Money Velocity in an Endogenous Growth Business Cycle with Credit Shocks *

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Abstract

The explanation of velocity has been based in substitution and income effects, since Keynes's (1923) interest rate explanation and Friedman's (1956) application of the permanent income hypothesis to money demand. Modern real business cycle theory relies on a goods productivity shocks to mimic the data's procyclic velocity feature, as in Friedman's explanation, while finding money shocks unimportant and not integrating financial innovation explanations. This paper sets the model within endogenous growth and adds credit shocks. It models velocity more closely, with significant roles for money shocks and credit shocks, along with the goods productivity shocks. Endogenous growth is key to the construction of the money and credit shocks since they have similar effects on velocity, through substitution effects from changes in the nominal interest rate and in the cost of financial intermediation, but opposite effects upon growth, through permanent income effects that are absent with exogenous growth.

Keywords: Velocity, business cycle, credit shocks, endogenous growth.

JEL: E13, E32, E44

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1 Introduction

The income velocity of money has been explained in quite a few different ways, leaving significant puzzles in the wake. Originally, during very low inflation times and stodgy banking technology eras, Fisher's (1911) assumption of an institutional fixity of velocity appeared reasonable. In contrast, fluctuations in velocity have been discussed as far back as Keynes (1923), whose proposed reform of money policy is perhaps his first activist stance: stabilize the price level by actively offsetting changes in velocity due to nominal interest rate changes that predictably affect money demand. Friedman (1959) takes a different tact in explaining velocity. While first famously restating money demand theory to emphasize both the substitution effects from interest rates and the income measures that might affect velocity (Friedman 1956), Friedman (1959) and Friedman and Schwartz (1963) apply the permanent income hypothesis of consumption (Friedman 1957) to money demand and emphasize the income effect. Money demand depends on permanent income; temporary income changes cause total income to rise while money demand does not, making the income velocity of money rise.

Many modern theories of money assume an exogenous velocity, for example as do Lucas (1988a), Ireland (1996), Alvarez, Lucas, and Weber (2001) and Cochrane (2005). But within the monetary business cycle framework, there has been an explanation of velocity as based on the income effect. Cooley and Hansen (1995) find evidence of a procyclic behavior of US velocity that they call "one of the most compelling features of aggregate data" (p.179). Using a standard, exogenous growth, monetary business cycle model, with goods productivity and money supply growth rate shocks, they find that their model shows considerable success in reproducing a procyclic velocity. The goods sector productivity shock drives velocity changes, since the money shock has little effect in the model. With exogenous growth, a positive goods productivity shock is temporary; income rises temporarily while money demand depends on consumption and is not much affected; and so the procyclic velocity occurs for the same reasons as in Friedman and Schwartz's (1963) application of the permanent income hypothesis. One problem is that the model's velocity tends to be too procyclic relative to the data.

A business cycle explanation of velocity as based in substitution effects has found no substantiation to date. Allowing money shocks has been found to have little impact on business cycles (Cooley and Hansen 1989) and also little role in explaining velocity over the business cycle [Cooley, Hansen, and Prescott (1995) and Benk, Gillman, and Kejak (2005a); see Wang and Shi (2006) for an exception]. Here, money shocks cause the inflation rate and nominal interest rate to be shocked, so their non-importance implies that the substitution effect of the nominal interest rate effect on money demand, emphasized by Keynes (1923) and many others since [for example McGrattan (1998) and Gillman and Kejak (2004)], is not important in explaining velocity. This creates somewhat of a puzzle: the most traditional explanation of velocity has no role in explaining velocity within a real business cycle framework.

The other explanation for velocity likewise not found in the monetary business cycle model is that increases in financial innovation cause substitution away from money and a higher velocity. For example, Friedman and Schwartz (1982) emphasized that shifts in velocity occur due to changes in financial innovation that cause money demand to shift down as interest-bearing instruments become popular, also a focus of Barnett (1997). For example, it is difficult to explain the recent upward movement in M1 velocity since 1994 without using financial deregulation of the banking system Gillman and Kejak (2004). Gillman and Kejak (2004) and Gillman and Nakov (2004) argue that shifts in financial sector productivity, due to banking law changes, can explain such shifts. Benk, Gillman, and Kejak (2005b) follow this direction by introducing, into an otherwise standard monetary business cycle, technology shocks to an exchange credit sector. An empirical construction of their shocks from data shows that the credit sector shocks contributed to explaining the observed GDP movements during the deregulatory era. This is consistent with an episodal description of cycles that are based in the finance sector, as overviewed in Plosser (1990). But the effect on velocity in a real business cycle setting, by such credit shocks, has to date not been established, leaving as a puzzle the unexplained velocity movements possibly due to financial deregulation.

The paper contributes to resolving the three velocity issues that are described above for stochastic dynamic general equilibrium models: movements that are too procyclic, no role for money shocks, and no role for financial deregulation. It does this by including credit shocks and by specifying an endogenous growth framework, in addition to standard goods productivity and money supply shocks. This combines all three explanations, based on the income effect and two substitution effects. Consider how this resolves the existing puzzles. First, a standard positive shock to goods production productivity causes not just a level effect on income, as does the temporary income effect that exists in the exogenous growth business cycle model. Instead, the real interest rate rises which in turn causes a higher growth rate. Part of the effect of the increased productivity is a temporary increase in the growth rate, and a permanent increase in the income level. Part is still just a temporary income increase. Velocity goes up by less from this shock than in the exogenous growth model: it does not increase at all from permanent income increases that also increase consumption and money demand. And so it is left to increase only from the part of the shock that remains as a temporary income effect. Velocity is less procyclic in the model due to this shock, and more consistent with the data.

Second, a positive credit shock within endogenous growth causes greater productivity of credit production, more credit use with less time required, less real money use, and more time available for other uses such as human capital investment. Also, for a given inflation rate, the greater credit use means the consumer need use leisure less to avoid the inflation tax, while using credit more, and the human capital utilization rate and the growth rate both increase. A positive temporary income effect also results from increased time productivity, as studied in Benk, Gillman, and Kejak (2005a).With less money use, velocity rises, and with a temporary income effect, velocity rises by more. The permanent income effect from higher growth additionally increases velocity, unlike the permanent income hypothesis applied to money demand, because both money and credit are used to buy the consumption that follows permanent income, and money demand goes down because of the substitution effect towards credit use. This provides a way to model deviations of velocity from its trend due to temporary changes in banking laws, such as deregulations, that affect GDP and velocity in ways that changes in the nominal interest rate, and typical cyclic fluctuations over the business cycle, cannot explain.

Third, money shocks and credit shocks are closely correlated within endogenous growth but not in exogenous growth, varying from 0.5 correlation with high persistence of the money shock (0.9) to 0.9 correlation with low persistence of the money shock (0.5). As a result money shocks become a non-trivial part of the velocity explanation, as they affect the growth rate and permanent income. A positive shock to the rate of growth in the money supply acts in a similar way to a credit shock, except for the effect on the growth rate. An increase in the money supply growth rate causes a positive shock to the inflation rate and the nominal interest rate. This reduces real money demand, causes substitution towards credit use, and increases velocity; a positive credit shock increases credit use, decreases money use and also increases velocity. However the money shock also causes substitution from inflation-taxed goods to non-inflation-taxed leisure, which reduces the human capital utilization rate and shocks down the growth rate (see gk 05A,B). This is opposite of the credit shock which decreases leisure use and increases the growth rate. Consequently the money shock decreases permanent income some, and consumption and real money demand; credit shocks cause the opposite changes. With exogenous growth, permanent income is not affected by such shocks and there is only substitution between money and credit, for a higher velocity. With endogenous growth, the increase in growth from a correlated credit shock is partially offset by the decrease in growth from the money shock. Together the money and credit shocks then can mimic better the actual change in growth and the change in permanent income, versus the temporary income effect. This extra degree of freedom in constructing the shocks, while taking into account growth rate and velocity changes, gives money shocks an importance not found in exogenous growth frameworks. A significant role of money in explaining velocity helps align common intuition with results of the business cycle model.

Section 2 sets out the endogenous growth economy with credit; Section 3 the calibration and solution methodology; and Section 4 the impulse responses. Sections 5 and 6 present the data and the construction of the model's shocks from the data, with full simulations and the explanation of the velocity given in Sections 7 and 8, followed by conclusions.

2 The Endogenous Growth Economy with Credit

The representative agent economy is an endogenous growth version of the monetary business cycle with credit of Benk, Gillman, and Kejak (2005b) and Benk, Gillman, and Kejak (2005a). Human capital investment causes growth as in Lucas (1988b).

2.1 Consumer Problem

Consider a representative agent that maximizes over an infinite horizon its expected lifetime utility over consumption c_t and leisure x_t. Utility is given by:

$$U = E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, x_t) \qquad 0 < \beta < 1, \quad u(c, x) = \frac{(cx^{\Psi})^{1-\theta}}{1-\theta}. \tag{1}$$

Utility maximization is subject to a cash stock constraint, an income flow constraint, and a human capital investment constraint, described below.

Output of goods (y_t), and human capital is produced with physical capital and effective labor each in Cobb-Douglas fashion, with functions denoted by G and H. Let k_t and h_t denote the stocks of physical capital and human capital, with the fixed depreciation rate of the capital stocks denoted by δ_k and δ_h. Let s_{Gt} and s_{Ht} denote the fraction of physical capital that the agent uses in the goods production and human capital investment, whereby

$$s_{Gt} + s_{Ht} = 1. \tag{2}$$

The agent allocates time (normalized to unity) amongst labor in goods production (l_t), leisure (x_t), time spent investing in the stock of human capital (n_t), and time spent in providing (producing) credit for exchange(f_t):

$$l_t + x_t + f_t + n_t = 1.$$

Then $l_t h_t$, $n_t h_t$, $f_t h_t$ are the effective labor employed in each sector.

Output of goods can be converted into physical capital without incurring any cost, and so is divided between consumption goods and investment net of capital depreciation. Thus, the capital stock used for production in the next period is given by:

$$k_{t+1} = (1 - \delta_k)k_t + i_t = (1 - \delta_k)k_t + y_t - c_t. \tag{3}$$

The human capital investment is produced with the following King and Plosser technology:

$$H(s_{Ht}k_t, n_t h_t) = A_H(s_{Ht}k_t)^{1-\eta}(n_t h_t)^{\eta}. \tag{4}$$

And the human capital flow constraint is:

$$h_{t+1} = (1 - \delta_h)h_t + H(s_{Ht}k_t, n_t h_t). \tag{5}$$

2.1.1 Exchange

The consumer can purchase the goods by using either money or credit services. Let $a_t \in (0, 1]$ denote the fraction of consumption goods that are purchased with money. The consumer's cash-in-advance constraint is

$$M_t + T_t \geq a_t P_t c_t, \tag{6}$$

where M_t is the money stock carried from the previous period and T_t is the nominal lump-sum money transfer received from the government.

