=0}^{\infty} \beta^t \Gamma_t \tag{141}$$

Taking a second order expansion of the per-period function Γ_t gives

$$\Gamma = \widehat{C} - \bar{N}^{1+\varphi}[\widehat{N} + \frac{1}{2}\widehat{N}^2(1+\varphi)] + tip \tag{142}$$

With the subsidies in place the linear terms will cancel (see below). It can be shown that

$$\begin{aligned}
\widehat{N}_N &= \widehat{Y}_N - a_N + \ln[\int_0^1 (\frac{P_N(i)}{P_N})^{-\epsilon} di] && (143)\\
&= \widehat{Y}_N - a_N + \frac{\epsilon_t}{2} var_i\{p_N(i)\} + O[3] && (144)
\end{aligned}$$

and,

$$\begin{aligned}
\widehat{N}_{T,H} &= \widehat{Y}_{T,H} - a_T + \ln[\int_0^1 (\frac{P_{T,H}(i)}{P_{T,H}})^{-\epsilon} di] && (145)\\
&= \widehat{Y}_{T,H} - a_T + \frac{\epsilon_t}{2} var_i\{p_{T,H}(i)\} + O[3] && (146)
\end{aligned}$$

where the price dispersion terms are of second order importance. We can therefore write the welfare function as,

$$\begin{aligned}
\Gamma = \; & \widehat{C} - \bar{N}^{1+\varphi}[\frac{(1-\alpha)\gamma}{1-\alpha\gamma}\left(\widehat{Y}_{T,H} + \frac{\epsilon_t}{2} var_i\{p_{T,H}(i)\}\right) && (147)\\
& + \frac{1-\gamma}{1-\alpha\gamma}\left(\widehat{Y}_N + \frac{\epsilon_t}{2} var_i\{p_N(i)\}\right) + \frac{1}{2}\widehat{N}^2(1+\varphi)] + tip
\end{aligned}$$

The log-linearised combined IRS-resource constraint is given by (which is exact),

$$\widehat{C} = (1-\alpha)\gamma(\widehat{Y}_{T,H}) + (1-\gamma)(\widehat{Y}_N) + \gamma\widehat{\varsigma} - (1-\alpha)\gamma\widehat{\phi} \tag{148}$$

From the social planner's problem we know that with the appropriate subsidies in place the following steady-state relationships hold,

$$\overline{N} = \overline{N}_N \frac{1-\alpha\gamma}{1-\gamma} = \overline{N}_{T,H} \frac{1-\alpha\gamma}{(1-\alpha)\gamma} \tag{149}$$

Which implies,using the focs,

$$\overline{N}^{1+\varphi} = (1-\alpha\gamma) \tag{150}$$

The loglinearised definition of aggregate employment is given by,

$$\widehat{N} = \left(\frac{1-\gamma}{1-\alpha\gamma}\right)\widehat{N}_N + \left(\frac{(1-\alpha)\gamma}{1-\alpha\gamma}\right)\widehat{N}_{T,H} \tag{151}$$

and to a second order can be written as,

$$\begin{aligned}\widehat{N} &= -\frac{1}{2}\widehat{N}^2 + \left(\frac{1-\gamma}{1-\alpha\gamma}\right)(\widehat{N}_N + \frac{1}{2}\widehat{N}_N^2) + \left(\frac{(1-\alpha)\gamma}{1-\alpha\gamma}\right)(\widehat{N}_{T,H} + \widehat{N}_{T,H}^2) \\ &= -\frac{1}{2}\widehat{N}^2 + \left(\frac{1-\gamma}{1-\alpha\gamma}\right)(\widehat{Y}_N + \frac{1}{2}\widehat{Y}_N^2) + \left(\frac{(1-\alpha)\gamma}{1-\alpha\gamma}\right)(\widehat{Y}_{T,H} + \widehat{Y}_{T,H}^2) \\ &+ \left(\frac{1-\gamma}{1-\alpha\gamma}\right)\frac{\epsilon_t}{2}var_i\{p_N(i)\} + \left(\frac{(1-\alpha)\gamma}{1-\alpha\gamma}\right)\frac{\epsilon_t}{2}var_i\{p_{T,H}(i)\} \quad (152)\end{aligned}$$

