Uninsured Risks, Loan Contracts and the Declining Equity Premium

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Abstract: Using a two period model with moral hazard and uninsured risk, we argue that the decline in equity premium from its historically high level is due to a gradual elimination of barriers to universal banking. The loan contracts set up by financial intermediaries became more complete in nature with the advent of universal banking in the 90s following the Gramm-Leach-Billy Act. Hence, it is the nature of the loan contracts, not just the borrowing constraint and uninsured risks that is more fundamental in explaining the size of the equity premium.

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1. Introduction

Two stylized facts are of considerable academic interest in finance. First, the historical US equity premium during the period 1889-1979 is too high to be consistent with a smooth aggregate consumption stream ((Mehra and Prescott, 1985). Second, the US equity premium showed a pronounced decline during recent years (Blanchard, 1993, Jagannathan et al, 2000, Fama and French, 2002). Jagannathan et al. (2000) attribute the recent decline in premium to a gradual elimination of market imperfections. Lettau et al. (2004) argue that the recent low premium is due to a decline in macroeconomic risk.

In this paper, we seek an alternative explanation of these two stylized facts. We argue that the decline in equity premium from its historically high level is due to a gradual elimination of barriers to universal banking. The loan contracts set up by financial intermediaries became more complete in nature with the advent of universal banking in the 90s following the Gramm-Leach-Billy Act.[1] The completeness of the loan contract lowered the uninsurable consumption risks of the households when they participate in the stock market vis-à-vis the bond market.

To demonstrate this we construct a model without aggregate risk but only with idiosyncratic project risks. When household's choice of effort in a project is hidden, banks while financing this project stipulate an incentive compatible contract only to partially insure individual's consumption to eliminate shirking. This makes the borrowing constraint endogenous. In a benchmark case of *grand or complete contracting* where banks monitor every financial transaction of the borrower, the equity premium is zero even though the borrowing constraint is binding.[2] A positive equity premium emerges in an *incomplete contracting* scenario where household's transactions in equity market are kept outside the purview of the contract exposing the household/shareholder to a greater uninsurable

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[1] In contrast with the extant literature focusing on aggregate risk (Lettau et al., 2004) and borrowing constraint (Constantinides et al., 2002), our exercise highlights the role of the banking environment in explaining the size of the premium. Our model has a direct bearing on a growing body of literature exploring the link between asset market frictions and the premium. Such frictions tend to arise out of incomplete markets or borrowing constraints. Mankiw (1986), Constantinides and Duffie (1996), Heaton and Lucas (1996, 1997) looked for explanations for a high premium in terms of incomplete markets where individuals fail to insure their income in the presence of permanent shocks.

[2] In contrast with Constantinides et al. (2002), in our model the borrowing constraint is endogenous driven by the incentive compatible constraint. Our exercise also illustrates that such a borrowing constraint alone cannot solve the equity premium puzzle.

consumption risk. The size of the premium depends on the degree of completeness of the contract as well as the extent of informational friction.

The comparison of these two contracting environments, *complete vs. incomplete*, is motivated by the degree of integration between commercial and investment banking in the United States during the post Glass-Steagall Act era**.** A regime of complete contracting requires financial institutions to have full ownership rights over firms managed by households meaning banks can explicitly control the number of shares issued by households via optimal contracts. However, in the United States the Glass-Steagall Act prohibited such cross ownership until 1999 when the Gramm-Leach-Billy Act effectively eliminated this barrier. Hence, during the Glass-Steagall era, banks or financial intermediaries were not legally allowed to stipulate stock market transactions of a firm. We envisage such an era as a regime of *incomplete contracting*. The repeal of this law permitting the cross-ownerships resembles an environment of *complete* contracting. The equity premium also sharply declined during this era of banking reforms. The testable hypothesis that emerges from our model is that the low equity premium in the 90s is due to a change in the nature of the loan contractual environment following these banking reforms. [3]

We perform two sets of quantitative exercises. First we compare our incomplete contract model with a standard representative agent model with aggregate risk. We demonstrate that even without any aggregate risk, our model with informational friction and incomplete contracts has the potential to outperform a standard representative agent model with aggregate risk in predicting the Mehra-Prescott historical equity premium. Second, we consider an intermediate contracting environment mixing the features of complete and incomplete contracts. Using this model, we calibrate the degree of contract completeness based on the low equity premium estimates in the post 1990 period.

[3] Though much of the literature on Glass-Steagall Act focuses on separation of investment and commercial banking preventing banks to underwrite securities of the borrowing firm, our emphasis here is on the prohibition of cross ownership among these two types of institutions that gives rise to incomplete contracting and emergence of equity of premium. Furthermore, empirical literature documents relaxation of this Act over a period of time leading to a final dismissal in 1999, suggesting a somewhat smooth transition from incomplete to complete contracting environment. For example, the Bank Holding Company Act allows financial firms to acquire 5% of the voting stock of a commercial firm. In 1987, the Bank holding companies and non bank subsidiaries were allowed given more freedom to participate in the equity markets. See Barth, Brumbaugh and Wilcox (2000). The equity premium also showed a gradual decline during this era.

The paper is organized as follows: In the following section we lay out the environment. Section 3 describes the model with moral hazard, but with complete contracts. Section 4 outlines an environment with incomplete contracts. Section 5 explores the equity premium puzzle and reports the calibration results. Section 6 concludes.