The amount of real credit used is equal to $c_t(1-a_t)$. The credit production function transforms the effective labor per unit of consumption into a certain share of credit in the total exchange for consumption, in a diminishing returns fashion, as given by

$$(1 - a_t) = F(\frac{f_t h_t}{c_t}, v_t) = A_F e^{v_t} \left(\frac{f_t h_t}{c_t}\right)^{\gamma}, \qquad A_F > 0, \quad \gamma \in (0, 1). \tag{7}$$

This makes $A_F e^{v_t}$ the productivity shift parameter. There exist credit productivity shocks that follow an autocorrelated process:

$$v_{t+1} = \varphi_v v_t + \epsilon_{vt}, \qquad \epsilon_{vt} \sim N(0, \sigma^2_{\epsilon v}), \quad 0 < \varphi_v < 1. \tag{8}$$

Note the microfoundations for the credit production function. Denoting total real credit by d_t, the full function can be written as

$$d_t = c_t(1 - a_t) = A_F e^{v_t} (f_t h_t)^{\gamma} c_t^{1-\gamma}, \tag{9}$$

so that it is CRS in effective labor $f_t h_t$ and consumption goods c_t. Gillman, Harris, and Kejak (2006) lay out a fully decentralized version of financial intermediation using a similar production function. It is micro-founded in the literature of Clark (1984) and Hancock (1985) who specify a third factor other than labor and capital for the production of financial services, this factor being the deposited funds. While capital is omitted here for simplicity, the goods consumption can be thought of as being equal to the deposited funds. This follows when all exchange means originate from the intermediary, both money deposits and credit, and is backed completely by deposits held in the bank. Since c_t is the total amount of goods bought with exchange means in the model, total deposits would equal c_t. Thus the production function is directly based upon the micro-banking function which is CRS in standard inputs and financial deposits. While the deposit structure is suppressed in this model for simplicity of presentation, the credit productivity shock can be thought of as a standard productivity shock to the financial intermediation sector.

2.1.2 Income

The period t budget constraint of the representative consumer is given by:

$$P_t w_t l_t h_t + P_t r_t s_{Gt} k_t + P_t(1 - \delta_k)k_t + T_t + M_t \geq P_t c_t + P_t k_{t+1} + M_{t+1}. \tag{10}$$

2.2 Government Money Supply

It is assumed that the government policy includes sequences of nominal transfers which satisfy:

$$T_t = \Theta_t M_t = (\Theta^* + e^{u_t} - 1)M_t, \qquad \Theta_t = [M_t - M_{t-1}]/M_{t-1}. \tag{11}$$

where Θ_t is the growth rate of money and Θ^* is the stationary growth rate of money. Transfer is subject to random shocks u_t which follow the autoregressive process:

$$u_{t+1} = \varphi_u u_t + \epsilon_{ut}, \qquad \epsilon_{ut} \,\tilde{}\, N(0, \sigma^2_{\epsilon u}), \quad 0 < \varphi_u < 1. \tag{12}$$

2.3 Producer Problem

The firm maximizes profit given by $y_t - w_t l_t h_t - r_t s_{Gt} k_t$, subject to a standard Cobb-Douglas production function in effective labor and capital. This is given as

$$y_t = G(s_{Gt}k_t, l_t h_t, z_t) = A_G e^{z_t}(s_{Gt}k_t)^{1-\alpha}(l_t h_t)^{\alpha}. \tag{13}$$

Technology shocks follow an autoregressive process:

$$z_{t+1} = \varphi_z z + \epsilon_{zt}, \qquad \epsilon_{zt} \sim N(0, \sigma^2_{\epsilon z}), \quad 0 < \varphi_z < 1. \tag{14}$$

The first order conditions for the firm's problem yield the following expressions for the wage rate and the rental rate of capital:

$$w_t = \alpha A_G e^{z_t} \left(\frac{s_{Gt}k_t}{l_t h_t} \right)^{1-\alpha}, \tag{15}$$

$$r_t = (1-\alpha) A_G e^{z_t} \left(\frac{s_{Gt}k_t}{l_t h_t} \right)^{-\alpha}. \tag{16}$$

2.4 Definition of Competitive Equilibrium

Denote the state of the economy by $s = (k, h, M, z, u, v)$ and by a prime (') the next-period values. A competitive equilibrium consists of a set of policy functions $c(s)$, $x(s)$, $l(s)$, $n(s)$, $f(s)$, $s_G(s)$, $a(s)$, $k'(s)$, $h'(s)$, $M'(s)$, pricing functions $P(s)$, $w(s)$, $r(s)$ and a value function $V(s)$, such that:

(i) households maximize utility: given the pricing functions and the policy functions, $V(s)$ solves the functional equation (17).

(ii) firms maximize profits, the functions w and r being given by (15) and (16).

(iii) goods and money markets clear, in equations (11)-(14).

The representative agent's optimization problem can be written in a recursive form as:

Ψ	θ	β	α	η	φ_z	φ_u	φ_v	$\sigma_{\epsilon z}$
3.2	2	0.986	0.6	0.99	0.95	0.90	0.95	0.75

$\sigma_{\epsilon u}$	$\sigma_{\epsilon v}$	A_G	A_H	A_F	γ	δ_k	δ_h
1.0	0.75	1	0.12	0.83	0.13	0.012	0.012

Table 1: Parameter Values Used in the Calibration

$$V(s) = \max_{c,x,l,n,f,s_G,a,k',h',M'} \{u(c,x) + \beta EV(s')\} \tag{17}$$

subject to the conditions (2)-(10).

3 Solution Methodology

In order to put the problem into a for for which sandard solution techniques an be applied, we transform the variables so that all variables in the der-erministic version of the model converge to a steady state. Define $\tilde{c} = c/h$, $\tilde{\imath} = i/h$, $\tilde{k} = k/h$, $\tilde{M} = M/Ph$ and thus $\tilde{s} = (\tilde{k}, 1, 1, z, u, v)$ so that all variables marked with $(\tilde{\ })$ follow a stationary process.

3.1 Calibration

To solve and simulate the model, the model parameters must be assigned values. We calibrate the model by mapping the model economy into observed features of the data; parameters are chosen so that certain features of the nonstochastic steady state of the model match average values from US quarterly time series between 1959:1-2004:4. The calibation process follows closely Benk, Gillman, and Kejak (2005a). Table 1 presents the parameter values used for calibration.

3.2 Numerical Approximation of Solution

In order to solve the model, we log-linearize the equilibrium conditions of the model around its deterministic steady state, and denote by $\bar{\xi}$ the steady state value of variable ξ, and by $\hat{\xi}$ its percentage deviation from the steady state ($\hat{\xi} = \log(\xi) - \log(\bar{\xi})$). Then we solve the resulting stochastic linear system of equations by using standard techniques described, for example as in Hartley, Sheffrin, and Salyer (1997).

4 Impulse responses

4.1 Goods Productivity

Productivity shocks (denoted TS below, Figure 1) cause a temporary increase in the growth rate, g_y below, and a permanent increase in consumption and real money balances normalized by human capital, c/H and m/H below, and to the real wage w. Normalized output y/H is higher for more than 50 periods, and leisure, x, is lower. The lower leisure causes the human capital utilization rate to be higher. The return on human capital depends positively on this utilization rate and the capital to effective labor ratio in human capital production, both of which rise.

4.2 Credit Productivity Shocks

The credit shock causes real money balances to fall, velocity to rise, the real interest rate to rise, the real wage rate to fall, and the growth rate to go up. It also causes the inflation rate and nominal interest rate to fall. The input price ratio change, or more expensive capital relative to labor, causes the investment ratio to fall and the consumption ratio to rise.

4.3 Money Supply Growth Rate Shocks

The money shock causes inflation and the nominal interest rate to rise, and velocity to rise, while causing a liquidity type effect of a decrease in the real interest rate, an increase in the real wage, and a decreased growth rate. As the input price ratio changes, the investment ratio increases and the consumption ratio decreases, a Tobin type effect.

5 Data

Listing of variables.

