Measuring Fiscal Sustainability

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Abstract

We propose an index of the fiscal stance that is convenient for practical use. It is based on a finite time horizon, not on an infinite time horizon like most tests. As it employs VAR analysis it is simple to compute and easily automated. We also show how it is possible to analyse a change of policy within a VAR framework. We use this methodology to examine the effect on fiscal sustainability of a change in policy. We then conduct an empirical examination of the fiscal stances of the US, the UK and Germany over the last 25 or more years, and we carry out a counter-factual analysis of the likely consequences for fiscal sustainability of using a Taylor rule to set monetary policy over this period. Among our findings are that the recent fiscal stances of all three countries are not sustainable, and that using a Taylor rule in the past would have improved the fiscal stances of the US and UK, but not that of Germany.

1 Introduction

Recent concerns in 2004 and 2005 about the fiscal stances of the US, France and Germany and of possible reforms to the EU's Stability and Growth Pact (largely due to the errant fiscal positions of France and Germany) have renewed interest in the issue of how to measure fiscal sustainability. In this paper we provide a critical review the literature on fiscal sustainability much of which is at least a decade old. We then propose a new way of measuring fiscal sustainability that avoids most of the limitations of those proposed in the literature. We use this to re-examine the fiscal stances of the US, the UK and Germany.

Determining whether a current fiscal stance is sustainable has proved both difficult and highly controversial. The Stability and Growth Pact attempts to resolve the problem by setting limits on the ratios of the government deficit to GDP and government debt to GDP. Such rules are, however, far too restrictive. Moreover, they can be shown to be neither necessary nor sufficient to achieve fiscal sustainability: a country could breach these limits and still have a sustainable fiscal policy, or it could satisfy the limits but not have a sustainable fiscal policy.

It is generally agreed that a fiscal stance is sustainable if it satisfies the government's inter-temporal budget constraint. In practice, this does not solve the problem either as the inter-temporal budget constraint is forward-looking over an infinite horizon. Most of the literature on fiscal sustainability focuses on past deficits and debts, but a government may attempt to circumvent such assessments by announcing the intention to offset current deficits and debts by generating future surpluses. This raises the question of whether such announcements are credible given the performance and structure of the economy. To answer this one would need a measure of the sustainability of the current fiscal stance based on a model of the economy.

In this paper we propose an index of the sustainability of the current fiscal stance derived from the inter-temporal budget constraint. The index is based on a comparison of the existing level of government debt with a forecast of the present value of current and future deficits and surpluses

derived from a simple VAR forecasting model of the economy. The time horizon for the present value is a matter of choice; it can be finite or infinite. The index is calculated as the ratio of this present value to the existing level of debt. If the index exceeds unity then the current fiscal stance is sustainable; if it is less than unity then a change in the fiscal stance must be considered.

The main attraction of this index is that it is easy to compute each period in a mechanical way, and it can be tailored to any time horizon. From a theoretical point of view it has a number of important advantages over existing procedures for determining fiscal sustainablility. It is more informative and general than simply looking at whether deficits and debts are stationary or non-stationary processes, or whether the deficit and debt are cointegrated. The greater generality arises from not assuming that interest rates, inflation and GDP growth are constant either in the past or over the forecast period, or that they may change but in a rigid predetermined way. Instead, all three variables are modelled in the VAR together with government expenditures, tax revenues and debt.

If the index indicates a lack of fiscal sustainability then a policy change may be required. It would then be desirable to know whether a particular policy change would achieve sustainability. The problem is that a policy switch would alter the model of the economy. This is known as the Lucas Critique. In a VAR forecasting model every equation would be affected. We therefore propose a new way to adjust a VAR forecasting model following a change to one of its equations that eliminates this problem for a VAR, though not a general structural general. As a result, we are able to compare fiscal sustainability under different fiscal and monetary policy regimes such as a move from discretionary to rules-based policies, or a change in policy rule.

Our main empirical findings are that during the period of strong economic growth in the 1990's the fiscal positions of the US, the UK and Geramny improved considerably, but in recent years the fiscal stance in all three countries has been steadily deteriorating. Our index indicates that a continuation of the present fiscal stances is leading to fiscal unsustainability in the three countries. We have shown that the German fiscal position has worsened steadily over the last thirty years

with only a brief respite in the mid 1990's and a sharp deterioration occurred after unification and again on joining EMU.

The paper is set out as follows. In Section 2 we examine a number of different ways of writing the government budget constraint and establish our notation. In Section 3 we analyse the conditions required for fiscal sustainability and provide an intuitive rationale for the various tests that have been proposed in the literature. We show that these tests have two main problems: due to the discount rate being time-varying and the analysis of fiscal sustainability is a non-linear problem; the tests relate to the very long run and hence are of limited practical use for short-term decision making. We address the first problem in Section 4 where we propose the use of a log-linear approximation to the government budget constraint. This enables linear methods of analysis to be used once more. And in Section 5 we propose a measure of the fiscal stance appropriate for the short run and show how this can implemented using VAR analysis. In Section 6 we explain how it is possible to use the same VAR after some of the original equations have replaced by new equations. We then show how this new methodology enables us to analyse the effect on fiscal sustainability of switching monetary policy to using a Taylor rule. Our empirical results are presented in Section 7 and our findings are summarized in Section 8.