2. Environment

The model adapts Kocherlakota (1998). [4] There are continuum of identical agents in the unit interval who live only for two periods. At date 1, a stand-in agent is endowed with y units of consumption goods, and an equity which represents a claim to date 2 output. The value of this equity is Q, which is basically the date 1 value of date 2 output. This Q can be divided in shares. Suppose there are $\bar{x}$ such shares in supply. Out of these $\bar{x}$ shares, the agent keeps x and sells $\bar{x}$ - x at the spot price Q. The buying and selling of shares takes place at date 1. Since $\bar{x}$ is a constant, it can be safely normalized to unity. The representative agent's own share (x) gives him proceeds in the second period. What proceeds he would get depends on the nature of the production technology to which we now turn.

The agent invests k units of capital at date 1 which goes through a production process and results in output depending on the interaction between idiosyncratic risks and the agent's choice of efforts. Individual's effort is a binary variable assuming 0 and 1 for no effort and positive efforts respectively. If individuals exert effort in period 1, then output will be $f(k)$ with probability p, and 0 with the complementary probability. This basically means that a fraction p of agents in the unit mass would succeed while the remaining 1-p will fail. If they do not exert effort, output will be $f(k)$ and 0 with probability q and $1-q$ respectively where $p > q$. The cost of effort is given by φ. The function $f(k)$ is increasing in k. All the risks in technology are idiosyncratic in nature. There is no aggregate risk.

[4] We introduce financial intermediaries, and loan contracts explicitly in Kocherlakota's (1998) setting. We analyze the historical equity premium as well as declining premium while Kocherlakota focuses only on the historical premium itself. See also Kahn (1990) for a model addressing related issues**.**

Let us next turn to the financing of projects. There are competitive banks which provide loans (*l*) to the agent in the first period and offer a safe rate *r* to the depositors. These loans are subject to default risk. If the project succeeds, the agent makes a repayment of *R* to the bank and if it fails he walks out paying nothing (due to limited liability). However, if project risks are independent and individuals are distributed in a continuum, intermediaries can generate a safe rate of return (r) by invoking the law of large numbers.[5] Hence, the expected profit of an intermediary assuming that agents have exerted efforts is:

$$p[R-(1+r)l]+(1-p)[0-(1+r)l]= pR-(1+r)l \, . \tag{1}$$

If there is free entry and exit, then zero expected profit of the intermediaries implies:

$$pR-(1+r)l=0 \, . \tag{2}$$

Since these banks are competitive, the individual just faces a menu of contracts *R* and *l* which satisfies this zero profit condition. The agent picks the *R* and *l* from this menu in such a way that it maximizes his expected utility.

Preferences

The utility function facing each agent is additively separable in consumption at each date and is of the form:

$$U = u(c_1)+v(c_2) \tag{3}$$

where where c_i =consumption in period *i*, *i*=1,2, *u*(.) and *v*(.) are: (a) three times continuously differentiable, (b) concave, and (c) has a convex form for the marginal utility. Hence, agents are risk-averse.

To sum up, the resource constraint of the individuals are given by:

$$c_1+s+k+xQ=y+Q+l \tag{4}$$

$$c_2^g = xf(k)-R+(1+r)s \quad \text{and} \quad c_2^b=(1+r)s \tag{5}$$

[5] The probability of all projects failing is close to zero because $(1-p)^n$ approaches zero as the number of independent projects, n approaches infinity. By this, we assume no-bankruptcy for the banks.

where $c_2^g =$ consumption in the second period when the project is successful. $c_2^b =$consumption in the second period when the project is unsuccessful, and s=individual's saving.

Hence, the expected utility of a representative agent is:

$U = u(c_1) + pv(c_2^g) + (1-p)v(c_2^b) - \varphi$, which can be rewritten as:

$$U = u(y + Q + l - s - k - xQ] + pv[xf(k) - R + (1+r)s] + (1-p)v[(1+r)s] - \varphi \qquad (6)$$

3. Moral Hazard and Endogenous Borrowing Constraint: The Case of Grand Contracting

We now introduce informational frictions due to moral hazard. Let the choice of entrepreneurial effort be a private knowledge to the household and unobserved by the financial intermediaries or banks. It is well known that complete smoothing of consumption (thus full insurance) will destroy the incentives to exert higher levels of effort. Hence, the intermediaries would issue a loan and charge a borrowing rate of interest such that consumption is only partially insured.[6]

The banks set up non-linear contracts with the households regarding the choice of all its financial variables, *R*, *l*, *x* and *s*. All these variables are determined by optimal contracts under which borrowers maximize their expected utility subject to a zero profit condition of the intermediary and the incentive compatibility condition. With the binary choice of efforts, such an incentive compatibility condition is:

$$v(xf(k) - R + (1+r)s) - v((1+r)s) \geq \frac{\varphi}{p-q} \qquad (7)$$

Since individuals exert effort at the beginning of the second period (before the resolution of uncertainty), this standard condition states that the gain in expected utility from zero to positive effort must be non-zero. Hence, the optimal contract problem can be written as:

[6] In the presence of full information about entrepreneurial effort, full consumption insurance takes place. All the idiosyncratic project risks will be transferred from the risk averse households to the risk neutral financial intermediaries. The banks pool the risk by redistributing consumption between the lucky and unlucky households in an actuarially fair fashion, meaning $c_2^g = c_2^b = pf(k)$. In fact, a social planner can also implement the same risk pooling.