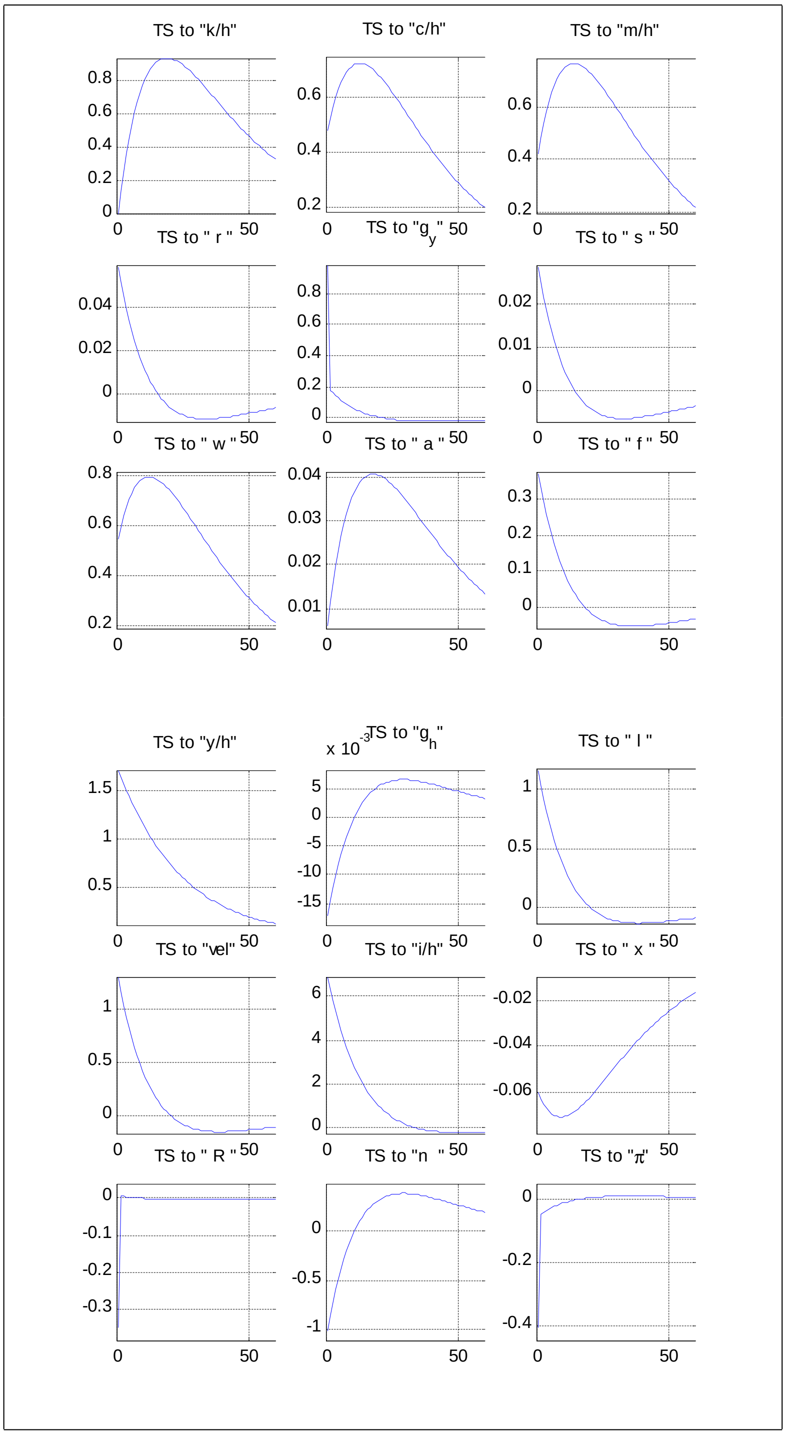

Figure 1: Impulse Responses to Productivity Shocks

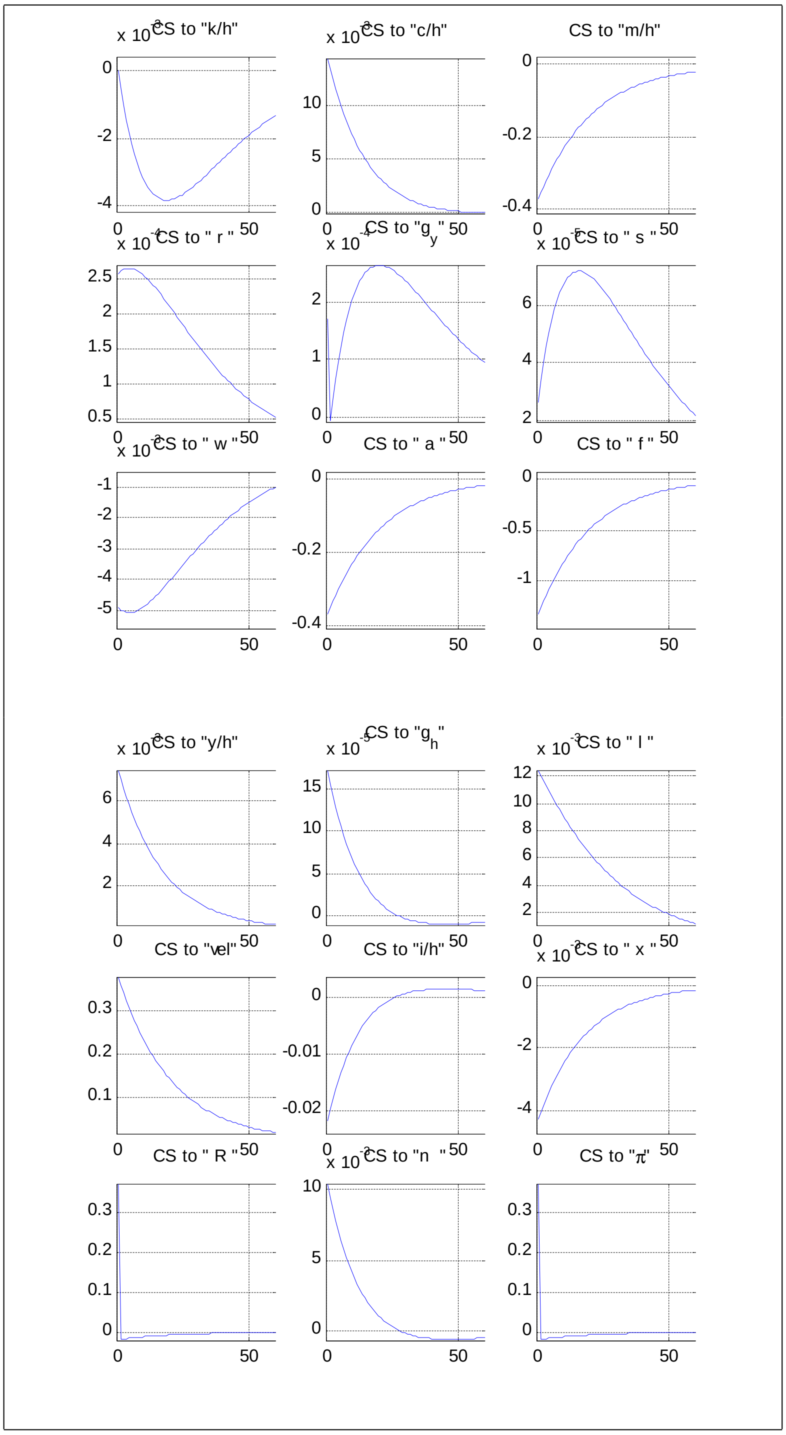

Figure 2: Impulse Responses to Credit Shocks

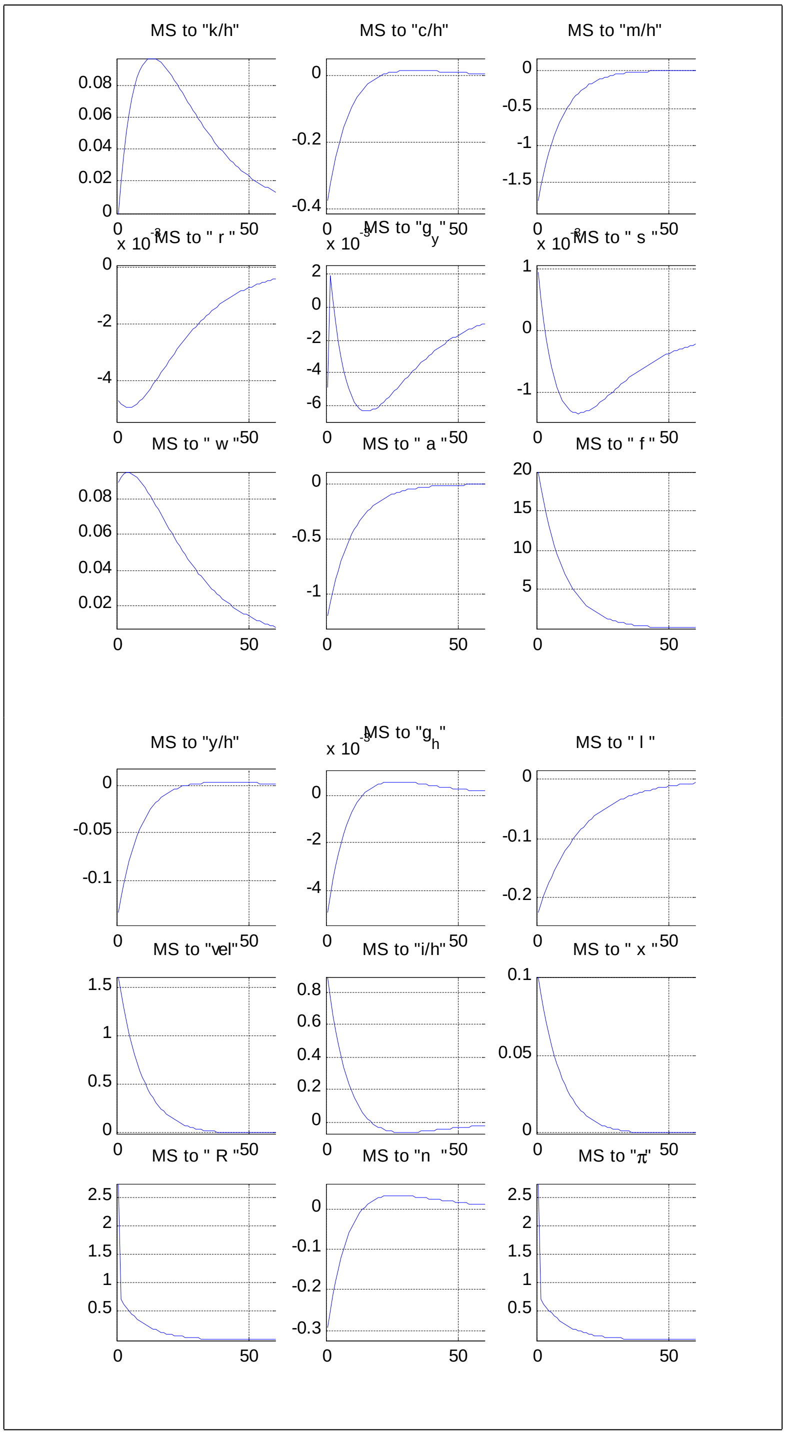

Figure 3: Impulse Responses to Money Shocks

5.1 Correlations with Output

Table 2 presents US data stylized facts. The numbers in the table represent the moments of the cyclical components of HP filtered series. Relative volatility is measured as the ratio of standard deviation of the series to the standard deviation of GDP.

Velocity's contemporaneous correlation is 0.26. Consumption and investment are positively correlated at 0.81 and 0.92. Inflation and the nominal interest rate are positively correlated contemporaneously but negatively at lags.

5.2 Correlation of Trends and Cycles in GDP and Velocity

Another perspective is just to look at the cyclical components of GDP and velocity together, or at the trends of GDP versus velocity (Figure 4). This shows that GDP and velocity cycled together in some periods, such as 1984-1986 and 1999-2001, but departures are many, leaving a potential role for credit and money shocks. Trendwise, the departures since the beginning of the high inflation era, starting say in 1973 with the collapse of Bretton Woods, are pronounced (Figure 5).

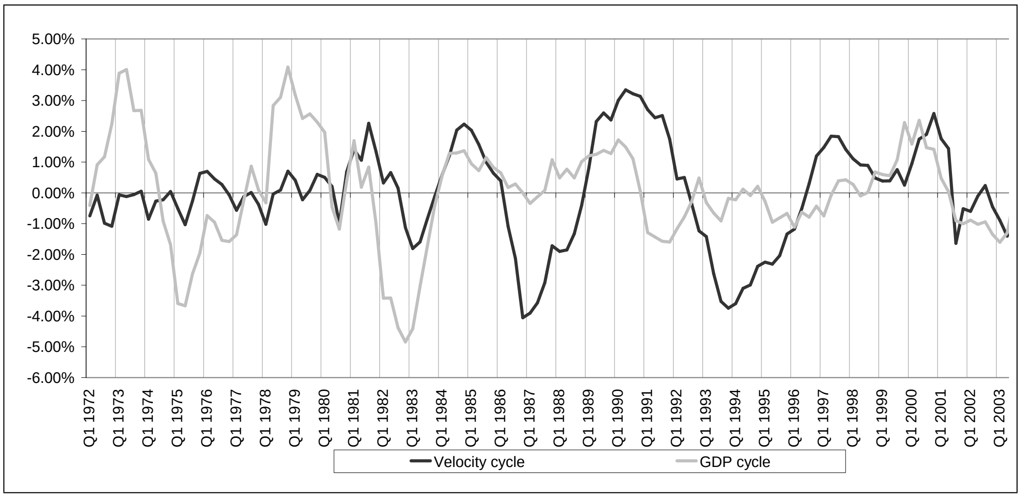

Figure 4: Cyclical Components of GDP and M1 Velocity

	Relative Volatility	Corr w/ Money growth	Corr w/ Output growth	corr(output(t), variable(t+i)) -5	-4	-3	-2	-1	0	1	2	3	4	5
GDP	1.00	-0.11	0.27	0.03	0.23	0.45	0.67	0.86	**1.00**	0.86	0.67	0.45	0.23	0.03
GDP growth	0.52	0.06	0.99	0.27	0.35	0.40	0.42	0.37	**0.30**	-0.23	-0.31	-0.38	-0.39	-0.36
Consumption	0.99	0.03	0.42	0.36	0.53	0.67	0.79	0.85	**0.81**	0.63	0.42	0.20	0.00	-0.18
Investment	2.50	-0.13	0.13	0.06	0.23	0.41	0.62	0.81	**0.92**	0.85	0.71	0.53	0.34	0.13
Banking hours	0.69	0.00	-0.39	-0.47	-0.46	-0.42	-0.32	-0.18	**0.00**	0.17	0.30	0.40	0.46	0.48
Nominal wages	0.42	0.01	-0.14	-0.45	-0.48	-0.51	-0.49	-0.41	**-0.32**	-0.30	-0.24	-0.17	-0.11	-0.04
Real wages	0.65	0.15	0.37	0.41	0.52	0.56	0.58	0.57	**0.49**	0.32	0.16	-0.01	-0.18	-0.30
CPI	0.80	-0.11	-0.36	-0.57	-0.67	-0.72	-0.72	-0.67	**-0.56**	-0.41	-0.25	-0.08	0.08	0.22
Inflation	0.29	-0.31	-0.19	-0.35	-0.28	-0.13	-0.01	0.15	**0.32**	0.41	0.45	0.46	0.48	0.39
Real money	1.93	0.21	0.28	0.29	0.39	0.44	0.47	0.43	**0.35**	0.22	0.10	-0.01	-0.10	-0.17
M1	1.48	0.21	0.15	0.04	0.11	0.15	0.18	0.16	**0.12**	0.05	-0.01	-0.05	-0.06	-0.07
M1 growth	0.65	1.00	0.06	0.10	0.13	0.10	0.07	-0.05	**-0.10**	-0.15	-0.12	-0.08	-0.04	-0.02
Tbill rate	0.76	-0.43	-0.28	-0.59	-0.51	-0.36	-0.18	0.08	**0.30**	0.42	0.47	0.47	0.45	0.41
Real interest	0.98	0.01	0.03	-0.04	-0.06	-0.13	-0.13	-0.11	**-0.15**	-0.16	-0.16	-0.18	-0.22	-0.14
Velocity	1.69	-0.26	-0.08	-0.19	-0.17	-0.11	-0.02	0.12	**0.26**	0.28	0.28	0.24	0.18	0.12

Note: See Appendix for data sources. Banking hours cover the period of 1972:1-2003:1.
All series represent the cyclical component of the HP filtered data. Series are in logs except those that represent rates.
Relative volatility is measured as the ratio of standard deviation of the series to the standard deviation of GDP.