Therefore we can rewrite the combined IRS-resource constraint as,

$$\begin{aligned}\widehat{C} &= (1-\alpha)\gamma(a_T) + (1-\gamma)(a_N) + \gamma\widehat{\varsigma} - (1-\alpha)\gamma\widehat{\phi} \\ &+ (1-\alpha\gamma)[\widehat{N} + \frac{1}{2}\widehat{N}^2 \\ &- \frac{1-\gamma}{1-\alpha\gamma}\widehat{N}_N^2 - \frac{(1-\alpha)\gamma}{1-\alpha\gamma}\widehat{N}_{T,H}^2] \quad (153)\end{aligned}$$

and can then rewrite welfare as,

$$\begin{aligned}\Gamma &= -\bar{N}^{1+\varphi}\frac{1}{2}[\varphi\widehat{N}^2 + \frac{1-\gamma}{1-\alpha\gamma}\widehat{N}_N^2 + \frac{(1-\alpha)\gamma}{1-\alpha\gamma}\widehat{N}_{T,H}^2 \quad (154) \\ &+ \left(\frac{1-\gamma}{1-\alpha\gamma}\right)\frac{\epsilon_t}{2}var_i\{p_N(i)\} + \left(\frac{(1-\alpha)\gamma}{1-\alpha\gamma}\right)\frac{\epsilon_t}{2}var_i\{p_{T,H}(i)\}] + tip\end{aligned}$$

From the social planner's problem we saw that the efficient level of employment in both sectors was invariant to shocks so that we can rewrite welfare in terms of efficiency gaps, $\widehat{X}^g = \widehat{X} - \widehat{X}^e$ to obtain,

$$\begin{aligned}\Gamma &= -\bar{N}^{1+\varphi}\frac{1}{2}[\varphi\widehat{N}^{g2} + \frac{1-\gamma}{1-\alpha\gamma}\widehat{N}_N^{g2} + \frac{(1-\alpha)\gamma}{1-\alpha\gamma}\widehat{N}_{T,H}^{g2} \quad (155) \\ &+ \left(\frac{1-\gamma}{1-\alpha\gamma}\right)\frac{\epsilon_t}{2}var_i\{p_N(i)\} + \left(\frac{(1-\alpha)\gamma}{1-\alpha\gamma}\right)\frac{\epsilon_t}{2}var_i\{p_{T,H}(i)\}] + tip\end{aligned}$$

However, it is more conventional to write this in terms of output gaps. From the efficiency solution derived above,

$$\widehat{Y}_N^e = a_N + \widehat{N}^e \quad (156)$$

and.

$$\widehat{Y}_{T,H}^e = a_T + \widehat{N}_{T,H}^e \quad (157)$$

Therefore from the production functions in both sectors these gaps will be proportional to the output gaps, such that welfare can be rewritten as,

$$\begin{aligned}\Gamma &= -\bar{N}^{1+\varphi}\frac{1}{2}[\varphi\left(\left(\frac{1-\gamma}{1-\alpha\gamma}\right)\widehat{Y}_N^g + \left(\frac{(1-\alpha)\gamma}{1-\alpha\gamma}\right)\widehat{Y}_{T,H}^g\right)^2 + \frac{1-\gamma}{1-\alpha\gamma}(\widehat{Y}_N^g)^2 + \frac{(1-\alpha)\gamma}{1-\alpha\gamma}(\widehat{Y}_{T,H}^g)^2 \\ &+ \left(\frac{1-\gamma}{1-\alpha\gamma}\right)\frac{\epsilon_t}{2}var_i\{p_N(i)\} + \left(\frac{(1-\alpha)\gamma}{1-\alpha\gamma}\right)\frac{\epsilon_t}{2}var_i\{p_{T,H}(i)\}] + tip \quad (158)\end{aligned}$$