2 The government budget constraint

We begin by considering the nominal government budget constraint (GBC), the sustainability of fiscal policy and the implications of various fiscal rules, such as the EU's Stability and Growth Pact.[1] The nominal GBC can be written

$$P_t g_t + (1 + R_t)B_{t-1} = B_t + \Delta M_t + P_t T_t \tag{1}$$

[1] There is a substantial literature on these issues. Most of it goes back some way in time. See, for example, Hamilton and Flavin (1986), Trehan and Walsh (1988, 1991), Kremers (1989), Wilcox (1989), Blanchard, Chouraqui, Hagemann and Sartor (1990), Bohn (1991, 1992, 1995, 1998, 2005), Hakkio and Rush (1991), Buiter, Corsetti and Roubini (1993), Ahmed and Rogers (1995) and Wickens and Uctum (2000). There is also a related literature on current account sustainability, see Wickens and Uctum (1993).

where g_t is real government expenditure including real transfers to households, T_t is total real taxes and M_t is the stock of outside nominal, non-interest bearing money in circulation that is supplied by the government (the central bank) at the start of period t, B_t is the nominal value of government bonds issued at the end of period t, R_t is the average interest rate on bonds issued at the end of period $t-1$ and $R_t B_{t-1}$ is total interest payments made in period t.[2] Thus the left-hand side of equation (1) is total nominal expenditures in period t and the right-hand side is total revenues plus additions to government current financial resources.

The equivalent real GBC can be derived from the nominal GBC by dividing through the nominal GBC by the general price level P_t. This gives

$$g_t + (1+R_t)\frac{P_{t-1}}{P_t}\frac{B_{t-1}}{P_{t-1}} = T_t + \frac{B_t}{P_t} + \frac{M_t}{P_t} - \frac{P_{t-1}}{P_t}\frac{M_{t-1}}{P_{t-1}}$$

or

$$g_t + (1+r_t)b_{t-1} = T_t + b_t + m_t - \frac{1}{1+\pi_t}m_{t-1} \tag{2}$$

where $\pi_t = \frac{\Delta P_t}{P_{t-1}}$ is the rate of inflation, b_t is the real stock of government debt, m_t is the real stock of money and r_t is the real rate of interest defined by

$$1 + r_t = \frac{1+R_t}{1+\pi_t}$$

and implying that approximately $r_t \simeq R_t - \pi_t$.

The GBC can also be expressed in terms of proportions of nominal or real GDP by dividing through the nominal GBC by nominal GDP $P_t y_t$, where y_t is real GDP. We obtain

$$\frac{g_t}{y_t} + \frac{1+R_t}{(1+\pi_t)(1+\gamma_t)}\frac{b_{t-1}}{y_{t-1}} = \frac{T_t}{y_t} + \frac{b_t}{y_t} + \frac{m_t}{y_t} - \frac{1}{(1+\pi_t)(1+\gamma_t)}\frac{m_{t-1}}{y_{t-1}} \tag{3}$$

where γ_t is the rate of growth of GDP and $\frac{T_t}{y_t}$ is the average tax rate.

[2] In practice governments issue bonds at a discount and redeem them at par. Thus if all bonds were for one period, then $B_t = P_t^B B_t^G$ where B_t^G is the number of bonds issued in period t each with price $P_t^B = \frac{1}{1+R_{t+1}}$ and $B_{t-1}^G = (1+R_t)B_{t-1}$.

The total nominal government deficit (or public sector borrowing requirement, PSBR) is defined as

$$P_t D_t = P_t g_t + R_t B_{t-1} - P_t T_t - \Delta M_t$$

hence $\frac{D_t}{y_t}$, the real government deficit as a proportion of GDP is

$$\begin{aligned} \frac{D_t}{y_t} &= \frac{g_t}{y_t} + \frac{R_t}{(1+\pi_t)(1+\gamma_t)}\frac{b_{t-1}}{y_{t-1}} - \frac{T_t}{y_t} - \frac{m_t}{y_t} + \frac{1}{(1+\pi_t)(1+\gamma_t)}\frac{m_{t-1}}{y_{t-1}} \\ &= \frac{b_t}{y_t} - \frac{1}{(1+\pi_t)(1+\gamma_t)}\frac{b_{t-1}}{y_{t-1}} \end{aligned}$$

The right-hand side shows the net borrowing required to fund the deficit expressed as a proportion of GDP.