$$Max_{\{l,s,k,R,x\}}\, U = u(y+Q+l-s-k-xQ] + pv(xf(k)-R+(1+r)s)+(1-p)v((1+r)s)-\varphi$$

subject to $pR = (1+r)l$ (8)

and

$$v(xf(k)-R+(1+r)s)-v((1+r)s) \geq \frac{\varphi}{p-q} \qquad (9)$$

which can be rewritten after substituting out *R* using (8):

$$L_{\max \cdot_{\{l,s,k\}}} = u(y+Q+l-s-k-xQ)+pv(xf(k)+(1+r)(s-\frac{l}{p}))+(1-p)v((1+r)s)-\varphi$$

$$+\mu[v(xf(k)+(1+r)(s-\frac{l}{p}))-v((1+r)s)-\frac{\varphi}{p-q}] \qquad (10)$$

First-order conditions are:

$$s: -u'(c_1)+(1+r)[\{pv'(c_2^g)+(1-p)v'(c_2^b)\}+\mu\{v'(c_2^g)-v'(c_2^b)\}]=0 \qquad (11)$$

$$l: u'(c_1)-(1+r)v'(c_2^g)[1+\frac{\mu}{p}]=0 \qquad (12)$$

$$k: -u'(c_1)+v'(c_2^g)xpf'(k)[1+\frac{\mu}{p}]=0 \qquad (13)$$

$$x: -u'(c_1)Q+pv'(c_2^g)f(k)[1+\frac{\mu}{p}]=0 \qquad (14)$$

Characterization of Equilibrium

1. Given *r* and *Q*, agents choose *l, s, R, x* optimally which satisfy the above first order conditions.

2. Loan and Equity markets clear meaning s=*l* and *x*=1.

From these first-order conditions, we immediately deduce the following proposition.

Proposition 1: The households are credit constrained and risks are uninsured so that $c_2^g > c_2^b$

Proof: Assuming (9) binds it follows immediately that $c_2^g > c_2^b$. In view of this, from (11) and (12), it follows that

$$\mu = \frac{p(1-p)[v'(c_2^b) - v'(c_2^g)]}{(1-p)v'(c_2^g) + pv'(c_2^b)} > 0 \qquad (15)$$

Verify now from (12) that $u'(c_1) - (1+r)v'(c_2^g) > 0$, implying that individuals would be better-off with additional borrowing.

The incentive compatible constraint deters full consumption insurance. The household would always wish that they could save and borrow more.[7] *The incentive compatible constraint is thus equivalent to a borrowing constraint.*

Equity Premium

We next turn our attention to pricing of equity and the resulting equity premium in this setting. We have the following proposition.

Proposition 2: The price of equity is: $Q = \frac{pf(k)}{1+r}$ and equity premium is zero

Proof: The proof directly follows from (12), and (14) and the expression of μ in proposition 1 as well as the equilibrium condition that $x = 1$.

Because of the incentive compatible constraint the marginal rates of substitution cannot be equalized state by state. Despite the presence of uninsurable consumption risk, the equity premium is zero. To see the intuition, note that the contracts in this model are Pareto optimal in the sense that all the individual risks are fully contracted. A social planner can also allocate the consumption risk for an economy like this. Once the social planner optimally allocates the risk, the marginal rates of substitution are not equal across states. The following proposition makes it evident.

Proposition 3: The following social planning problem is isomorphic to the present optimal contract environment.

[7] See equation 11, which illustrates that the marginal benefit of saving exceeds its marginal cost because μ>0.

$$\text{Max} \quad u(c_1) + pv(c_2^g) + (1-p)v(c_2^b) \qquad \text{(P)}$$

$$\text{s.t.} \quad c_1 + k = y;$$

$$pc_2^g + (1-p)c_2^b = pf(k); \text{ and}$$

$$v(c_2^g) - v(c_2^b) \geq \frac{\varphi}{p-q}.$$

Proof: Plug the equilibrium conditions *s=l*, *x=1*, into the household's sequential budget constraints (4) and (5) and then multiply the second period budget constraints (5) for good and bad states by *p* and *(1-p)* respectively, add them up to get the social planner's resource constraints.

In the next step, check that the first order condition of the social planning problem (P) is given by:

$$\frac{1}{u'(c_1)} = \frac{1}{pf'(k)}\left[\frac{p}{v'(c_2^g)} + \frac{1-p}{v'(c_2^b)}\right] \qquad (16)$$

Next combine the first order condition (13) of the optimal contract problem, substitute out the lagrange multiplier μ using (15) and verify that it reduces to (16). [8]

The zero equity premium results from the fact that there is no aggregate risk in this model. All the idiosyncratic individual risks are properly contracted. The presence of borrowing constraint and uninsurable risk, *per se*, thus cannot explain the existence of equity premium, as long as all project risks are contracted in advance. [9]

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[8] It is instructive to note that the first order condition of this social planning problem resembles the Pareto optimal contract condition in Rogerson (1985) although Rogerson's setting is quite different from ours.

[9] One may be curious to know what happens to equity premium in the presence of aggregate risk and moral hazard. In the presence of aggregate risk, the equity premium would be of course positive because it will reflect the non-diversifiable aggregate uncertainty. The issue is whether the presence of moral hazard would make any difference in the size of the equity premium. We have an example (available from the authors upon request) illustrating that the moral hazard does not make any difference to the size of the equity premium even if aggregate risk is present.