which simplifies to,

$$\begin{aligned}\Gamma &= -\bar{N}^{1+\varphi}\frac{1}{2}[\left(\frac{1-\gamma}{1-\alpha\gamma}\right)\left(1+\varphi\left(\frac{1-\gamma}{1-\alpha\gamma}\right)\right)(\widehat{Y}_N^g)^2+\frac{(1-\alpha)\gamma}{1-\alpha\gamma}(1+\varphi\frac{(1-\alpha)\gamma}{1-\alpha\gamma})(\widehat{Y}_{T,H}^g)^2 \\ &+2\varphi\left(\frac{1-\gamma}{1-\alpha\gamma}\right)\frac{(1-\alpha)\gamma}{1-\alpha\gamma}(\widehat{Y}_N^g)(\widehat{Y}_{T,H}^g) \qquad (159)\\ &+\left(\frac{1-\gamma}{1-\alpha\gamma}\right)\frac{\epsilon_t}{2}var_i\{p_N(i)\}+\left(\frac{(1-\alpha)\gamma}{1-\alpha\gamma}\right)\frac{\epsilon_t}{2}var_i\{p_{T,H}(i)\}]+tip\end{aligned}$$

Woodford (2006, Chapter 6) shows that,

$$\sum\beta^t varvar_i\{p_{N,t}(i)\}=\frac{1}{\lambda_N}\sum\beta^t\pi_{N,t}^2 \qquad (160)$$

and,

$$\sum\beta^t varvar_i\{p_{T,H,t}(i)\}=\frac{1}{\lambda_T}\sum\beta^t\pi_{T,H,t}^2 \qquad (161)$$

so that we can write out welfare measure as equation (112) in the main text.

Appendix II - Linking the Relative Price Term to the Output Gaps.

Log-linearising,

$$Y_{T,H} = \gamma S C^{*}[(1-\alpha)\varsigma_t + \alpha] \tag{162}$$

yields,

$$\widehat{Y}_{T,H} = \widehat{S} + \widehat{\phi} \tag{163}$$

and,.

$$T = S^{\alpha}\left(\frac{1-\gamma}{\gamma}\right)\left(\frac{C_N}{C_T}\right)^{-1} \tag{164}$$

log-linearises as,

$$\widehat{T} = \alpha\widehat{S} - \left(\widehat{C}_N - \widehat{C}_T\right) \tag{165}$$

Solving simultaneously,

$$\widehat{T} = \alpha(\widehat{Y}_{T,H} - \widehat{\phi}) - \left(\widehat{C}_N - \widehat{C}_T\right) \tag{166}$$

Using the definition of tradeable consumption,

$$\widehat{T} = \alpha(\widehat{Y}_{T,H} - \widehat{\phi}) - \left(\frac{1}{\gamma}\widehat{Y}_N - \frac{1}{\gamma}\widehat{C}\right) \tag{167}$$

Using IRS/AD condition (55) to eliminate aggregate consumption we obtain,

$$\widehat{T} = \alpha(\widehat{Y}_{T,H} - \widehat{\phi}) - \left(\frac{1}{\gamma}\widehat{Y}_N - \frac{1}{\gamma}\left((1-\alpha)\gamma\widehat{Y}_{T,H} + (1-\gamma)\widehat{Y}_N + \gamma\widehat{\varsigma} - (1-\alpha)\gamma\widehat{\phi}\right)\right) \tag{168}$$

Simplifying,

$$\widehat{T} = \widehat{Y}_{T,H} - \widehat{Y}_N + \alpha\widehat{\varsigma} \tag{169}$$

Using the definition of efficient variables this can be rewritten in gap form as equation (125) in the main text.