Table 2: US business Cycle Facts: 1959:1 to 2004:4

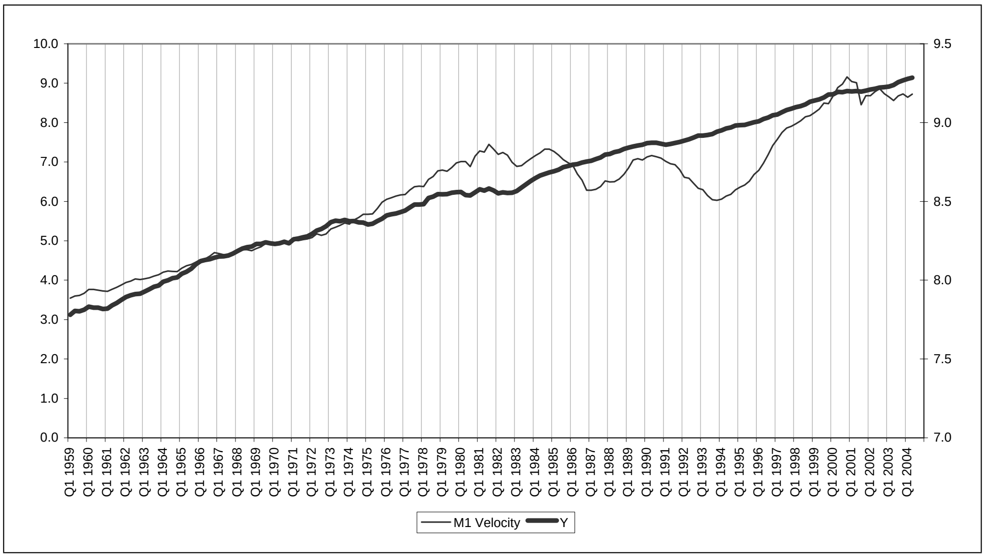

Figure 5: Actual US GDP versus M1 Velocity

And Figure 6 shows that while a positive GDP and velocity correlation characterizes much of the first half of the sample, this breaks down in the second half. After 1990, the credit and money shocks, which are constructed in the next section, also appear to show relatively more effect on velocity than does the productivity shock (see Section **??** below).

6 Construction of shocks

Based on the solution of the model from section 2, the log-deviations of the model variables be written as linear functions of the state $s = (\widehat{\tilde{k}}, z, u, v)$. By stacking the equations, the solution can be written in matrix form as follows:

$$X_t = A \left[\begin{array}{c} \widehat{\tilde{k}}_t \end{array} \right] + B \left[\begin{array}{ccc} z_t & u_t & v_t \end{array} \right]', \tag{18}$$

where $X = \left[\begin{array}{ccccccc} \widehat{\tilde{c}} & \hat{x} & \hat{l} & \hat{n} & \hat{f} & \hat{s}_G & \hat{a} \end{array} \right]'$. From (18), one can construct the solution of any variable of the model, by forming the appropriate linear combination of the appropriate rows of (18), the linear combinations being given by the linearized versions of equations (2)-(10).

Given the model solution (18) (that is, knowing the value of matrices A and B), the series of shocks $\left[\begin{array}{ccc} z_t & u_t & v_t \end{array} \right]$ can be constructed by using data on

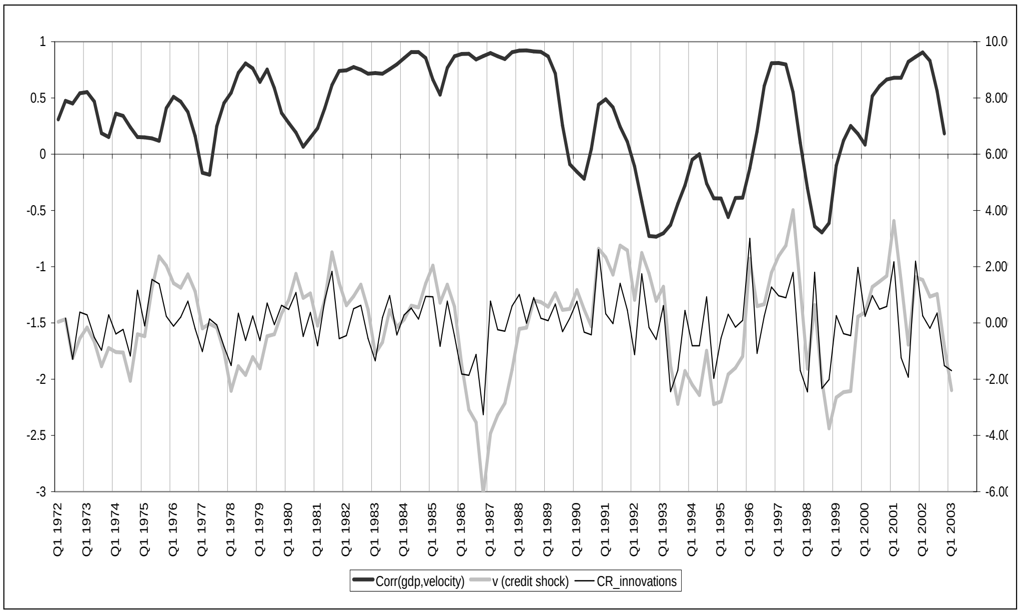

Figure 6: Actual Velocity-GDP Correlation Versus Credit Shock and Credit Shock Innovations

X_t and $\widehat{\widetilde{k}}_t$ and "solving" the system of linear equations (18). It can be easily seen, that in order to identify the three series of shocks, we need data on at least three variables from X_t. In a three-variable case the shocks represents the solution of a system of three linear equation. If more that three variables are used, then the shocks are "overidentified" as we have more equations than unknowns. In such a case we apply a least-square procedure as we illustrate below.

In the procedure of constructing the shocks, we employ the variables on which we were able to find reliable data. However, as human capital is involved in the model, we were forced to use human capital data as well. We used the human capital series compiled by Jorgenson and Stiroh (2000) that we extrapolated over our horizon until 2003. Although this human capital series is the best we know at this moment, we are confident that measurement errors in this series induce errors in our procedure of constructing the shocks. Therefore, we seek to minimize the use of such human capital series, and we construct stationary variables c/y, i/y and m/y that do not depend on h, and on which we use data to construct the shocks. We also use data on labor hour in banking sector f. and on the wage rate in banking - the latter series being used as a proxy for the marginal product of labor in banking ($mplb$) Thus, the only place where we are constrained to employ data on human capital, is the construction of the state variable $\widehat{\tilde{k}}_t$ ($\tilde{k} = k/h$). The data series on $\widehat{\tilde{k}}$ is constructed by using the capital accumulation equation (3), data on investment to compute $\widehat{\tilde{\imath}}_t$ and the initial condition $\widehat{\tilde{k}}_{-1} = 0$.

Having the data series on $\widehat{\tilde{k}}$, $\widehat{c/y}$, $\widehat{i/y}$, $\widehat{m/y}$, $\hat{f}$ and $\widehat{mplb}$, we set up a system of linear equations:

$$XX_t = AA \left[\begin{array}{c} \widehat{\tilde{k}}_t \end{array} \right] + BB \left[\begin{array}{ccc} z_t & u_t & v_t \end{array} \right]', \tag{19}$$

where $XX = \left[\begin{array}{ccccc} \widehat{c/y} & \widehat{i/y} & \widehat{m/y} & \widehat{f} & \widehat{mplb} \end{array} \right]'$ and the rows of the matrices AA and BB result from the linear combinations of the corresponding rows of matrices A and B, the appropriate linear combinations being given by the linear equations that define the variables from XX as functions of the variables from X. The marginal product of labor in banking, is derived from equation (7), while the definition of the other terms of the matrix XX is straightforward.

The least square "estimates" for the shock series are computed as follows:

$$est \left[\begin{array}{ccc} z_t & u_t & v_t \end{array} \right]'_t = (BB'BB)^{-1}BB'(XX_t - AA \left[\begin{array}{c} \widehat{\tilde{k}}_t \end{array} \right]).$$

In this approach we used five variables to construct the economy's three shocks. To test for the robustness of the process of shock construction, we repeated the computation by using combinations of five variables taken four at a time, and five taken three at a time, allowing for fifteen more possible ways to construct the shocks. Figure 8 graphs seven of the computed credit shock series of these along with the baseline of five variables. The results show that all combinations that include m/y, either c/y or i/y, and either f or $mplb$ generate nearly the same shock series, while other combinations (not graphed) show randomness and lack of conformity. Thus, we found that the results are robust as long as the variables are included that correspond to the model's three sectors in which the three shocks occur.

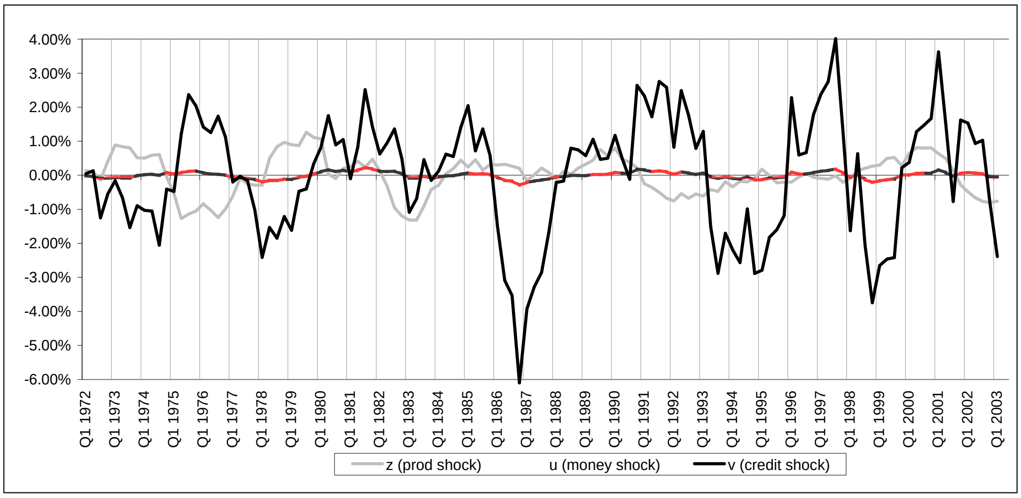

Figure 7: The Constructed Producticity, Money and Credit Shocks

Figure 7 shows the baseline shocks as constructed by the above methods.

A crosscheck of the model calibration is to estimate the shock persistence parameters φ_z, φ_u and φ_v from the constructed shock series. For this reason we estimate a system formed by equations (14), (12) and (8) by the method of seemingly unrelated regressions (SUR). The resulting estimates 0.89 (0.04), 0.88 (0.03) and 0.86 (0.04) (standard errors in paranthesis) lie close to the assumed values from Table 1, which validates our calibration. The estimated cross-correlations of the error terms are $corr(z,u) = -0.06$, $corr(z,v) = -0.11$ and $corr(u,v) = 0.94$. These estimated correlations are then used to simulate the model in section 7.