4. **Incomplete Contracts**

We now consider a contractual arrangement in which a positive equity premium emerges. Consider a contracting environment where household's issue of shares (*x*) is not monitored by the bank, and hence it is outside the purview of the contract. In this sense this contractual arrangement is incomplete as opposed to the grand contracting described earlier. Households make decision about purchase of shares (*x*) without taking into account the incentive compatibility condition for positive effort. The competitive banks, on the other hand, design an optimal contract about the deposit (*s*), loans (*l*), repayment (*R*), and project (*k*) which are incentive compatible for the household regarding the choice of positive effort. Both households and banks move simultaneously and thus banks cannot observe the household's choice of shares.[10] In a Nash equilibrium, all these variables are determined simultaneously which is formalized as follows.[11]

Characterization of Equilibrium

1. Given *r, Q s, l, k, R,* the household chooses the share holding *x,* which maximizes its expected utility (6) subject to the bank's zero profit condition (2).
2. Given *r, Q* and *x* competitive banks offer a menu of contracts, *s, l, k, R* which maximize household's expected utility (6) subject to the bank's zero profit condition (2) and incentive compatibility condition (9).
3. The share and loan markets clear meaning $x=1$ *and* $s=l$.

The first order condition for the household's issue of shares is:

[10] The enforceability of financial contracts between an individual and an intermediary depends on the degree of commitment by both parties to adhere to contracts. Since savings are held as deposits with the intermediary and there could be an element of irreversibility in the choice of capital, mechanism of commitment could work perfectly with these variables. On the other hand, individuals could buy or sell shares from the market (not from the intermediaries), the degree of commitment is lesser with the amount of shares transacted.

[11] Since households and banks move simultaneously, banks do not observe household's issue of shares. The household thus does not need to take into account the effect of purchase of shares (*x*) on the IC condition. An alternative contractual arrangement would be that the household moves first about the issue of shares (*x*) and then banks sign contracts with the households about the remaining variables upon observing *x*. Such a contracting environment would give rise to a zero equity premium because the households will be able to arrive at the same Pareto optimal contracts as in the previous section by the optimal choice of *x*. The assumption that banks do not observe household's issuance of shares is motivated by the separation between commercial and investment banking during the Glass-Steagall era.

$$x: -u'(c_1)Q + pv'(c_2^g)f(k) = 0 \qquad (17)$$

The optimal contract problem for the bank is now written as:

$$Max_{\{l,s,k,R\}}\, U = u(y + Q + l - s - k - xQ) + pv(xf(k) - R + (1+r)s) + (1-p)v((1+r)s) - \varphi$$

subject to $pR = (1+r)l$

and (7).

The problem can be rewritten as:

$$L_{\max_{\{l,s,k\}}} = u(y + Q + l - s - k - xQ) + pv(xf(k) + (1+r)(s - \frac{l}{p})) + (1-p)v((1+r)s) - \varphi$$
$$+ \mu\{v(xf(k) + (1+r)(s - \frac{l}{p})) - v((1+r)s) - \frac{\varphi}{p-q}\}$$

First-order conditions:

$$s: -u'(c_1) + (1+r)[\{pv'(c_2^g) + (1-p)v'(c_2^b)\} + \mu\{v'(c_2^g) - v'(c_2^b)\}] = 0 \qquad (18)$$

$$l: u'(c_1) - (1+r)v'(c_2^g)[1 + \frac{\mu}{p}] = 0 \qquad (19)$$

$$k: -u'(c_1) + pv'(c_2^g)xf'(k)[1 + \frac{\mu}{p}] = 0 \qquad (20)$$

Equity Premium

Denote the proportional equity premium in this incomplete contract economy as EP^{INC}. The proportional equity premium is the ratio of the gross expected return on stock and gross expected return on riskfree saving deposits. In other words,

$$EP^{INC} = \frac{(pf(k)/Q)}{1+r} \qquad (21)$$

We have the following proposition.

Proposition 4: $EP^{INC} = 1 + \frac{\mu}{p}$

Proof: Using (17) and (19), we get:

$$Q = \frac{pf(k)}{[1 + \frac{\mu}{p}](1 + r)} \tag{22}$$

which immediately proves the proposition.

The equity premium is thus determined by the shadow price of the incentive constraint and it is positive. [12] Households while participating in the stock market bear a greater uninsurable consumption risk than when they participate in the bond market. This is because the bond market transactions are under the purview of the optimal contract while the stock market transactions are not. The lagrange multiplier, which is basically the shadow price of incentive compatible constraint, drives a wedge between the perceived intertemporal marginal rate of substitution (IMRS) of the consumer/shareholders and consumer/bondholders. This cross sectional heterogeneity of the IMRS gives rise to a positive equity premium.

In an influential paper Constantinides and Duffie (1996) show that the partial consumption insurance and the consumer heterogeneity together could explain the equity premium. In our model this partial consumption insurance arises due to moral hazard which gives rise to an endogenous borrowing constraint. However, this alone cannot explain the equity premium if this partial consumption insurance is contracted in advance. One also needs additional heterogeneity of IMRS, which is driven in our model by the nature of the contract.

The upshot is that the contractual environment is of paramount importance in driving the premium in the stock market. To make this point more transparent, we next develop a special example economy in which the real allocations are identical in both grand contract and incomplete contract environments but the equity premium differs.

[12] Note that a unit value of the proportional equity premium means a zero equity premium.