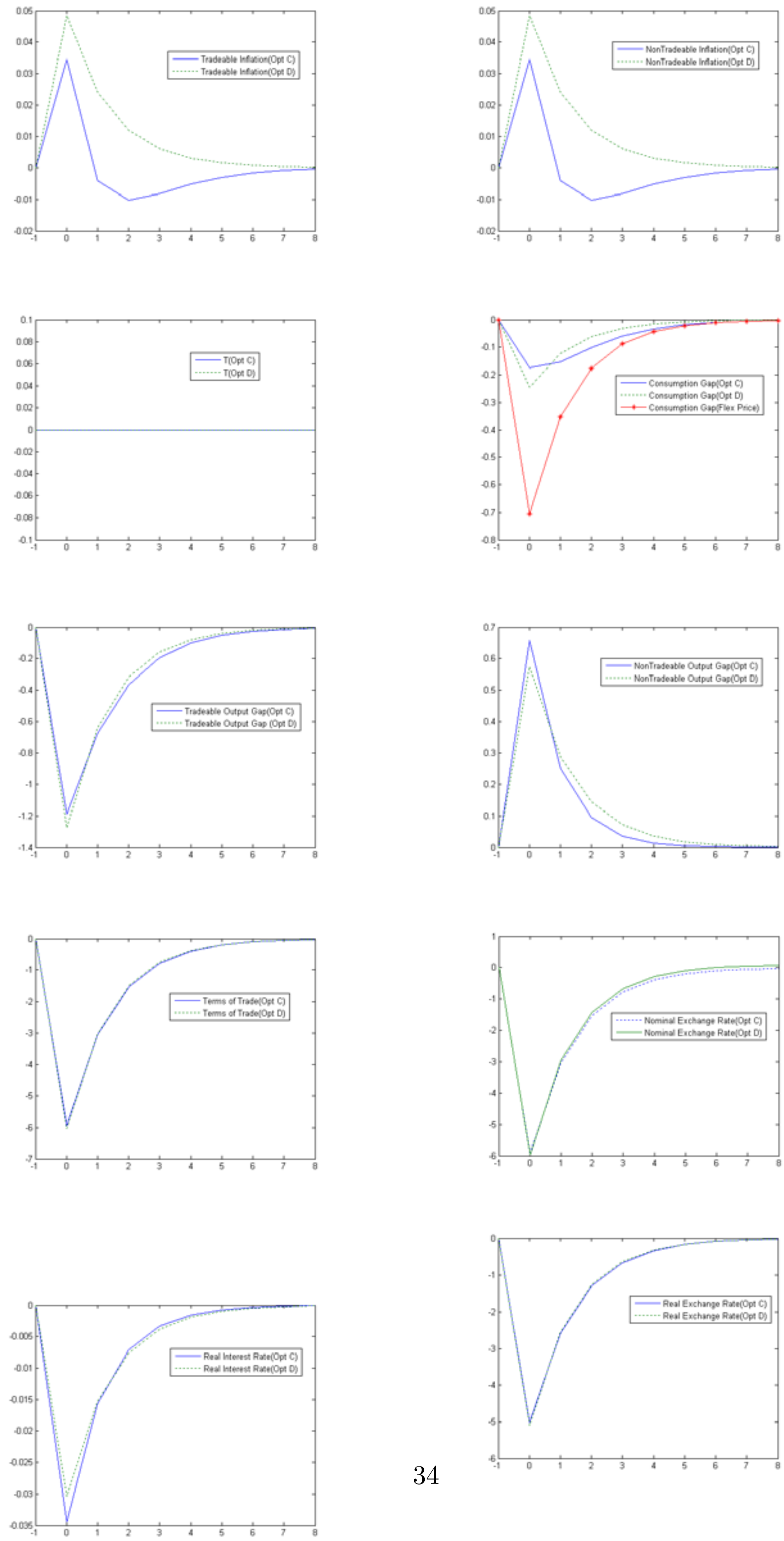

Figure 1: Optimal Response to a 1 Std Dev IRS Shock.