6.1 Sensitivity of the shock construction

For robustness, all combinations of variables were experimented with in constructing the shocks. Figure 7 shows the results of some of these experiments for alternatively constructing the credit shock; a similar profile results in each of these cases, which include the baseline case of using all five variables.

Another test of the robustness was to use data on banking productivity instead of banking hours; both variables enter the model and are alternatives. For this purpose, we replaced the equation for $\hat{f}$ in the system (19) with the

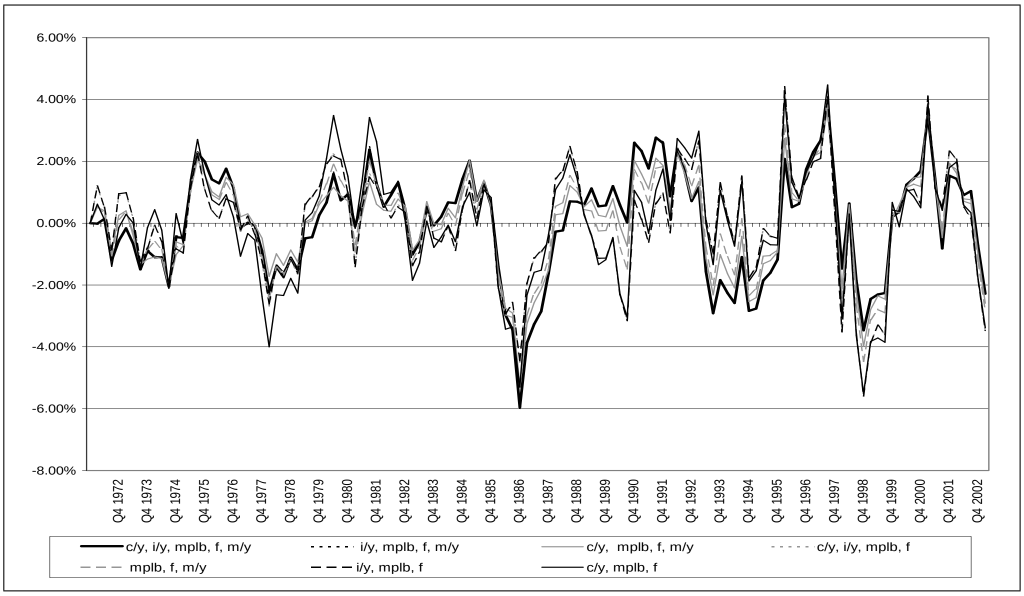

Figure 8: Credit Shocks Constructed by Using Various Combinations of Variables

equation describing the solution for the marginal product of labor in banking, derived from equation (7). The data series for banking productivity was proxied with data on the wage rate in the banking sector. The constructed shocks by using this new data on banking wages proved to be very similar to those constructed by using labor hours in banking.

6.2 Exogenous versus Endogenous Growth: Construction of the Shocks

Figures 9, 10 and 11 show that the main difference in the shock construction between exogenous growth and endogenous growth models is the construction of the money shock. This leads to the result that the money shock is important in velocity movements in the endogenous growth model.

6.3 Sensitivity of Shock Construction to Calibration

A larger "a" (inverse velocity) makes money shocks smaller and increases their correlation with the credit shocks. A bigger γ makes the contribution

Figure 9: Productivity Shocks Estimated from the Endogenous and the Exogenous Growth Model

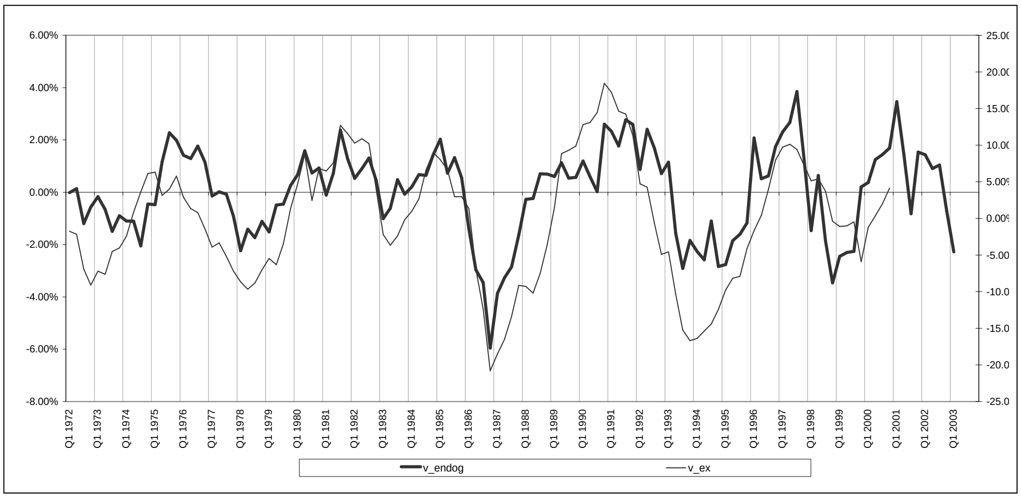

Figure 10: Credit Shocks Estimated from the Endogenous and the Exogenous Growth Model

Figure 11: Money Shocks Estimated from the Endogenous and the Exogenous Growth Model

of money to velocity relative to credit bigger.

7 Simulations

Here the calibrated model is used to simulate variables that can be compared to the actual US correlation experience at leads and lags with real GDP. The model is simulated by constructing shock processes according to equations (14), (12) and (8) and imposing the correlations among the error terms that have been estimated in section 6.

7.1 All shocks

The simulated moments are summarized in Table 3. Consumption and investment, normalized by the level of human capital, are correlated contemporaneously at 0.88 and 0.91, which compare fairly well to 0.81 and 0.92 of the actual data. Output growth correlation with output is 0.30 in the data and 0.48 in the simulation. The real wage is positively correlated in both data and the simulation, but the real interest rate is negatively correlated in the data, while positively correlated in the simulation.

The velocity contemporaneous correlation is 0.29 as compared to 0.26 in the data. This is almost exact, and it reflects our setting the persistence of the goods productivity shock a bit higher than indicated by the seemingly unrelated regressions that were run to estimate the persistences of the shocks. Note that the velocity correlation is significantly lower than in the exogenous growth model, in which we estimated it at 0.60 in Benk, Gillman, and Kejak (2005a). The simulated volatility of velocity, with all shocks operative, is 65% of that in the data, at 1.09 as compared to 1.69 in the data; this compares to less than half in the exogenous growth economy in Benk et al, and 57% for the comparable case of a relative risk aversion coefficient of 2 in (Table 3 of) Wang and Shi (2006). Also the money growth rate contemporaneous correlation with output is -0.10 in the data and -0.11 in the simulation, a match not found in other models; Cooley and Hansen (1995) and Benk, Gillman, and Kejak (2005a) both have a -0.01 correlation here. The M1 correlation in the data is 0.12 while for m/h the simulated correlation is 0.57. One clear failing of the model is that the relative volatilities for the simulated nominal interest rate and inflation rate are much too high as compared to the data.

7.2 Isolating the Effects of the Different Shocks

And examining Table 4, the velocity correlation, when only the goods productivity shock is operative, is almost three times as high ath 0.78. With only the credit shock, this correlation is 1 (Table 5), and with only the productivity and credit shocks, this is 0.70 (Table 6). These results indicate that the money shock is instrumental in getting a contemporaneous correlation of velocity that matches the data. And the credit shock allows certain periods of velocity changes, which are not due directly to income changes, to be modeled closely.

The model without credit shocks, but with credit still in the model, gives a velocity correlation that is about 40% higher than in the data, at 0.37 compared to 0.26. Also without the credit shock the simulated velocity volatility is 0.87, as compared to 1.09 with all shocks, and 1.69 in the data. Most of the other correlations are similar. So the credit shock helps mainly in getting a better simulated velocity and its volatility.

Without the money shock, as in Table 6, the simulated volatilities of the interest rate and inflation rate are much closer to those in the data. This suggests that the money shock is introducing excess volatility of the money supply.

8 The effect of shocks on velocity

Write the line of the system of equations (18) that corresponds to velocity ($\widehat{vel} = \widehat{y/m}$) in the following form:

$$\widehat{y/m} = \phi_k \hat{\tilde{k}}_t + \phi_z z_t + \phi_u u_t + \phi_v v_t + error$$

Then the terms $\phi_z z_t$, $\phi_u u_t$ and $\phi_v v_t$ indicate the contribution to productivity, money and credit shocks to the cyclical component of velocity $(\widehat{y/m})$. Figure 12 then shows how each shock affects velocity fluctuations. Credit shocks are rather unimportant in the early part of the sample before deregulation occured. But in the early 1980s, the downturn in 1986-1988, and the upturn following the McFadden Act repeal in 1994, the credit shock appears to have had the largest impact on velocity of the three shocks.

8.1 Contribution of credit versus money shocks

Credit and money shocks have some high correlation with respect to their effects on velocity but opposite effects on output growth. Figure 13 shows how the credit and money shocks have opposite effects on output.So when GDP growth is positively linked to credit, as in the Figure 14, the credit shock will be more important in the effect on velocity. When the credit shock moves opposite of the GDP growth, the money shock will be moving with the GDP growth and will be more important in the velocity effect.

8.2 Variance Decomposition Of Velocity

The decomposition of the variance of the velocity for each the endogenous and exogenous growth models was conducted for comparison, for the baseline

	Relative Volatility	Corr w/ Money growth	Corr w/ output growth	corr(output(t), variable(t+i)) -5	-4	-3	-2	-1	0	1	2	3	4	5
k/h	0.84	0.02	0.12	0.48	0.52	0.56	0.60	0.64	**0.69**	0.73	0.77	0.80	0.82	0.84
m/h	1.21	-0.81	0.03	0.39	0.42	0.46	0.49	0.53	**0.57**	0.57	0.56	0.55	0.55	0.54
c/h	0.67	-0.27	0.24	0.62	0.67	0.72	0.77	0.82	**0.88**	0.88	0.87	0.87	0.86	0.85
l	0.49	-0.29	0.53	0.56	0.59	0.63	0.66	0.70	**0.74**	0.62	0.52	0.42	0.33	0.26
z	0.54	-0.06	0.52	0.74	0.78	0.83	0.88	0.94	**1.00**	0.93	0.87	0.81	0.75	0.70
u	0.47	1.00	0.16	-0.06	-0.07	-0.08	-0.09	-0.10	**-0.11**	-0.10	-0.09	-0.08	-0.07	-0.06
v	0.53	0.77	0.05	-0.06	-0.07	-0.08	-0.09	-0.10	**-0.11**	-0.11	-0.10	-0.09	-0.09	-0.08
s	0.01	-0.05	0.54	0.40	0.42	0.44	0.46	0.48	**0.51**	0.38	0.26	0.16	0.07	-0.01
r	1.06	-0.18	0.53	0.44	0.46	0.49	0.52	0.54	**0.57**	0.44	0.33	0.23	0.14	0.05
w	0.72	0.05	0.31	0.63	0.68	0.73	0.77	0.83	**0.88**	0.88	0.88	0.88	0.87	0.86
y/h	1.00	-0.11	0.48	0.74	0.78	0.83	0.89	0.94	**1.00**	0.94	0.89	0.83	0.78	0.74
k/l	0.94	0.16	-0.17	0.13	0.15	0.16	0.18	0.20	**0.21**	0.32	0.40	0.48	0.54	0.60
i/h	3.00	0.06	0.61	0.68	0.72	0.76	0.80	0.85	**0.90**	0.80	0.71	0.62	0.55	0.48
x	0.08	0.60	-0.17	-0.56	-0.60	-0.65	-0.70	-0.74	**-0.79**	-0.78	-0.76	-0.75	-0.73	-0.71
R	20.74	0.79	0.44	-0.07	-0.08	-0.09	-0.09	-0.10	**-0.11**	-0.05	-0.04	-0.04	-0.04	-0.03
n	0.49	-0.19	-0.47	-0.18	-0.19	-0.19	-0.20	-0.20	**-0.21**	-0.08	0.03	0.12	0.20	0.28
h growth	0.01	-0.18	-0.47	-0.18	-0.18	-0.18	-0.19	-0.19	**-0.20**	-0.08	0.03	0.13	0.21	0.28
velocity	1.02	0.85	0.44	0.25	0.25	0.26	0.27	0.28	**0.29**	0.24	0.19	0.15	0.11	0.08
a	0.72	-0.98	-0.13	0.09	0.10	0.11	0.12	0.14	**0.15**	0.14	0.13	0.13	0.12	0.11
f	8.92	1.00	0.18	-0.05	-0.06	-0.07	-0.08	-0.08	**-0.10**	-0.09	-0.08	-0.07	-0.06	-0.06
inflation	60.04	0.80	0.42	-0.08	-0.09	-0.10	-0.11	-0.12	**-0.13**	-0.06	-0.05	-0.04	-0.04	-0.03
y growth	0.09	0.16	1.00	0.38	0.40	0.42	0.44	0.46	**0.48**	0.62	0.53	0.44	0.36	0.29