We also define the nominal primary deficit $P_t d_t$ (the total deficit less debt interest payments) as

$$P_t d_t = P_t D_t - R_t B_{t-1}$$

which implies that

$$\frac{d_t}{y_t} = \frac{D_t}{y_t} - \frac{R_t}{(1+\pi_t)(1+\gamma_t)}\frac{b_{t-1}}{y_{t-1}}$$

Hence the ratio of the primary deficit to GDP is

$$\begin{aligned} \frac{d_t}{y_t} &= \frac{g_t}{y_t} - \frac{T_t}{y_t} - \frac{m_t}{y_t} + \frac{1}{(1+\pi_t)(1+\gamma_t)}\frac{m_{t-1}}{y_{t-1}} \\ &= \frac{b_t}{y_t} - \frac{1+R_t}{(1+\pi_t)(1+\gamma_t)}\frac{b_{t-1}}{y_{t-1}} \end{aligned} \tag{4}$$

This is a non-linear difference equation in $\frac{b_t}{y_t}$. If we define

$$1+\rho_t = \frac{1+R_t}{(1+\pi_t)(1+\gamma_t)}$$

where approximately, $\rho_t = R_t - \pi_t - \gamma_t = r_t - \gamma_t$, the real interest rate adjusted for economic growth, then equation (4) can be written as

$$\frac{b_t}{y_t} = (1+\rho_t)\frac{b_{t-1}}{y_{t-1}} + \frac{d_t}{y_t} \tag{5}$$

This is the key equation for determining the sustainability of fiscal policy. We note that the evolution of $\frac{b_t}{y_t}$ can also be written in terms of the total deficit since

$$\frac{b_t}{y_t} = \frac{1}{(1+\pi_t)(1+\gamma_t)}\frac{b_{t-1}}{y_{t-1}} + \frac{D_t}{y_t} \tag{6}$$

For positive inflation and growth this is a stable difference equation

3 Fiscal sustainability

Fiscal sustainablity concerns the evolution of $\frac{b_t}{y_t}$ and whether it remains finite or explodes. The fiscal stance is said to be sustainable if $\frac{b_t}{y_t}$ is finite and if financial markets are willing to hold the level of debt that emerges. Before describing our proposed new procedure for determining whether the fiscal stance is sustainable we review the principal methods available in the literature. All of these methods take equation (5) as their starting point. In discussing sustainability it is convenient to distinguish between two cases: where the discount rate ρ_t (and hence R_t, π_t and γ_t) is assumed to be constant and where it is allowed to be time varying.[3]

3.1 Constant discount rate

If ρ_t is assumed to be constant then from equation (5) $\frac{b_t}{y_t}$ evolves according to the difference equation

$$\frac{b_t}{y_t} = (1+\rho)\frac{b_{t-1}}{y_{t-1}} + \frac{d_t}{y_t} \tag{7}$$

where $1+\rho = \frac{1+R}{(1+\pi)(1+\gamma)}$ or, approximately, $\rho = R - \pi - \gamma$. The solution for $\frac{b_t}{y_t}$ depends on whether the equation (7) is stable or unstable. We consider both cases.

Case1: $\rho < 0$ (stable case)

[3] Ahmed and Rogers (1995) and Bohn (1995, 2005) argue that the appropriate discount rate to use for discounting future primary surpluses is the inter-temporal marginal rate of substitution. In a complete markets full general equilibrium model this would be the real rate of return used here.

In this case $\frac{\mathbf{1+R}}{(\mathbf{1+\pi})(\mathbf{1+\gamma})} < \mathbf{1}$ and equation (7) is a stable difference equation, and hence can be solved *backwards* by successive substitution. The expected value of the debt-GDP ratio in n period's time conditional on information at time t is

$$E_t(\frac{b_{t+n}}{y_{t+n}}) = (1+\rho)^n \frac{b_t}{y_t} + \sum_{s=0}^{n-1} (1+\rho)^{n-s} E_t(\frac{d_{t+s}}{y_{t+s}}) \tag{8}$$

Taking the limit as $n \to \infty$ gives the transversality condition

$$\lim_{n\to\infty} (1+\rho)^n \frac{b_t}{y_t} = 0 \tag{9}$$

If this holds then we obtain

$$\lim_{n\to\infty} E_t(\frac{b_{t+n}}{y_{t+n}}) = \lim_{n\to\infty} \sum_{s=1}^{n} (1+\rho)^{n-s} E_t(\frac{d_{t+s}}{y_{t+s}}) \tag{10}$$

The evolution of the debt-GDP ratio depends on that of $\frac{d_t}{y_t}$. Suppose that $\frac{d_t}{y_t}$ may be stochastic but is expected to grow at the rate λ, then

$$E_t(\frac{d_{t+s}}{y_{t+s}}) = (1+\lambda)^s \frac{d_t}{y_t} \tag{11}$$

It follows that

$$\begin{aligned} \lim_{n\to\infty} E_t(\frac{b_{t+n}}{y_{t+n}}) &= \lim_{n\to\infty} \sum_{s=1}^{n} (1+\rho)^{n-s} (1+\lambda)^s \frac{d_t}{y_t} \\ &= \lim_{n\to\infty} (1+\lambda) \left(\frac{(1+\lambda)^n - (1+\rho)^n}{\lambda - \rho} \right) \frac{d_t}{y_t} \\ &= -\frac{1}{\rho} \frac{d_t}{y_t} \qquad if\ \lambda = 0 \end{aligned} \tag{12}$$

If $\rho, \lambda < 0$ then $\lim_{n\to\infty} E_t(\frac{b_{t+n}}{y_{t+n}}) = 0$ and it will explode if $\lambda > 0$. Thus, the debt-GDP ratio will remain finite and positive if the ratio of the primary surplus to GDP $(-\frac{d_t}{y_t})$ does not explode. We note that if $\lambda < 0$ then $\frac{d_t}{y_t}$ is a stationary I(0) process and the expected, or long-run value of the debt-GDP ratio is zero. And if $\lambda = 0$ then $\frac{d_t}{y_t}$ is a non-stationary I(1) process, and hence $\frac{b_t}{y_t}$ will also be I(1). Moreover, $\frac{b_t}{y_t}$ and $\frac{d_t}{y_t}$ will be cointegrated with cointegrating vector $(1, \frac{1}{\rho})$. Fiscal policy is therefore sustainable provided $\frac{b_t}{y_t}$ does not grow over time.