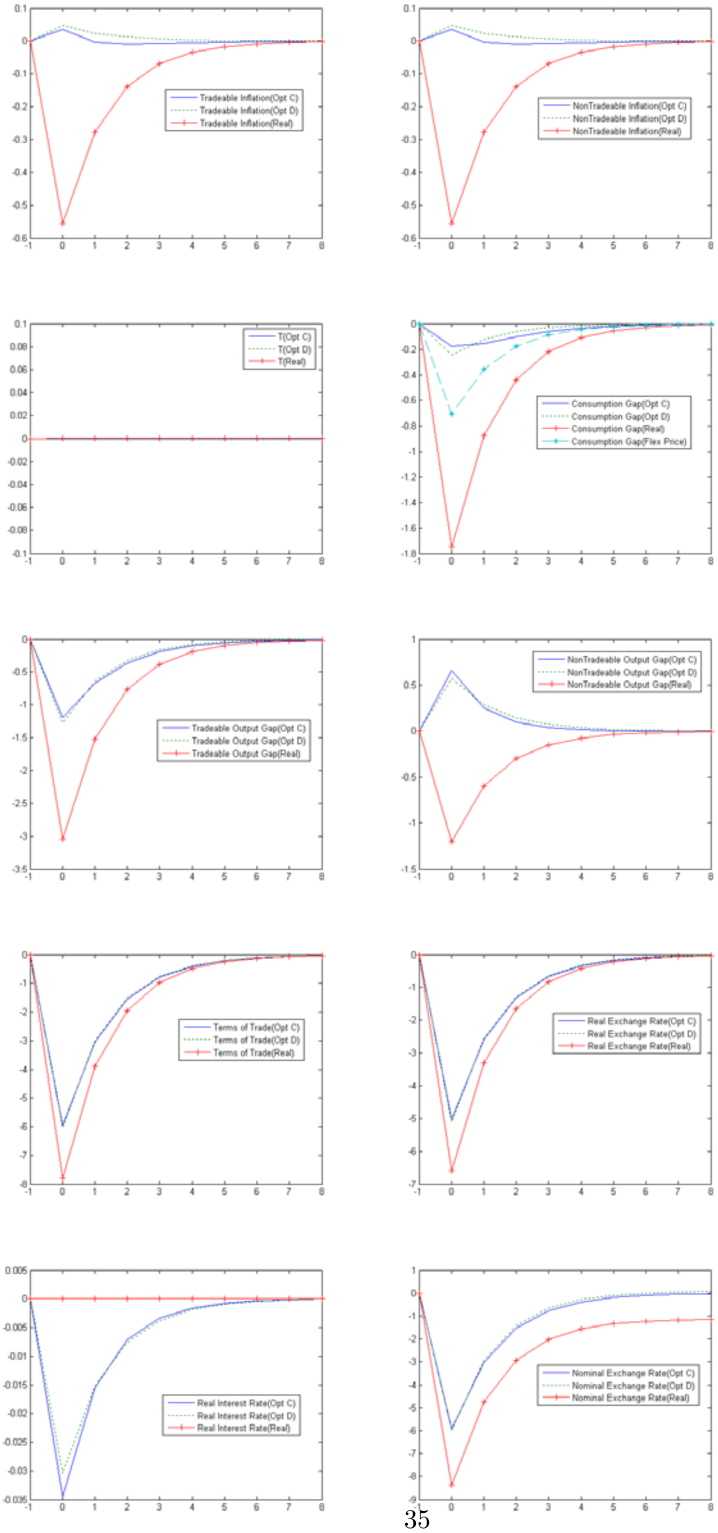

Figure 2: Optimal Policy and Constant Real Interest Rate Policy