An Example

One can further rewrite the equity premium by substituting out the lagrange multiplier μ. Use (18) and (19) to get the same expression as in (15), which upon substitution in (22) yields:

$$EP^{INC} = 1 + \frac{(1-p)[v'(c_2^b) - v'(c_2^g)]}{(1-p)v'(c_2^g) + pv'(c_2^b)} \tag{23}$$

Assume the following parametric specifications of the utility function and the production function:

$U = \ln C_1 + \ln C_2$ and $f(k) = ak$.

where *a* is a positive total factor productivity (TFP) term. Using this specification, we get the following closed form solution for the proportional equity premium EP^{INC}. The appendix provides an outline of the derivation.

$$EP^{INC} = 1 + (1-p)[\frac{(\lambda - 1)}{1 + p(\lambda - 1)}] \tag{24}$$

where $\lambda = \exp(\frac{\psi}{p-q})$. (24a)

and the riskfree rate is given by:

$$1 + r = ap \tag{25}$$

The lagrange multiplier is directly proportional to the ratio of consumption in good and bad states which keeps the household just indifferent between shirking and not shirking. In the context of the logarthmic utility function this ratio is λ which is positively related to the disutility of effort ψ as shown in (24a). The higher the disutility of effort ψ, the greater the λ. λ is thus a measure of informational friction. Note that a higher informational friction raises the uninsurable risk of all the households. Since the household while participating in the share market bears even a greater uninsurable consumption risk, the equity premium is monotonically increasing in the informational friction parameter λ. When ψ is zero, λ equals unity, in which case the equity premium vanishes because the informational friction is absent. [13]

We next compare the real allocations and the expected welfare in this incomplete contract economy and the grand contract economy described in section 3. Let both these economies share the same logarithmic preference and the linear technology. The Appendix A shows that the real allocations and the welfare are identical in both these scenarios even though the equity premium differs. Thus it is the contractual environment not the real allocations that drives the equity premium.

5 Equity Premium Puzzle

The purpose of this section is to demonstrate that the incomplete contract version of our model may be more effective in explaining the historically observed equity premium than a representative agent (RA) model with aggregate and idiosyncratic risk. Consider the following representative agent who is subject to an aggregate output risk and idiosyncratic risk as follows. The idiosyncratic risk is the same as in the earlier sections, meaning that with probability p, an agent can succeed in producing a positive output if he is in a good state. However, there is an aggregate risk in that with probability π, the output of all the agents can be higher.

The aggregate production function is, therefore, given by:

$$y = \left| \begin{array}{lll} \varepsilon^G f(k) & prob & \pi \\ \varepsilon^B f(k) & prob & 1-\pi \end{array} \right. \qquad (26)$$

where $\varepsilon^G > \varepsilon^B$ and G, B stand for aggregate good and bad states respectively.

[13] The capital structure is endogenous here reflecting the nature of risk that individuals undertake in our set up. They tend to invest in risky technology in excess of loans that they take from the intermediary. This excess amount $(k-l)$ represents risk because in the bad state of nature, this amount is not recovered and the individuals alone bear this loss. Hence, the premium is also proportional to this amount. Note that $k-l=[\frac{p(\lambda-1)}{1+p(\lambda-1)}]\frac{y}{2}>0$.

There are four states of nature, *(g,G), (b,G), (g, B), (b, B).* Assume that the agent contracts in advance in contingent claims markets by trading in Arrow-Debreu securities. Let q^j be the price of a claim that pays one unit of consumption if the aggregate state is *j* (=*G*, *B*). Define the price of a claim that pays one unit of consumption when the individual state is *i (=g or b)* and the aggregate state is *j* as $p^{i,j}$. By law of large number, it follows that $p^{i,j} = prob(i).\ q^j$.[14]

The representative agent thus solves the following program.

$$L = u(c_1) + \pi p v(c_2^{g,G}) + \pi(1-p)v(c_2^{b,G}) + (1-\pi)pv(c_2^{g,B}) + (1-\pi)(1-p)v(c_2^{b,B})$$

$$+\lambda[y + pq_G f(k)\varepsilon^G + pq_B f(k)\varepsilon^B - c_1 - pq_G c_2^{g,G} - (1-p)q_G c_2^{b,G} - pq_B c_2^{g,B} - (1-p)q_B c_2^{b,B} - k] \quad (27)$$

where $c_2^{i.j}$ is the consumption in the second period when the individual state is *i* and the aggregate state is *j, i=g,b* and *j=G,B.*

It is straightforward to verify that the equilibrium consumption allocation is given by:

$$c_2^{g,G} = c_2^{b,G} = p\varepsilon^G f(k) \quad (28)$$

$$c_2^{g,B} = c_2^{b,B} = p\varepsilon^B f(k) \quad (29)$$

In other words, consumption will be equalized across individual states but not across aggregate states because aggregate risks cannot be pooled.[15]

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[14] See Kocherlakota (1998) for a similar representation of Arrow-Debreu Pricing in the presence of aggregate and idiosyncratic risks.

[15] To see this observe that the equilibrium allocation of this economy can be solved by a social planning problem where the planner maximizes the same utility function subject to the following state contingent resource constraints: $c_1 + k = y,\ \ pc_2^{g,j} + (1-p)c_2^{b,j} = p\varepsilon^j f(k)$, with j=G, B.

Define $c_2^{g,G} = c_2^{b,G} = c_2^G$ and $c_2^{g,B} = c_2^{b,B} = c_2^B$. Stock prices and risk free interest rate are given by:

$$Q = q_G pf(k)\varepsilon^G + q_B pf(k)\varepsilon^B \tag{30}$$

$$1 + r = \frac{1}{q^G + q^B} \tag{31}$$

To make a valid cross-model comparison assume that the utility function is logarithmic (*u(c) = ln c*) and the production function is linear, i.e. *f(k)=k.* The appendix shows that that the gross expected return on stock (call it R_m) and risk free rate (call it $R_f = 1 + r$) are given by:

$$R_m = p[\pi\varepsilon^G + (1-\pi)\varepsilon^B] \tag{32}$$

$$R^f = \frac{p\varepsilon^G \varepsilon^B}{\pi\varepsilon^B + (1-\pi)\varepsilon^G} \tag{33}$$

The expected return on the market portfolio is proportional to the expected total factor productivity (TFP hereafter) in this economy. The proportional equity premium (EP^{RA}) defined as the ratio of R_m to R_f is given by:

$$EP^{RA} = \frac{[1+\pi(\omega-1)][\omega-\pi(\omega-1)]}{\omega} \tag{34}$$

where $\omega = \dfrac{\varepsilon^G}{\varepsilon^B}$.