Case 2: $\rho > 0$ **(unstable case)**

In this case $0 < \frac{(1+\pi)(1+\gamma)}{1+R} < 1$ and equation (7) is an unstable difference equation and hence must be solved *forwards*, not backwards as follows:

$$\begin{aligned} \frac{b_t}{y_t} &= \frac{1}{1+\rho} E_t(\frac{b_{t+1}}{y_{t+1}} - \frac{d_{t+1}}{y_{t+1}}) \\ &= (1+\rho)^{-n} E_t(\frac{b_{t+n}}{y_{t+n}}) - \sum_{s=1}^{n} (1+\rho)^{-s} E_t(\frac{d_{t+s}}{y_{t+s}}) \end{aligned} \tag{13}$$

Taking limits as $n \to \infty$ gives the transversality condition

$$\lim_{n\to\infty} (1+\rho)^{-n} E_t(\frac{b_{t+n}}{y_{t+n}}) = 0 \tag{14}$$

which implies that

$$\frac{b_t}{y_t} = \sum_{s=1}^{\infty} (1+\rho)^{-s} E_t(\frac{-d_{t+s}}{y_{t+s}}) \tag{15}$$

We note that the right-hand side of equation (15) is the expected present value of current and future primary surpluses expressed as a proportion of GDP. This condition implies that current and future surpluses will be sufficient to pay-off current debt.

Suppose once more that $\frac{d_t}{y_t}$ is expected to evolve according to equation (11) then

$$\begin{aligned} \frac{b_t}{y_t} &= \sum_{s=1}^{\infty} (1+\rho)^{-s} (1+\lambda)^s (\frac{-d_t}{y_t}) \\ &= \frac{1+\lambda}{\rho-\lambda}(\frac{-d_t}{y_t}) \qquad if \ -1 < \lambda < \rho, \ \rho > 0 \end{aligned} \tag{16}$$

Thus, provided that the current level of the debt-GDP ratio does not exceed the right-hand side, fiscal policy is sustainable and the debt-GDP ratio will grow at the rate λ, the same rate as $\frac{-d_t}{y_t}$.

If $\frac{-d_t}{y_t}$ is stationary then $-1 < \lambda < 0$ and $\frac{b_t}{y_t}$ will also be stationary. If $\lambda = 0$, so that $\frac{-d_t}{y_t}$ is I(1) then we obtain the same condition as equation (12)

$$\frac{b_t}{y_t} = \frac{1}{\rho}(\frac{-d_t}{y_t}) \tag{17}$$

implying that $\frac{b_t}{y_t}$ will be I(1) and cointegrated with $\frac{-d_t}{y_t}$.

These results can be compared with a number of well-known empirical tests for fiscal sustainability and provide some insight into the rationale behind the tests. The test of Hamilton and Flavin (1986) is based on the following version of equation (13)

$$\frac{b_t}{y_t} = A_0 (1+\rho)^{-t} - \sum_{s=1}^{\infty} (1+\rho)^{-s} E_t(\frac{d_{t+s}}{y_{t+s}})$$

except that real debt and the real primary deficit is used rather than $\frac{b_t}{y_t}$ and $\frac{d_t}{y_t}$. On the null hypothesis that the transversality condition holds $A_0 = 0$.

Trehan and Walsh (1988) propose a cointegration test for fiscal sustainability. They measure debt and the primary deficit in real terms rather than as proportions of GDP, but Hakkio and Rush (1991) employ the test expressing the variables as proportions of GDP. If the variables have unit roots and are cointegrated with cointegrating vector $(\rho, 1)$ then fiscal policy is sustainable. (Or, if government expenditures and revenues are I(1), then the cointegrating vector with debt must be $(\rho, 1, -1)$.) This result follows immediately from equations (12) and (17). Alternatively, if the cointegrating relation between debt and the primary deficit is

$$\frac{d_t}{y_t} + \alpha \frac{b_t}{y_t} = u_t$$

where u_t is I(0), then from equation 7),

$$(1+\alpha)\frac{b_t}{y_t} = (1+\rho)\frac{b_{t-1}}{y_{t-1}} + u_t$$

It follows that $\frac{b_t}{y_t}$ has a unit root if $\alpha = \rho$.