Table 3: Business cycle properties - simulations: all shocks, with persistent (0.9) money

	Relative Volatility	Corr w/ Money growth	Corr w/ output growth	corr(output(t), variable(t+i)) -5	-4	-3	-2	-1	0	1	2	3	4	5
k/h	0.82	NaN	0.16	0.47	0.51	0.55	0.59	0.64	**0.69**	0.74	0.78	0.81	0.83	0.84
m/h	0.68	NaN	0.34	0.60	0.65	0.70	0.75	0.80	**0.86**	0.87	0.88	0.88	0.87	0.87
c/h	0.64	NaN	0.37	0.62	0.67	0.72	0.77	0.83	**0.89**	0.89	0.89	0.88	0.88	0.86
l	0.48	NaN	0.70	0.58	0.62	0.65	0.69	0.72	**0.76**	0.64	0.53	0.43	0.34	0.26
z	0.55	NaN	0.63	0.73	0.78	0.83	0.88	0.94	**1.00**	0.93	0.87	0.81	0.75	0.69
u	0.00	NaN	NaN	NaN	NaN	NaN	NaN	NaN	**NaN**	NaN	NaN	NaN	NaN	NaN
v	0.00	NaN	NaN	NaN	NaN	NaN	NaN	NaN	**NaN**	NaN	NaN	NaN	NaN	NaN
s	0.01	NaN	0.63	0.42	0.44	0.46	0.49	0.51	**0.53**	0.40	0.28	0.18	0.08	0.00
r	1.06	NaN	0.65	0.46	0.49	0.51	0.54	0.56	**0.59**	0.46	0.34	0.24	0.14	0.06
w	0.71	NaN	0.38	0.63	0.68	0.73	0.78	0.84	**0.89**	0.90	0.89	0.89	0.88	0.86
y/h	1.00	NaN	0.60	0.73	0.78	0.83	0.88	0.94	**1.00**	0.94	0.88	0.83	0.78	0.73
k/l	0.91	NaN	-0.22	0.12	0.13	0.15	0.17	0.19	**0.21**	0.31	0.41	0.49	0.55	0.61
i/h	3.05	NaN	0.70	0.69	0.73	0.77	0.82	0.87	**0.92**	0.82	0.72	0.64	0.56	0.49
x	0.06	NaN	-0.43	-0.66	-0.71	-0.76	-0.81	-0.87	**-0.93**	-0.92	-0.91	-0.89	-0.87	-0.85
R	1.86	NaN	0.33	-0.23	-0.24	-0.26	-0.27	-0.28	**-0.30**	0.03	0.03	0.02	0.02	0.01
n	0.48	NaN	-0.51	-0.23	-0.24	-0.24	-0.25	-0.26	**-0.27**	-0.14	-0.02	0.08	0.17	0.25
h growth	0.01	NaN	-0.51	-0.22	-0.23	-0.24	-0.24	-0.25	**-0.26**	-0.13	-0.01	0.09	0.18	0.26
velocity	0.53	NaN	0.70	0.59	0.63	0.66	0.70	0.74	**0.78**	0.65	0.54	0.45	0.36	0.28
a	0.04	NaN	0.21	0.51	0.55	0.59	0.64	0.69	**0.74**	0.78	0.81	0.83	0.85	0.86
f	0.15	NaN	0.69	0.55	0.58	0.61	0.64	0.67	**0.71**	0.58	0.47	0.37	0.28	0.19
inflation	6.44	NaN	0.07	-0.34	-0.35	-0.37	-0.39	-0.41	**-0.43**	-0.12	-0.08	-0.06	-0.03	-0.01
y growth	0.08	NaN	1.00	0.46	0.49	0.51	0.54	0.57	**0.60**	0.74	0.62	0.52	0.43	0.35

Table 4: Business Cycle Properties - Simulations: with Only the Productivity Shock

	Relative Volatility	Corr w/ Money growth	Corr w/ output growth	corr(output(t), variable(t+i)) -5	-4	-3	-2	-1	0	1	2	3	4	5
k/h	0.88	NaN	-0.15	-0.33	-0.37	-0.42	-0.47	-0.52	**-0.57**	-0.63	-0.67	-0.71	-0.74	-0.76
m/h	53.69	NaN	0.25	-0.67	-0.72	-0.79	-0.85	-0.92	**-1.00**	-0.93	-0.87	-0.82	-0.76	-0.71
c/h	1.79	NaN	-0.33	0.68	0.73	0.79	0.86	0.92	**0.99**	0.91	0.83	0.75	0.68	0.61
l	2.06	NaN	-0.20	0.64	0.70	0.76	0.83	0.90	**0.97**	0.93	0.89	0.84	0.80	0.76
z	0.00	NaN	NaN	NaN	NaN	NaN	NaN	NaN	**NaN**	NaN	NaN	NaN	NaN	NaN
u	0.00	NaN	NaN	NaN	NaN	NaN	NaN	NaN	**NaN**	NaN	NaN	NaN	NaN	NaN
v	143.38	NaN	-0.26	0.67	0.72	0.79	0.85	0.92	**1.00**	0.93	0.87	0.81	0.76	0.71
s	0.02	NaN	0.06	0.43	0.48	0.53	0.59	0.65	**0.71**	0.74	0.77	0.79	0.80	0.80
r	2.49	NaN	-0.10	0.58	0.64	0.70	0.77	0.83	**0.91**	0.89	0.87	0.85	0.83	0.80
w	1.10	NaN	0.10	-0.58	-0.64	-0.70	-0.77	-0.83	**-0.91**	-0.89	-0.87	-0.85	-0.83	-0.80
y/h	1.00	NaN	-0.29	0.67	0.73	0.79	0.86	0.93	**1.00**	0.93	0.86	0.79	0.73	0.67
k/l	2.76	NaN	0.10	-0.58	-0.64	-0.70	-0.77	-0.83	**-0.91**	-0.89	-0.87	-0.85	-0.83	-0.80
i/h	2.46	NaN	0.41	-0.65	-0.70	-0.75	-0.81	-0.86	**-0.93**	-0.82	-0.71	-0.62	-0.53	-0.46
x	0.59	NaN	0.28	-0.67	-0.73	-0.79	-0.86	-0.93	**-1.00**	-0.93	-0.86	-0.80	-0.74	-0.69
R	523.50	NaN	-0.99	0.15	0.17	0.18	0.19	0.20	**0.21**	-0.16	-0.15	-0.15	-0.14	-0.13
n	1.17	NaN	-0.40	0.65	0.70	0.76	0.82	0.88	**0.94**	0.83	0.73	0.64	0.55	0.48
h growth	0.02	NaN	-0.41	0.65	0.70	0.75	0.81	0.87	**0.93**	0.82	0.72	0.63	0.54	0.47
velocity	54.69	NaN	-0.26	0.67	0.72	0.79	0.85	0.92	**1.00**	0.93	0.87	0.82	0.76	0.71
a	52.81	NaN	0.26	-0.67	-0.72	-0.79	-0.85	-0.92	**-1.00**	-0.93	-0.87	-0.81	-0.76	-0.71
f	189.96	NaN	0.26	-0.67	-0.72	-0.79	-0.85	-0.92	**-1.00**	-0.93	-0.87	-0.81	-0.76	-0.71
inflation	1505.84	NaN	-0.99	0.15	0.16	0.18	0.19	0.20	**0.21**	-0.17	-0.16	-0.15	-0.15	-0.14
y growth	0.59	NaN	1.00	-0.21	-0.23	-0.24	-0.26	-0.27	**-0.29**	0.08	0.09	0.09	0.10	0.10

Table 5: Business Cycle Properties - Simulations: with Only the Credit Shock

	Relative Volatility	Corr w/ Money growth	Corr w/ output growth	corr(output(t), variable(t+i)) -5	-4	-3	-2	-1	0	1	2	3	4	5
k/h	0.83	NaN	0.16	0.48	0.52	0.56	0.60	0.65	**0.70**	0.74	0.78	0.81	0.83	0.85
m/h	0.73	NaN	0.34	0.60	0.64	0.69	0.73	0.78	**0.84**	0.84	0.85	0.84	0.84	0.83
c/h	0.65	NaN	0.36	0.63	0.68	0.73	0.78	0.83	**0.89**	0.89	0.89	0.88	0.88	0.86
l	0.47	NaN	0.70	0.57	0.61	0.64	0.68	0.71	**0.75**	0.63	0.52	0.42	0.33	0.25
z	0.55	NaN	0.62	0.73	0.78	0.83	0.88	0.94	**1.00**	0.93	0.87	0.81	0.75	0.70
u	0.00	NaN	NaN	NaN	NaN	NaN	NaN	NaN	**NaN**	NaN	NaN	NaN	NaN	NaN
v	0.55	NaN	-0.08	-0.09	-0.10	-0.10	-0.10	-0.10	**-0.11**	-0.10	-0.09	-0.09	-0.08	-0.07
s	0.01	NaN	0.63	0.40	0.42	0.45	0.47	0.49	**0.52**	0.39	0.27	0.16	0.07	-0.01
r	1.05	NaN	0.65	0.44	0.47	0.49	0.52	0.55	**0.58**	0.45	0.33	0.22	0.13	0.05
w	0.72	NaN	0.37	0.64	0.68	0.73	0.78	0.84	**0.90**	0.90	0.89	0.89	0.87	0.86
y/h	1.00	NaN	0.60	0.73	0.78	0.83	0.89	0.94	**1.00**	0.94	0.89	0.83	0.78	0.73
k/l	0.92	NaN	-0.21	0.14	0.16	0.17	0.19	0.21	**0.23**	0.34	0.42	0.50	0.57	0.62
i/h	3.02	NaN	0.70	0.68	0.73	0.77	0.82	0.86	**0.92**	0.81	0.72	0.64	0.56	0.48
x	0.06	NaN	-0.42	-0.66	-0.71	-0.76	-0.82	-0.87	**-0.93**	-0.92	-0.91	-0.89	-0.87	-0.85
R	2.82	NaN	0.22	-0.16	-0.17	-0.17	-0.18	-0.19	**-0.20**	0.03	0.03	0.03	0.02	0.02
n	0.48	NaN	-0.51	-0.20	-0.21	-0.22	-0.23	-0.24	**-0.24**	-0.12	0.00	0.10	0.19	0.27
h growth	0.01	NaN	-0.50	-0.20	-0.21	-0.21	-0.22	-0.23	**-0.24**	-0.11	0.01	0.11	0.20	0.28
velocity	0.55	NaN	0.64	0.52	0.55	0.59	0.62	0.66	**0.70**	0.58	0.48	0.39	0.30	0.23
a	0.21	NaN	0.12	0.18	0.19	0.20	0.21	0.22	**0.23**	0.23	0.22	0.22	0.22	0.21
f	0.76	NaN	0.22	0.20	0.20	0.21	0.22	0.23	**0.24**	0.21	0.18	0.15	0.13	0.10
inflation	8.91	NaN	0.06	-0.25	-0.26	-0.27	-0.28	-0.29	**-0.31**	-0.07	-0.05	-0.03	-0.01	0.01
y growth	0.08	NaN	1.00	0.45	0.48	0.51	0.54	0.57	**0.60**	0.73	0.61	0.51	0.42	0.34