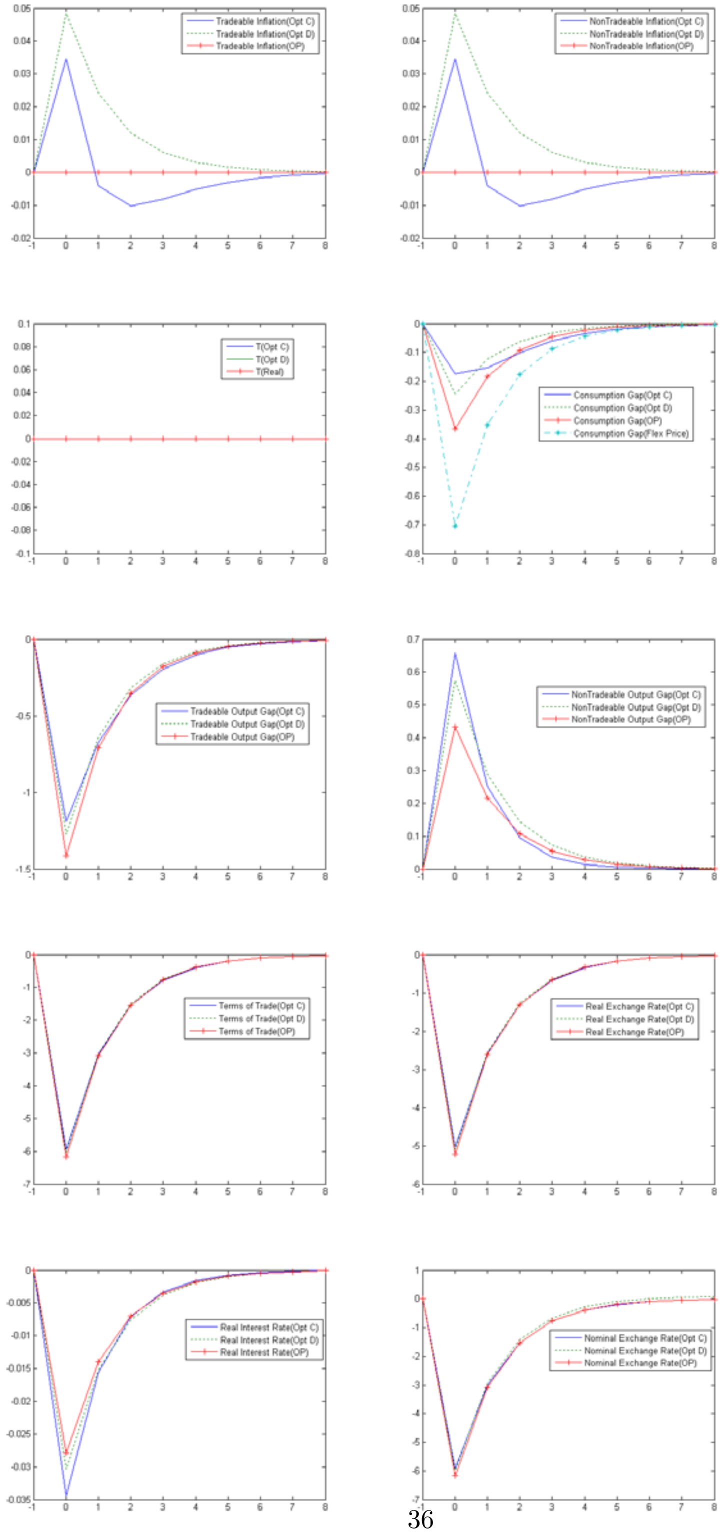

Figure 3: Optimal Policy and Strict Output Inflation Targeting