Note that the equity premium is independent of the probability (*p*) of individual success because the risks are idiosyncratic and wash out in the aggregate.[16]

[16] The same does not happen in the incomplete contract economy because the individual risks do not go away in equilibrium.

Cross Model Comparison of Equity Premia

We are now ready to compare the equity premium in the incomplete contract economy (INC hereafter) in (24) with that of the representative agent (RA hereafter) economy in (34). Note that there is no aggregate consumption risk in the incomplete contract economy which means variance of consumption is zero meaning a perfectly smooth aggregate consumption in this environment . On the other hand, in the RA economy there is aggregate risk, which means that the variance of aggregate consumption is positive.

Why do we compare two models: one without aggregate risk and other with aggregate risk? We do this comparison simply to identify the role of individual uninsurable risk in explaining the historical equity premium. If we can replicate the historical equity premium using a model without aggregate risk, such a model will do even better if we add aggregate risk to it. We demonstrate now that this simple INC model with zero aggregate consumption risk but with informational frictions could outperform a standard RA model with aggregate risk in terms of reproducing the historical equity premium.

To make a fair cross model comparison, we pose the following question. Suppose we aim to replicate the historical equity premium and the riskfree rate in both INC and RA models. What range of values of the parameters in each of these models will accomplish this goal? Are these parameter values empirically plausible?

There are two important parameters, i.e., λ and ω. The parameter λ is a measure of information friction in the INC model while ω is a measure of aggregate consumption risk in the RA model. In order to replicate the historical equity premium and the riskfree rate in each of these models λ and ω have to be calibrated. We proceed as follows.

Calibration of λ

We first turn our attention to the equation for equity premium (24) in the incomplete contract economy. Following Kocherlakota (1998), we assume that aggregate and individual states are equally likely, meaning $p=\pi=1/2$. Moreover, we assume that individual and aggregate states are mutually independent. We then calibrate the TFP parameter a of the INC model by setting the riskfree rate ap in (25) equal to the historical average riskfree rate. In this way, the INC model exactly replicates the historical riskfree rate, and thus there is no riskfree rate puzzle. Finally using (24) and the historical equity premium, we can easily calculate λ.

Calibration of ω

We next turn our attention to the RA model. In order to calculate ω, we need to know the aggregate states, ε^G and ε^B. Setting $p=\pi=1/2$, and R_m and R_f at the historical levels, we use (32) and (33) to solve these two aggregate states. This solution strategy ensures that the historical equity premium and riskfree rate are perfectly replicated by the RA model.

Results

Table 1 summarizes the results for various available datasets for the US equity premium. The value of ω ranges from 1.49 to 1.77. Following Kocherlakota (1998) the parameter ω which is the ratio of per capita consumption in good and bad aggregate states is actually 1.073.[17] The RA model requires at least 50% higher aggregate consumption variability than it is observed in the data to replicate the historical equity premium and the low riskfree rate.[18]

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[17]These numbers are taken from Kocherlakota (1998). Note that Kocherlakota's specification of per capita consumption in good and bad aggregate states exactly apply to our RA model here.

[18] Since the RA model has aggregate uncertainty while the INC model does not have, in principle the total factor productivity estimates are different in these two models. Note that a is the TFP in the INC model which we calibrated using the observed riskfree rate. On the other hand, the TFP stochastically fluctuates between ε^G and ε^B in the RA model. We have calculated the expected TFP in the RA model which can be used as a reasonable benchmark of comparison with the TFP term a in the INC model. For alternative data sets a ranges from 2.008 to 2.05 while the expected TFP ranges from 2.14 to 2.17. The TFP estimates are not wildly different in these two models.

Table 1: Comparison of Incomplete Contract and Representative Agent Models

Data Set	%Mean Riskfree Rate	% Mean Equity Premium	λ in the INC Economy	ω in the RA Economy
1802-1998 (Siegel)	2.9	4.1	1.08	1.49
1871-1999 (Shiller)	1.74	5.75	1.12	1.60
1889-2000 (Mehra-Prescott)	1.14	6.92	1.15	1.68
1926-2000 (Ibbotson)	0.4	8.4	1.18	1.77

Note: Summary of the various data sets came from Mehra and Prescott (2003).

On the other hand, the INC model does not require any aggregate consumption variation, which means that ω is unity by construction. What is the price we pay to replicate the historical equity premium and the low riskfree rate?. One requires some degree of informational friction summarized by the parameter λ as opposed to aggregate consumption risk in the RA model. The range of variation of λ is from 1.08 to 1.18. Recall that a unit value of λ means no informational friction. Although we do not have any readily available estimate of λ based on microeconomic evidence, it is noteworthy that for INC model, only a moderate dose of uninsured consumption risk (8% to 18%) can replicate the historical equity premium and the risk free rate without invoking any aggregate consumption risk at all. [19] Our incomplete contract model outperforms a standard RA model in reproducing the historical equity premium.[20]

[19] In both models, we assume log utility, which means we do not require a high degree of risk aversion.