Table 6: Business Cycle Properties - Simulations: with Only the Productivity and Credit Shocks

	Relative Volatility	Corr w/ Money growth	Corr w/ output growth	corr(output(t), variable(t+i)) -5	-4	-3	-2	-1	0	1	2	3	4	5
k/h	1.51	0.51	-0.16	-0.08	-0.12	-0.17	-0.23	-0.30	**-0.37**	-0.46	-0.53	-0.58	-0.62	-0.65
m/h	13.67	-1.00	-0.42	0.46	0.54	0.64	0.74	0.86	**1.00**	0.87	0.77	0.68	0.59	0.52
c/h	2.71	-0.97	-0.49	0.48	0.56	0.65	0.75	0.86	**0.99**	0.84	0.72	0.60	0.51	0.42
l	2.25	-0.97	-0.27	0.39	0.47	0.56	0.66	0.78	**0.92**	0.85	0.79	0.74	0.68	0.63
z	0.00	NaN	NaN	NaN	NaN	NaN	NaN	NaN	**NaN**	NaN	NaN	NaN	NaN	NaN
u	8.07	1.00	0.39	-0.45	-0.54	-0.63	-0.73	-0.85	**-0.99**	-0.88	-0.78	-0.69	-0.61	-0.54
v	0.00	NaN	NaN	NaN	NaN	NaN	NaN	NaN	**NaN**	NaN	NaN	NaN	NaN	NaN
s	0.02	-0.21	0.32	-0.07	-0.05	-0.03	0.00	0.03	**0.06**	0.20	0.31	0.40	0.48	0.54
r	3.15	-0.85	-0.11	0.29	0.36	0.44	0.53	0.64	**0.76**	0.75	0.74	0.73	0.71	0.69
w	1.39	0.85	0.11	-0.29	-0.36	-0.44	-0.53	-0.64	**-0.76**	-0.75	-0.74	-0.73	-0.71	-0.69
y/h	1.00	-0.99	-0.45	0.47	0.55	0.65	0.75	0.86	**1.00**	0.86	0.75	0.65	0.55	0.47
k/l	3.48	0.85	0.11	-0.29	-0.36	-0.44	-0.53	-0.64	**-0.76**	-0.75	-0.74	-0.73	-0.71	-0.69
i/h	6.24	0.94	0.51	-0.48	-0.56	-0.65	-0.74	-0.85	**-0.98**	-0.82	-0.68	-0.56	-0.46	-0.37
x	0.78	1.00	0.42	-0.46	-0.55	-0.64	-0.74	-0.86	**-1.00**	-0.87	-0.77	-0.67	-0.59	-0.51
R	330.64	0.83	0.84	-0.40	-0.47	-0.55	-0.63	-0.73	**-0.85**	-0.47	-0.42	-0.38	-0.33	-0.29
n	2.04	-0.89	-0.54	0.48	0.55	0.63	0.72	0.82	**0.95**	0.77	0.62	0.49	0.38	0.28
h growth	0.03	-0.88	-0.54	0.48	0.55	0.63	0.72	0.82	**0.94**	0.76	0.61	0.49	0.37	0.28
velocity	12.67	1.00	0.41	-0.46	-0.54	-0.64	-0.74	-0.86	**-0.99**	-0.87	-0.77	-0.68	-0.60	-0.52
a	9.54	-1.00	-0.40	0.45	0.54	0.63	0.73	0.85	**0.99**	0.88	0.78	0.69	0.61	0.54
f	162.01	1.00	0.40	-0.45	-0.54	-0.63	-0.73	-0.85	**-0.99**	-0.88	-0.78	-0.69	-0.61	-0.54
inflation	954.89	0.83	0.83	-0.40	-0.47	-0.55	-0.63	-0.73	**-0.85**	-0.47	-0.43	-0.38	-0.34	-0.29
y growth	0.81	0.39	1.00	-0.24	-0.27	-0.31	-0.34	-0.39	**-0.45**	0.05	0.05	0.06	0.07	0.08

Table 7: Business Cycle Properties - Simulations: with Only the Money Shock

	Relative Volatility	Corr w/ Money growth	Corr w/ output growth	corr(output(t), variable(t+i)) -5	-4	-3	-2	-1	0	1	2	3	4	5
k/h	0.82	0.03	0.13	0.46	0.50	0.54	0.58	0.63	**0.68**	0.73	0.76	0.79	0.82	0.83
m/h	1.06	-0.76	0.05	0.43	0.47	0.50	0.54	0.58	**0.62**	0.62	0.62	0.61	0.60	0.59
c/h	0.67	-0.27	0.25	0.61	0.66	0.71	0.76	0.82	**0.88**	0.88	0.87	0.86	0.85	0.84
l	0.50	-0.30	0.52	0.56	0.60	0.63	0.67	0.71	**0.75**	0.63	0.52	0.42	0.34	0.26
z	0.54	-0.06	0.52	0.73	0.77	0.83	0.88	0.94	**1.00**	0.93	0.86	0.80	0.74	0.69
u	0.46	1.00	0.16	-0.07	-0.08	-0.09	-0.09	-0.10	**-0.11**	-0.10	-0.09	-0.08	-0.07	-0.06
v	0.00	NaN	NaN	NaN	NaN	NaN	NaN	NaN	**NaN**	NaN	NaN	NaN	NaN	NaN
s	0.01	-0.07	0.53	0.41	0.43	0.45	0.48	0.50	**0.53**	0.40	0.28	0.17	0.08	-0.01
r	1.07	-0.19	0.52	0.45	0.47	0.50	0.53	0.56	**0.59**	0.46	0.34	0.24	0.14	0.06
w	0.71	0.05	0.32	0.62	0.66	0.71	0.77	0.82	**0.88**	0.88	0.88	0.87	0.86	0.85
y/h	1.00	-0.11	0.49	0.73	0.77	0.83	0.88	0.94	**1.00**	0.94	0.88	0.83	0.77	0.73
k/l	0.93	0.19	-0.17	0.11	0.12	0.14	0.15	0.17	**0.19**	0.29	0.38	0.46	0.53	0.58
i/h	3.02	0.05	0.61	0.67	0.72	0.76	0.81	0.85	**0.90**	0.80	0.71	0.62	0.55	0.48
x	0.08	0.61	-0.18	-0.56	-0.60	-0.64	-0.69	-0.74	**-0.79**	-0.78	-0.76	-0.74	-0.72	-0.70
R	19.28	0.82	0.44	-0.08	-0.08	-0.09	-0.10	-0.11	**-0.12**	-0.05	-0.05	-0.04	-0.04	-0.04
n	0.49	-0.17	-0.47	-0.20	-0.20	-0.21	-0.22	-0.22	**-0.23**	-0.10	0.01	0.11	0.19	0.27
h growth	0.01	-0.16	-0.46	-0.19	-0.20	-0.20	-0.21	-0.21	**-0.22**	-0.10	0.02	0.11	0.20	0.27
velocity	0.87	0.79	0.49	0.29	0.31	0.32	0.34	0.35	**0.37**	0.31	0.25	0.20	0.15	0.11
a	0.55	-1.00	-0.15	0.11	0.11	0.12	0.13	0.15	**0.16**	0.15	0.14	0.13	0.12	0.12
f	9.26	1.00	0.17	-0.06	-0.07	-0.08	-0.08	-0.09	**-0.10**	-0.09	-0.08	-0.07	-0.07	-0.06
inflation	55.88	0.82	0.42	-0.09	-0.10	-0.11	-0.12	-0.13	**-0.14**	-0.07	-0.06	-0.05	-0.04	-0.04
y growth	0.09	0.16	1.00	0.38	0.40	0.42	0.44	0.46	**0.49**	0.63	0.53	0.45	0.36	0.29

Table 8: Business Cycle Properties - Simulations: with Only the Productivity and Money Shocks

	Relative Volatility	Corr w/ Money growth	Corr w/ output growth	corr(output(t), variable(t+i)) -5	-4	-3	-2	-1	0	1	2	3	4	5
k/h	1.55	0.52	-0.15	-0.07	-0.11	-0.16	-0.22	-0.29	**-0.37**	-0.45	-0.52	-0.57	-0.61	-0.64
m/h	17.38	-0.99	-0.39	0.43	0.51	0.60	0.71	0.83	**0.97**	0.86	0.76	0.67	0.60	0.53
c/h	2.75	-0.96	-0.49	0.46	0.54	0.63	0.74	0.86	**1.00**	0.84	0.71	0.59	0.49	0.41
l	2.27	-0.97	-0.28	0.37	0.46	0.55	0.65	0.77	**0.91**	0.84	0.78	0.72	0.67	0.62
z	0.00	NaN	NaN	NaN	NaN	NaN	NaN	NaN	**NaN**	NaN	NaN	NaN	NaN	NaN
u	8.51	1.00	0.39	-0.43	-0.52	-0.61	-0.72	-0.84	**-0.98**	-0.87	-0.77	-0.68	-0.60	-0.54
v	9.92	0.77	0.19	-0.27	-0.34	-0.41	-0.49	-0.58	**-0.69**	-0.65	-0.61	-0.57	-0.53	-0.50
s	0.02	-0.20	0.33	-0.08	-0.06	-0.04	-0.02	0.00	**0.03**	0.18	0.29	0.39	0.46	0.52
r	3.20	-0.85	-0.11	0.27	0.34	0.43	0.52	0.63	**0.75**	0.74	0.73	0.71	0.70	0.67
w	1.42	0.85	0.11	-0.27	-0.34	-0.43	-0.52	-0.63	**-0.75**	-0.74	-0.73	-0.71	-0.70	-0.67
y/h	1.00	-0.98	-0.46	0.46	0.54	0.63	0.74	0.86	**1.00**	0.86	0.74	0.63	0.54	0.46
k/l	3.54	0.85	0.11	-0.27	-0.34	-0.43	-0.52	-0.63	**-0.75**	-0.74	-0.73	-0.71	-0.70	-0.67
i/h	6.41	0.94	0.51	-0.47	-0.54	-0.63	-0.73	-0.85	**-0.98**	-0.82	-0.68	-0.56	-0.45	-0.36
x	0.79	1.00	0.43	-0.45	-0.53	-0.62	-0.73	-0.85	**-1.00**	-0.87	-0.76	-0.66	-0.58	-0.50
R	371.72	0.80	0.86	-0.38	-0.45	-0.52	-0.61	-0.71	**-0.82**	-0.43	-0.38	-0.34	-0.30	-0.26
n	2.07	-0.88	-0.55	0.46	0.53	0.62	0.71	0.82	**0.95**	0.77	0.62	0.48	0.37	0.28
h growth	0.03	-0.87	-0.55	0.46	0.53	0.61	0.71	0.82	**0.94**	0.76	0.61	0.48	0.36	0.27
velocity	16.41	0.99	0.38	-0.42	-0.50	-0.60	-0.70	-0.83	**-0.97**	-0.86	-0.76	-0.68	-0.60	-0.53
a	13.10	-0.98	-0.36	0.41	0.49	0.58	0.69	0.81	**0.95**	0.85	0.76	0.68	0.61	0.55
f	160.68	1.00	0.40	-0.44	-0.52	-0.61	-0.72	-0.85	**-0.99**	-0.87	-0.77	-0.68	-0.60	-0.53
inflation	1072.97	0.80	0.85	-0.38	-0.45	-0.52	-0.61	-0.71	**-0.82**	-0.43	-0.38	-0.34	-0.30	-0.27
y growth	0.81	0.39	1.00	-0.23	-0.27	-0.31	-0.35	-0.40	**-0.46**	0.05	0.06	0.07	0.07	0.08