3.2 Time-varying discount rate

In practice, ρ_t will be time-varying, not constant and so these tests will in general be invalid. We therefore revert to the original budget constraint, equation (5). This may be solved forwards to obtain

$$\frac{b_t}{y_t} = E_t[(\Pi_{s=1}^{n} \frac{1}{1+\rho_{t+s}})\frac{b_{t+n}}{y_{t+n}}] - E_t[\sum_{s=1}^{n}(\Pi_{i=1}^{s} \frac{1}{1+\rho_{t+i}})\frac{d_{t+s}}{y_{t+s}}] \tag{18}$$

if

$$\delta_{t,s} = \Pi_{i=1}^{s} \frac{1}{1+\rho_{t+i}} \leq 1 \quad for\ all\ s \geq 1$$

Hence fiscal solvency depends on the transversality condition

$$\lim_{n\to\infty} E_t[(\Pi_{s=1}^{n} \frac{1}{1+\rho_{t+s}})\frac{b_{t+n}}{y_{t+n}}] = 0 \tag{19}$$

which implies that

$$\frac{b_t}{y_t} = E_t[\sum_{s=1}^{\infty}(\Pi_{i=1}^{s} \frac{1}{1+\rho_{t+i}})(\frac{-d_{t+s}}{y_{t+s}})] \tag{20}$$

Like equation (15), equation (20) says that the present value of current and future primary surpluses must be sufficient to offset current debt liabilities. The difference is that the discount rate is compounded from time-varying rates.

In order to analyse sustainability we define the variables

$$\begin{aligned} x_t &= \delta_{t,n}\frac{b_t}{y_t} \\ z_t &= \delta_{t,n}\frac{d_t}{y_t} \end{aligned}$$

We may now write equation (5) as

$$\Delta x_t = z_t$$

Fiscal sustainability now requires the transversality condition

$$\lim_{n\to\infty} E_t(x_{t+n}) = 0$$

and implies that

$$x_t = -\lim_{n\to\infty} E_t[\sum_{s=1}^{n} z_{t+s}]$$

Wilcox (1989) shows that fiscal sustainability is satisfied if x_t is a zero-mean stationary process. Wickens and Uctum (2000) prove a more general result that does not require x_t to be stationary. They show that fiscal sustainability is satisfied if z_t is a zero-mean stationary process. It then follows that x_t will be an I(1) process. Trehan and Walsh (1991) argue that fiscal policy is sustainable with a variable discount rate if the *total* deficit is stationary. This result follows

directly from equation (6). As it is a stable difference equation if nominal growth is positive, $\frac{b_t}{y_t}$ is finite (and stationary) if $\frac{D_{t+s}}{y_{t+s}}$ is stationary.

3.3 Stability and Growth Pact (SGP)

This was based on the original Maastricht conditions that $\frac{b_t}{y_t}$ must be less than 0.6 and $\frac{D_t}{y_t}$ must be less than 0.03. It can be shown that these conditions are neither necessary, nor sufficient for fiscal sustainability. Much has already written on the issue of necessity. To show insufficiency, consider equation (6) assuming that inflation and growth are constant and $\frac{b_t}{y_t}$ and $\frac{D_t}{y_t}$ are constant at these maximum values. Hence we obtain the condition

$$\begin{aligned} \frac{b}{y} &= \frac{(1+\pi)(1+\gamma)}{(1+\pi)(1+\gamma)-1}\frac{D}{y} \\ &\simeq \frac{1}{\pi+\gamma}\frac{D}{y} \end{aligned}$$

It follows, therefore, that

$$\pi+\gamma \simeq \frac{\frac{D}{y}}{\frac{b}{y}}$$

Thus, given the limits on debt and deficits specified under the SGP, the nominal rate of growth must not be less than $\frac{0.03}{0.6} \equiv 5\%$. If nominal growth were less than this then debt would rise above 60% even if the deficit limit were satisfied. Conversely, even if the deficit or debt limits were exceeded, the appropriate rate of nominal growth would still be consistent with fiscal sustainability. For example, if the deficit exceeds 3% it is still possible for the debt-GDP ratio to satisfy the 60% limit if nominal growth exceeds 5%. This shows that, in general, the SGP is neither necessary nor sufficient for fiscal sustainability.

3.4 Assessment

These measures of fiscal sustainability are of limited practicality. First, it is necessary to forecast future deficits, inflation, growth and interest rates in order to compute the present value of expected future deficits. It may sometimes be possible to use official forecasts as in Wickens and

Uctum (2000). If these are not available, or to provide an independent check, other means must be found to construct the forecasts. A structural economic model is a possibility, but has the disadvantage of embodying prior information that may prove contentious and difficult for outsiders to replicate. In view of this, in this paper we propose using a VAR to provide the forecasts. This has the merit of being easily understood and replicable.

Second, the time horizon in these tests is so distant that the tests provide an ineffective constraint on fiscal policy in the short run. Like Wickens and Uctum (2000) we therefore examine fiscal sustainability over a much shorter, finite, time horizon.

Third, and related to the second point; a government running persistent, and even large deficits, may simply claim that they expect, or will generate, offsetting surpluses at some point in the future. It is therefore desirable to be able to evaluate fiscal sustainability under alternative policies. In general, this cannot be accomplished in a VAR by simply replacing some of the equations (for example, by the new policy rules) and leaving the others unaltered, as all equations are affected. We therefore devise a valid way to do this, creating a new VAR from the old.

Fourth, since the main policy instruments are interest rates and taxes, and the present value condition is a nonlinear function of these, we recast the analysis of fiscal sustainability in the form of a model linear in logarithms. It is then straightfoward to relate the present value calculation to the VAR, and to incorporate changes to the structure of the VAR.