[20] The issue arises whether we can use a two period model to calibrate the historical average that is based on many periods. The equity premium in our model is averaged across states while in the data it is time averaged. Are these two averages comparable? We assume that the model economy is stationary in the sense that the

Declining Equity Premium

We now turn our attention to declining equity premium. The model described so far reflects two polar environments: (i) grand contract, and (ii) incomplete contract. These two extremes can be thought of as two banking regimes: (i) a regime of full integration between commercial and investment banking, and (ii) a regime of no integration between these two types of banking. To bring more realism to the banking contractual environment, we consider an intermediate scenario. Define a new parameter $\theta \in (0,1)$ as a measure of the degree of completeness of the contract. A higher value of θ means a greater degree of integration between commercial and investment banking. In our model context, a higher θ means that the contract is more complete.

The optimal contract problem can be now rewritten as:

$$L_{\max_{\{l,s,k\}}} = u(y+Q+l-s-k-xQ) + pv(xf(k)+(1+r)(s-\frac{l}{p})) + (1-p)v((1+r)s) - \varphi$$

$$+\mu\theta\{v(xf(k)+(1+r)(s-\frac{l}{p})) - pv((1+r)s) - \frac{\varphi}{p-q}\}$$

The household's share valuation equation (17) now changes to:

$$x: -u'(c_1)Q + pv'(c_2^g)\, f(k)(1+\frac{\theta\mu}{p}) = 0 \qquad (35)$$

The lagrange multiplier μ now enters the household's share valuation equation only to the extent the contract is complete in nature. The grand contract and incomplete contract are two special cases when θ equals 1 and 0 respectively.

It is straightforward to verify that in this mixed contracting environment, the proportional equity premium (which we now call EP^{mixc}) in Proposition 4 changes to:

$$EP^{mixc} = \frac{1+\dfrac{\mu}{p}}{1+\dfrac{\theta\mu}{p}} \qquad (36)$$

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probability distribution of states satisfies the ergodicity properties. Thus the time average of any relevant variable (say equity premium) is the same as the ensemble average (defined over the state space).

In the context of our parametric example with logarthmic utlity and linear production function, (36) reduces to:

$$EP^{mixc} = \frac{1+(1-p)[\frac{(\lambda-1)}{1+p(\lambda-1)}]}{1+\theta(1-p)[\frac{(\lambda-1)}{1+p(\lambda-1)}]} \qquad (37)$$

It is easy to verify that the real allocations in this special logarithmic example is invariant to the banking integration parameter θ. Different contracting environments (θ) engender different equity premia without disturbing the real allocations and without bringing any aggregate risk. This makes the calibration of θ easy.

We calibrate the degree of banking integration parameter θ during the 90s when the progress towards universal banking was nearly complete. The parameter *p* is fixed at the same level as in Table 1. The informational friction parameter is fixed at the benchmark level, 1.13 which is the average of the λ values calibrated in Table 1. The historical equity premium estimates during 1889-1979 came from Mehra and Prescottt (1985) while estimates of the equity premium during the 90s based on S&P data came from Jagannathan et. al. Table 4. [21] Plugging the recent equity premium estimates into (37), we calculate the value of θ. The results are summarized in Table 2.

Table 2: The Calibrated Degree of Contract Completeness Based on US Equity Premium

Period	US Equity Premium	θ
1889-1979	6.18%	.25
1990-99	2.51%	.60
December 1999	1.26%	.80

Note: The average equity premium estimates for 1889-1979 is from Mehra and Prescott (1985) and the equity premium estimates during the 90s came from Jagannathan et al. (2000).

[21]According Jagannathan et al. (2000) estimates, the decline in equity premium started from 1970. Since we argue that banking reform in the 90s is a potential candidate for the decline in premium, we only focus on the estimates of equity premium in the 90s for the purpose of calibration. The question remains: why did the equity premium decline earlier? One may argue that the financial markets anticipated these reforms way ahead of time. Moreover, the progress towards universal banking was rather gradual suggesting a somewhat smooth transition from incomplete to complete contracting environment. See footnote 3 for documentation of some of the earlier banking reforms.

The historical equity premium of 6.79% corresponds to a contract completeness of 0.18 which may be interpreted as a banking integration of about 18% During the 90s, the equity premium averaged about 2.51% meaning a value of θ equal to 0.6. It is noteworthy that the equity premium reached a value of 1.26% (lowest in the 90s) in December 1999 which coincided with the enactment of the Gramm-Leach-Billy Act on November 12, 1999. This Act virtually eliminated all barriers to the integration between commercial and investment banking making the contract nearly complete. Based on our model we can infer that θ was close to 0.8 while a unit value of θ means full contracting or universal banking.

6. Conclusion

A number of recent papers make the point that the equity premium is traceable to uninsurable risk and borrowing constraint. Using a simple two period setting, we show that the nature of contracting between the financial intermediary and the household/entrepreneur is crucial for determining the equity premium. Informational frictions such as moral hazard may lead to a borrowing constraint and uninsured risk but whether this will translate into a premium in the stock market depends on whether financial intermediaries such as banks exert any control over individuals' transactions in the equity market. Our calibration exercise shows that a simple model with incomplete contract where the bank has no control over the stock trading can outperform a representative agent model with aggregate risk. With slight modification, such an incomplete contract environment can explain the post 90s decline in the equity premium.