Table 9: Business Cycle Properties - Simulations: with Only the Credit and Money Shocks

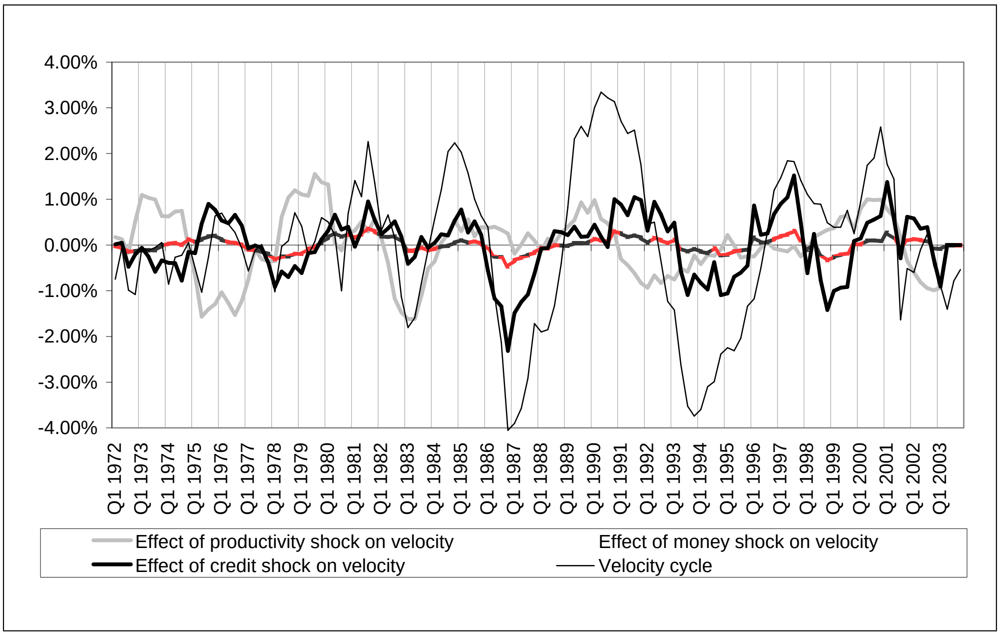

Figure 12: Effect of shocks on velocity cycle.

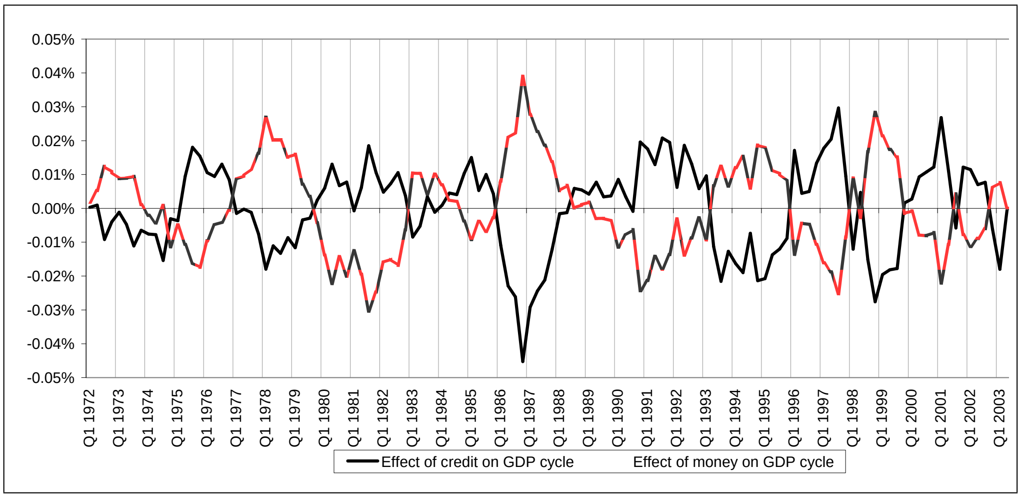

Figure 13: Effect of credit and money shocks on GDP cycle.

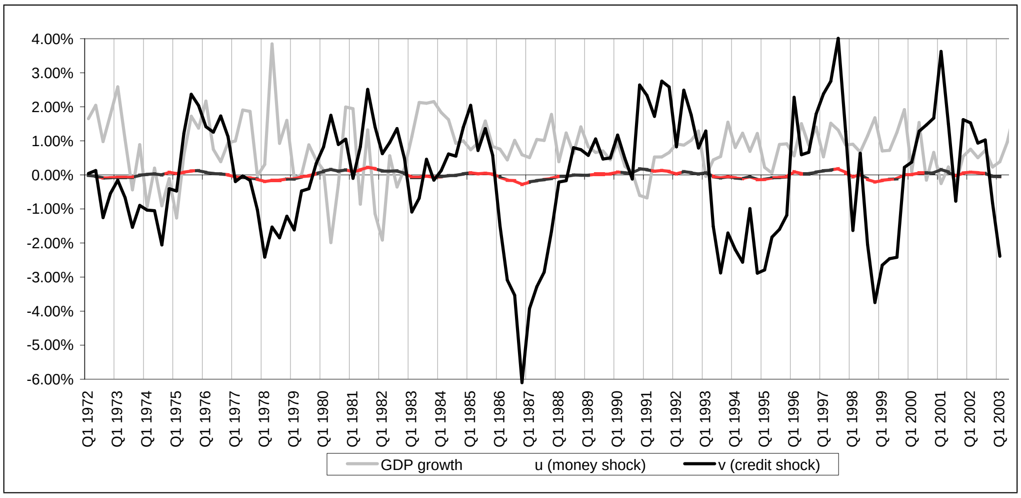

Figure 14: GDP growth, money and credit shocks

Shock ordering			Endogenous growth			Exogenous growth		
CR	PR	M	46.5%	20.0%	1.2%	78.1%	14.5%	0.1%
CR	M	PR	45.4%	1.0%	19.0%	78.3%	2.6%	7.8%
PR	CR	M	9.3%	59.0%	3.1%	1.2%	89.5%	0.8%
M	CR	PR	37.5%	12.8%	19.2%	2.0%	82.8%	10.3%
M	PR	CR	38.1%	16.4%	8.5%	1.4%	9.6%	79.3%
PR	M	CR	9.4%	52.7%	10.5%	1.4%	10.0%	74.0%

Table 10: The decomposition of the variance of the velocity, based on various shocks orderings

five variable case of the shock construction. There are six possible orderings of the shocks and each is reported in Table 10. The variance is decomposed as in Ingram, Kocherlakota, and Savin (1994) and Benk, Gillman, and Kejak (2005b), the technique is described in the Appendix.

9 Conclusions

The paper extends a standard monetary real business cycle by setting it within endogenous growth and adding credit sector shocks. The result is that velocity can be better explained by some criteria. The correlation of velocity with output is smaller and closer to that of the data. This was the main problem found in previous work, a correlation that is too high. At the same time, two other factors enter to explain velocity besides the goods productivity shocks. Substitution effects from the money supply growth rate shocks, and the consequent, but small, growth rate effect of the shocks, have a significant impact on velocity in some periods, such as the strong cyclical increase in velocity during the high inflation period of 1980-1981. Credit shocks, found to have an important impact on GDP during the deregulatory era, in Benk, Gillman, and Kejak (2005b), also effect velocity strongly during this period. Thus while temporary income deviations can be important, as in Friedman (1959) and Friedman and Schwartz's (1963) permanent income hypothesis explanation of velocity, during times when money supply growth rates and credit markets are significantly shocked, these other factors can dominate the swings in velocity.

Meanwhile, the use of a variable velocity in the monetary policy debate appears sporadically with the monetary, velocity feedback, rule of McCallum (1990) and the policy rules of Alvarez, Lucas, and Weber (2001). However, the monetary business cycle with endogenous growth leaves open the possibility of deriving from general equilibrium a system of equations that defines a policy regime in which velocity can play a role in keeping to the target inflation level, while letting the nominal interest rate fluctuate in line with the real interest rate changes over the business cycle.

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10 Appendix

10.1 Data Sources and Definitions

Data used in this paper came from the sources below (all seasonally adjusted). Data is on quarterly frequency and cover the 1959:Q1 - 2004:Q4 time period, unless otherwise indicated.

1. IMF - International Financial Statistics:

   GDP: GDP volume at 2000 prices.
   GDP growth: $\Delta \log(GDP)$.
   GDP at current prices.
   CPI: Consumer Prices.
   Consumption: Household Consumption Expenditures, deflated with CPI.
   Investment: Gross Fixed Capital Formation.deflated with GDP deflator.
   Nominal wages: Wages: Hourly Earnings.
   Real wages: Nominal wages deflated with CPI.
   Inflation: $\Delta \log(CPI)$.
   M1.
   M1 growth: $\Delta \log(M1)$.
   Real money: M1 deflated with CPI.
   Tbill rate: Treasury Bill Rate.
   Real interest rate: TBill rate-Inflation.
   Velocity: GDP at current prices/M1.

2. Bureau of Labor Statistics (1972:Q1-2003:Q1) - quarterly data calculated from the average of three months:

Banking hours: Hours worked in commercial banking=Production workers*Average weekly hours of production workers.

Wages in banking: Average weekly earnings of production workers in commercial banking.

Data series used to construct the shocks are the following:

c/y: Consumption/GDP.
i/y: Investment/GDP.
m/y: M1/GDP at current prices.
f: Banking hours.
$mplb$: Wages in banking.

10.2 Variance Decomposition

The decomposition of the variance of the velocity from section 8.2 has been done as desribed by Ingram, Kocherlakota, and Savin (1994), pp. 424:

Let z, v and u be the three correlated shocks. Let's assume the ordering z-v-u, that is, the movements in z are responsible for any comovements between z and v or z and u, and that movements in v are responsible for any comovements between v and u. We can formalize this notion by defining v^e_{t-s} to be the residuals in a regression of v_{t-s} on the vector $(z_t, ..., z_{t-s})$ and u^e_{t-s} to be the residuals in a regression of u_{t-s} on the vector $(z_t, ..., z_{t-s}, v_t, ..., v_{t-s})$. Thus we interpret v^e_{t-s} as capturing the movements of v that are not associated with current, future, or past movements in z.

Given this particular ordering, the decomposition of the variance of velovity vel_t into the components due to the various shocks can be obtained by running the regression:

$$vel_t = \underbrace{\sum_{s=0}^{S} \beta_{z,s} z_{t-s}}_{vel^z_t} + \underbrace{\sum_{s=0}^{S} \beta_{v,s} v^e_{t-s}}_{vel^v_t} + \underbrace{\sum_{s=0}^{S} \beta_{u,s} u^e_{t-s}}_{vel^u_t} + \varepsilon_t$$

Then the fraction of the variance of vel_t explained by each shock is given by: $P^z = \frac{Var(vel_t^z)}{Var(vel_t)}$, $P^v = \frac{Var(vel_t^v)}{Var(vel_t)}$, $P^u = \frac{Var(vel_t^u)}{Var(vel_t)}$. The results are sensitive to the ordering adopted.