Fifth, by comparing the projected outcome for the debt-GDP ratio constructed in this way over any given time horizon with its initial level, we are able to provide an index of the current fiscal stance and, by altering policy rules, of any other policy stance. In this way policy comparisons may be made.

Taken together, we believe these changes to standard practice constitute a considerable advance. Moreover, as the whole procedure can easily by automated, it has the potential to become a standard desciptive statistic for the fiscal stance.

4 A log-linear approach to fiscal sustainability

4.1 The log-linearized GBC

The first step is to log-linearize the government budget constraint. As the primary deficit can take negative values, it is necessary to write the GBC in terms of total expenditures g_t and total revenues v_t both of which are strictly positive. We therefore re-write the GBC, equation (3), as

$$\frac{b_t}{y_t} = \frac{g_t}{y_t} - \frac{v_t}{y_t} + (1+\rho_t)b_{t-1}$$

where

$$\frac{v_t}{y_t} = \frac{T_t}{y_t} + \frac{m_t}{y_t} - \frac{1}{(1+\pi_t)(1+\gamma_t)}\frac{m_{t-1}}{y_{t-1}}$$

Next we approximate the GBC about the steady-state solution in which we assume that all variables are constant. The steady-state solution to the GBC therefore satisfies

$$\rho\frac{b}{y} = -\frac{g}{y} + \frac{v}{y}$$

The GBC may be re-written as

$$f(x_t) = \exp\left[\ln\frac{b_t}{y_t}\right] - \exp\left[\ln\frac{g_t}{y_t}\right] + \exp\left[\ln\frac{v_t}{y_t}\right] - \exp\left[\ln(1+\rho_t) + \ln\frac{b_{t-1}}{y_{t-1}}\right] = 0$$

Noting that a first-order Taylor series approximation to $h(x_t) = \exp[\ln x_t]$ about $\ln x$ is

$$h(x_t) = x[1 + (\ln x_t - \ln x)]$$

a log-linear approximation to the GBC is given by

$$\begin{aligned} \ln\frac{b_t}{y_t} &\simeq c + \frac{g}{b}\ln\frac{g_t}{y_t} - \frac{v}{b}\ln\frac{v_t}{y_t} + (1+\rho)\ln(1+\rho_t) + (1+\rho)\ln\frac{b_{t-1}}{y_{t-1}} \qquad (21) \\ c &= -\rho\ln\frac{b}{y} - \frac{g}{b}\ln\frac{g}{y} + \frac{v}{b}\ln\frac{v}{y} - (1+\rho)\ln(1+\rho) \end{aligned}$$

As $\ln(1+\rho_t) \simeq \rho_t$, in effect the discount rate is an additional variable in the equation. Thus, by employing a log-linear transformation of the GBC, we may analyse fiscal sustainability when the

deficit and discount rate are time-varying using, once more, a constant coefficient linear difference equation.

Whether the difference equation is a stable or unstable depends on the sign of ρ. Assuming that $\rho > 0$, we solve the equation forwards to obtain

$$\ln \frac{b_t}{y_t} = (1+\rho)^{-n} E_t(\ln \frac{b_{t+n}}{y_{t+n}}) - \sum_{s=1}^{n} (1+\rho)^{-s} E_t(k_{t+s}) \tag{22}$$

$$k_t = c + \frac{g}{b} \ln \frac{g_t}{y_t} - \frac{v}{b} \ln \frac{v_t}{y_t} + (1+\rho) \ln(1+\rho_t) \tag{23}$$

where k_t is, in effect, the logarithmic equivalent of the primary deficit. The transversality condition is therefore

$$\lim_{n\to\infty} (1+\rho)^{-n} E_t(\ln \frac{b_{t+n}}{y_{t+n}}) = 0 \tag{24}$$

which implies that

$$\ln \frac{b_t}{y_t} = -\sum_{s=1}^{\infty} (1+\rho)^{-s} E_t(k_{t+s}) \tag{25}$$

If k_t is stationary then $\ln \frac{b_t}{y_t}$, and hence $\frac{b_t}{y_t}$, remains finite and stationary. This may occur due to the individual terms of k_t being stationary, or due to some being I(1) but being cointegrated with the approporiate contegrating vector. If k_t and each component of k_t are I(1) then, if they also cointegrated with cointegrating vector given by the coefficients in the definition of c, then fiscal sustainability is still satisfied.

4.2 Fiscal sustainability over a finite time horizon

So far we have discussed fiscal sustainability over an infinite time horizon. This could remove much of the immediate relevance of the issue as fiscal correction could be deferred to a distant future. Using a finite time horizon would avoid this. Suppose that the time horizon is n periods. Equations (8), (13) and (18) show the evolution of $\frac{b_t}{y_t}$ over this horizon and equation (22) shows the evolution of $\ln \frac{b_t}{y_t}$. Wickens and Uctum (2000) argued that such equations can be used to determine whether projected values of the primary surplus and discount rate are consistent with

the desired change in the debt-GDP ratio, or in the discounted debt-GDP ratio. We are now able to improve on this by endogenising the primary surplus and the discount rate forecasts.