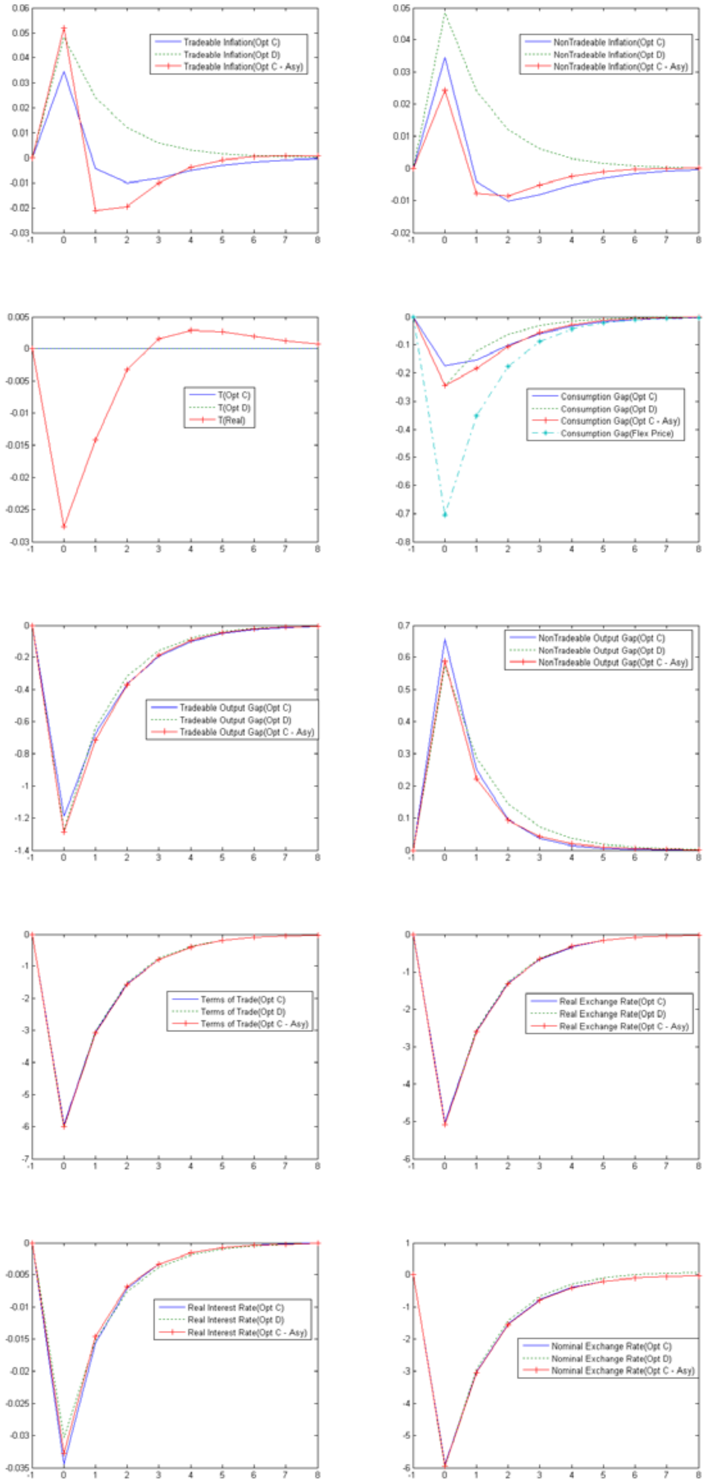

Figure 4: Optimal Policy with Symmetric and Asymmetric Price Stickiness across Sectors.