Appendix A

Derivation of (24)

Using (18) and (19), we could collapse the first-order conditions to:

$$\frac{(1+r)}{u'(c_1)} = \frac{p}{v'(c_2^g)} + \frac{(1-p)}{v'(c_2^b)},$$

Then by using the logarithmic utility we get:

$$pc_2^g + (1-p)c_2^b = (1+r)c_1 \tag{A.1}$$

Use (2), (5) and the loan market clearing condition, $s=l$ to obtain

$$c_2^g = f(k) + (1+r)(1-\frac{1}{p})s = ak + (1+r)(1-\frac{1}{p})s \tag{A.2}$$

$$c_2^b = (1+r)s \tag{A.3}$$

and

$$c_1 = y - k \tag{A.4}$$

Plugging (A.2), (A.3) (A.4) into (A.1), and using (25) we get:

$$k = \frac{y}{2} \tag{A.5}$$

From the incentive constraint (9), we get:

$$c_2^g = \lambda c_2^b, \tag{A.6}$$

where $\lambda = \exp(\psi/(p-q)) > 1$

Plug (A.2), (A.3) into (A.6) and use the loan market equilibrium condition $s = l$ to obtain the equilibrium loan amount:

$$l = \frac{pak}{(1+r)[1+p(\lambda-1)]} \tag{A.7}$$

Next use the expression for equity premium in (23) and use (A.2), (A.3), (A.7) to obtain (24).

Comparing allocations in grand contract and incomplete contract environments

We show now that in the context of the log utility example, the equilibrium allocations are identical in both the grand contract and incomplete contract environments. First observe that that the first order conditions for *s*, *l* and *k* are identical in both environments (see (11) through (13) and (18) through (20)). This means (A.1) holds for both environments. The immediate implication is that (A.4) and (A.5) hold as well. Using (A.2) through (A.7), and the loan market clearing condition, *s*=*l*, we get:

$$c_2^g = \frac{ap\lambda}{1+p(\lambda-1)} \cdot \frac{y}{2} \tag{A.8}$$

$$c_2^b = \frac{ap}{1+p(\lambda-1)} \cdot \frac{y}{2} \tag{A.9}$$

Since the equilibrium allocations c_1, c_2^g, c_2^b are the same in both environments, it means that the expected utility is the same in both. However, this is true only in the context of the logutility example.

Appendix B
Derivation of (32), (33) and (34)

The first order conditions based on (27) are:

$$\frac{\partial L}{\partial c_1} = u'(c_1) - \lambda = 0 \tag{B.1}$$

$$\frac{\partial L}{\partial c_2^{g,G}} = \pi p v'(c_2^{g,G}) - \lambda p q_G = 0 \qquad \text{(B.2)}$$

$$\frac{\partial L}{\partial c_2^{b,G}} = \pi(1-p)v'(c_2^{b,G}) - \lambda(1-p)q_G = 0 \qquad \text{(B.3)}$$

$$\frac{\partial L}{\partial c_2^{g,B}} = (1-\pi)pv'(c_2^{g,B}) - \lambda p q_B = 0 \qquad \text{(B.4)}$$

$$\frac{\partial L}{\partial c_2^{b,B}} = (1-\pi)(1-p)v'(c_2^{b,B}) - \lambda(1-p)q_B = 0 \qquad \text{(B.5)}$$

$$\frac{\partial L}{\partial k} = -\lambda + \lambda p q_G \varepsilon^G f'(k) + \lambda p q_B f'(k)\varepsilon^B = 0 \qquad \text{(B.6)}$$

Using (B.1) and (B.6), we get:

$$u'(c_1) = \left[\pi v'(c_2^{g,G}) + (1-\pi)v'(c_2^{g,B})\right] p f'(k) \qquad \text{(B.7)}$$

Next note that the economy-wide resource constraints and social plannin optima are:

$$c_1 + k = y \qquad \text{(B.8)}$$

$$c_2^{g,G} = c_2^{b,G} = p\varepsilon^G f(k) \qquad \text{(B.9)}$$

$$c_2^{g,B} = c_2^{b,B} = p\varepsilon^B f(k) \qquad \text{(B.10)}$$

Using (B.7),(B.8), (B.9) and (B.10) we get:

$$c_1 = k = \frac{y}{2} \qquad \text{(B.11)}$$

The price of equity is:

$$Q = q_G pf(k)\varepsilon^G + q_B pf(k)\varepsilon^B \tag{B.12}$$

We price the Arrow-Debreu securities in such a way that it supports the economy wide resource constraints (B.8) and (B.9) and (B.10). It is easy to verify that

$$q_G = \left(\frac{\pi}{p\varepsilon^G}\right) \tag{B.13}$$

$$q_B = \left(\frac{1-\pi}{p\varepsilon^B}\right) \tag{B.14}$$

The expected return on market portfolio (R_m) is given by:

$$R_m = \frac{p\varepsilon^G \pi f(k) + p(1-\pi)\varepsilon^B f(k)}{Q} \tag{B.15}$$

which after substituting (B.12) yields:

$$R_m = p.\left(\pi\varepsilon^G + (1-\pi)\varepsilon^B\right) \tag{B.16}$$

The risk free rate (R_f) is given by:

$$R_f = \frac{1}{q_G + q_B} \tag{B.17}$$

Using (B.13) and (B.14),

$$R_f = p.\left(\frac{\varepsilon^G \varepsilon^B}{\pi \varepsilon^B + (1-\pi)\varepsilon^G} \right) \tag{B.17}$$

Dividing (B.16) by (B.17) we get the proportional equity premium given by (34).

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