4.3 An index of sustainability

The use of an index of sustainability was initially proposed by Blanchard, Chouraqui, Hagemann, and Sartor (1990) and Buiter, Corsetti and Roubini (1993). Their indices are based on a comparison of the current debt-GDP ratio and that n periods ahead with given fixed values of the deficit and discount rate. We generalize this, allowing the deficit and discount rate to be time-varying and endogenous, and the target level of the debt-GDP ratio to be a choice variable.

Equation (25) is the logarithmic equivalent of the earlier result that fiscal policy is sustainable if matched by the present value of future primary deficits. Equation (22) can be re-written as

$$(1+\rho)^{-n} E_t(\ln \frac{b_{t+n}}{y_{t+n}}) - \ln \frac{b_t}{y_t} = \sum_{s=1}^{n} (1+\rho)^{-s} E_t(k_{t+s})$$

which can be interpreted as determining in logarithmic terms the present value of primary deficits required to achieve an expected change in discounted debt. If we replace $E_t[\ln \frac{b_{t+n}}{y_{t+n}}]$ by a target level $\ln(\frac{b_{t+n}}{y_{t+n}})^*$ then we can determine whether future values of k_t are consistent with satisfying a particular target change in discounted debt given by

$$(1+\rho)^{-n} \ln(\frac{b_{t+n}}{y_{t+n}})^* - \ln \frac{b_t}{y_t} = \sum_{s=1}^{n} (1+\rho)^{-s} E_t(k_{t+s}) \tag{26}$$

A measure of sustainability may be constructed by comparing the two sides of equation (26). If, for example, the aim is to decrease discounted debt then the left-hand side will be negative and the right-hand side gives the present value of the primary surplus required to achieve this reduction in debt. An increase in discounted debt requires a lower primary surplus.

Consider, therefore, the following measure of fiscal sustainability for an $n-$period horizon:

$$FS(t,n) = [(1+\rho)^{-n} \ln(\frac{b_{t+n}}{y_{t+n}})^* - \ln \frac{b_t}{y_t}] - \sum_{s=1}^{n} (1+\rho)^{-s} E_t(k_{t+s}) \gtreqless 0$$

If $FS(t,n) = 0$ then the current fiscal stance is forecast to achieve the debt target. If $FS(t,n) > 0$ then the present value of the primary surplus will exceed the desired change in discounted debt. The current fiscal stance is then sustainable. And if $FS(t,n) < 0$ the forecasted present value of the primary surplus is insufficient to achieve the desired change in the debt-GDP ratio. In this case the current fiscal stance would not be sustainable. Consequently, we propose the following index of fiscal sustainability, denoted by $FSI(t,n)$,

$$FSI(t,n) = \exp[FS(t,n)]$$

where $FS(n) \gtreqless 0$ is equivalent to $FSI(n) \gtreqless 1$.

We have constructed the index so that it provides a comparison between the desired change in discounted debt and the present value of primary surpluses over a given horizon. However, it also provides a comparison with the existing level of the debt-GDP ratio since

$$\begin{aligned} FSI(t,n) &= \frac{K_{t,n}}{b_t/y_t} \\ \ln K_{t,n} &= (1+\rho)^{-n} \ln(\frac{b_{t+n}}{y_{t+n}})^* - \sum_{s=1}^{n} (1+\rho)^{-s} E_t(k_{t+s}) \end{aligned}$$

As $n \to \infty$ the first term in $\ln K_{t,n}$ tends to zero and the index can be interpreted as comparing the the existing level of the debt-GDP ratio with the resources to pay it off.

In practice, the special case considered by Buiter and Blanchard of maintaining a constant debt-GDP ratio over the planning horizon will usually be of most interest. In this case

$$FS(t,n) = [(1+\rho)^{-n} - 1] \ln \frac{b_t}{y_t} - \sum_{s=1}^{n} (1+\rho)^{-s} E_t(k_{t+s}) \gtreqless 0$$

The index of fiscal sustainability then becomes

$$\begin{aligned} FSI(t,n) &= \exp[FS(t,n)] \\ &= \frac{K_{t,n}}{b_t/y_t} \\ \ln K_{t,n} &= (1+\rho)^{-n} \ln \frac{b_t}{y_t} - \sum_{s=1}^{n} (1+\rho)^{-s} E_t(k_{t+s}) \end{aligned} \tag{27}$$

This is the case we consider below.

We note that in the special case of achieving a constant debt-GDP ratio over an $n-$period horizon equation (22) can also be written as

$$\begin{aligned}
\ln \frac{b_t}{y_t} &= (1+\rho)^{-n} \ln \frac{b_t}{y_t} - \sum_{s=1}^{n} (1+\rho)^{-s} E_t(k_{t+s}) \\
&= -\frac{1}{1-(1+\rho)^{-n}} \sum_{s=1}^{n} (1+\rho)^{-s} E_t(k_{t+s}) \\
&\simeq -\frac{1}{n\rho} \sum_{s=1}^{n} (1+\rho)^{-s} E_t(k_{t+s})
\end{aligned}$$

In this case the index could be calculated as

$$\begin{aligned}
FSI(t,n) &= \frac{K^*_{t,n}}{b_t/y_t} \\
\ln K^*_{t,n} &= -\frac{1}{1-(1+\rho)^{-n}} \sum_{s=1}^{n} (1+\rho)^{-s} E_t(k_{t+s})
\end{aligned}$$

where the numerator is now proportional to the present value of primary surpluses. We use the previous method of calculating the index, equation (27).