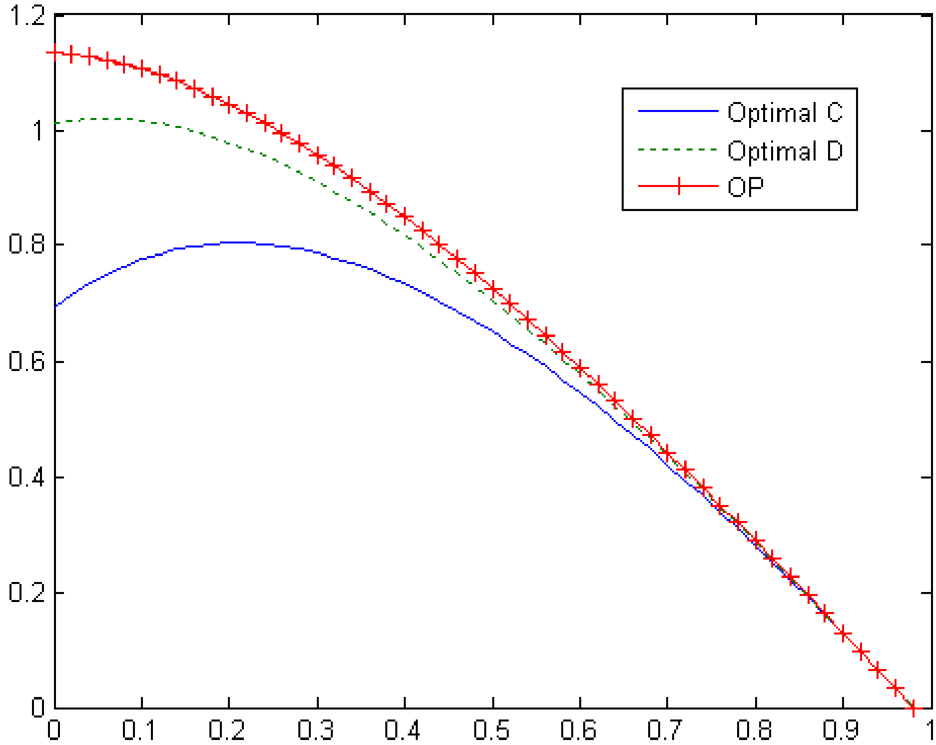

Figure 5: Welfare under Alternative Policies against the Proportion of Non-Tradeables.

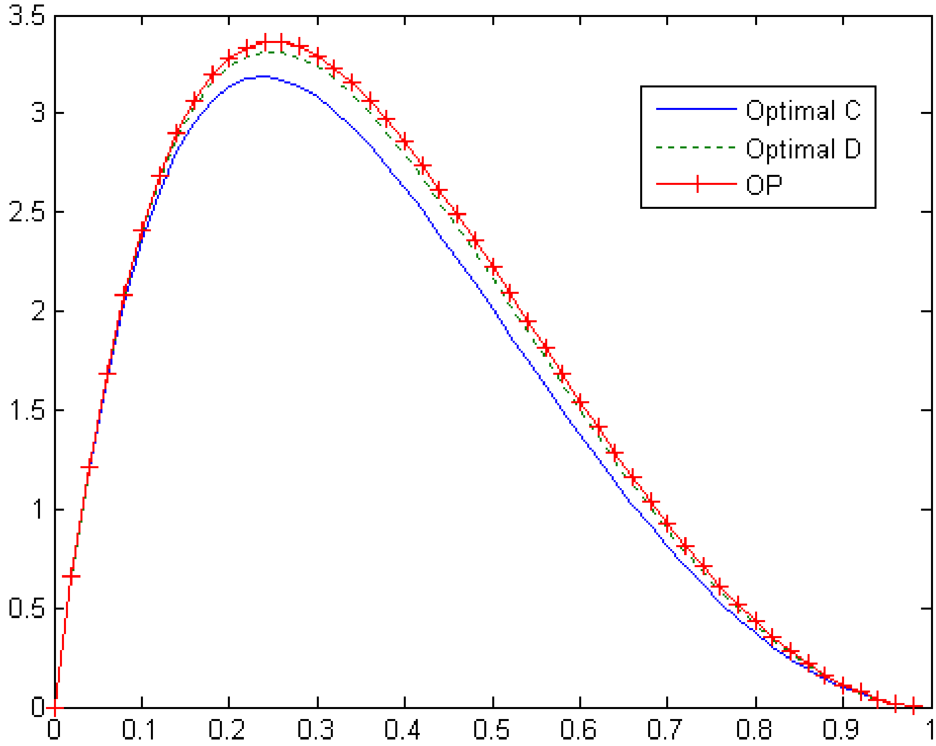

Figure 6: Welfare Under Alternative Policies Against Degree of Home Bias