4.4 Forecasting the fiscal variables

We require forecasts of the vector

$$z_t = \left(ln\frac{b_t}{y_t}, ln\frac{g_t}{y_t}, ln\frac{v_t}{y_t}, ln\left(1+\rho_t\right), ln(1+\gamma_t), ln\left(1+\pi_t\right) \right)$$

Later we add two variables to this vector. This is explained below. We propose the use of a VAR(p) to obtain these forecasts. This is a simple forecasting scheme that is easily implemented and is theory free. Given the VAR

$$\mathbf{z}_t = \mathbf{A}_0 + \sum_{i=1}^{p} \mathbf{A_i z}_{t-i} + \mathbf{e}_t, \tag{28}$$

where $\mathbf{e}_t \sim i.i.d.[\mathbf{0}, \Sigma]$. The vector of variables $\mathbf{z}_t$ may be I(0) or I(1). For forecasting purposes it is unnecessary to take account any non-stationarity or cointegration among the variables. Equally, if cointegration exists, a cointegrated VAR could be estimated instead of a levels VAR and the

cointegrated VAR could then be written in levels to obtain (28). We also note that to improve the forecasts $\mathbf{z}_t$ may contain additional variables to those that appear in the budget constraint.

$n-$period ahead forecasts may be obtained using the companion form

$$\mathbf{Z}_t = \mathbf{B}_0 + \mathbf{B}\mathbf{Z}_{t-1} + \mathbf{u}_t.$$

where $\mathbf{Z}_t'$=$[\mathbf{z}_t', \mathbf{z}_{t-1}', ..., \mathbf{z}_{t-p+1}]$, $\mathbf{u}_t'$=$[\mathbf{e}_t', \mathbf{0}, ..., \mathbf{0}]$, $\mathbf{B}_0' = [\mathbf{A}_0', 0, ..., 0]$ and

$$\mathbf{B} = \begin{bmatrix} \mathbf{A}_1 & \mathbf{A}_2 & . & . & \mathbf{A}_{p-1} \\ \mathbf{0} & \mathbf{I} & \mathbf{0} & . & . \\ \mathbf{0} & . & \mathbf{I} & \mathbf{0} & . \\ . & . & . & . & . \\ . & . & \mathbf{0} & \mathbf{I} & \mathbf{0} \end{bmatrix}$$

The forecast of $\mathbf{Z}_{t+n}$ is

$$E_t[\mathbf{Z}_{t+s}] = \sum_{i=0}^{s-1} \mathbf{B}^i \mathbf{B}_0 + \mathbf{B}^s \mathbf{Z}_t$$

Expressing k_t as the following linear function of $\mathbf{z}_t$

$$k_t = a + \boldsymbol{\beta}' \mathbf{z}_t$$

and defining the selection matrix $\mathbf{S}$ =$[\mathbf{I}, \mathbf{0}, \mathbf{0}, .., \mathbf{0}]$ such that

$$\mathbf{z}_t = \mathbf{S}\mathbf{Z}_t$$

we obtain

$$\begin{aligned} FS(t,n) &= \ln K_{t,n}^* - \ln \frac{b_t}{y_t} \\ &= -\frac{1}{1-(1+\rho)^{-n}} \sum_{s=1}^{n} \{(1+\rho)^{-s} \left[a + \boldsymbol{\beta}' \mathbf{S} (\sum_{i=0}^{s-1} \mathbf{B}^i \mathbf{B}_0 + \mathbf{B}^s \mathbf{Z}_t)\right]\} - \ln \frac{b_t}{y_t} \end{aligned}$$

As the last term $\ln \frac{b_t}{y_t}$ is also a linear function of $\mathbf{Z}_t$, $FS(t,n)$ could just be written as

$$FS(t,n) = a_n + \mathbf{b}_n' \mathbf{Z}_t$$

where a_n is a scalar dependent on the time horizon and $\mathbf{b}_n$ is a vector. This emphasizes that $FS(t,n)$ is predetermined, being based solely on information available at time t.

To implement this in practice it will be necessary to estimate a_n and $\mathbf{b}_n$ from the VAR estimates. The choice of ρ and c could be based, for example, on the average values in the sample, their time t values or their average values over the forecast period. A time series for $FS(t,n)$ could be calculated from the sample either using all of the sample observations to estimate the VAR, or recursively using only observations up to period t.

5 Evaluating fiscal sustainability under alternative policy rules

Finding that fiscal sustainability is not satisfied may prompt a change of policy. But before adopting a new policy it would be helpful to estimate its likely effect on fiscal sustainability. Moreover, it is desirable to do this using the same theory-free VAR framework. The problem is that, in general, a change of policy would alter the VAR. As a result, the VAR based on the historic data would be invalid for carrying out the evaluation of the new policy. In this section we show how this problem may be overcome, and how a VAR may still be used to analyze fiscal sustainability. The methodology is based on Wickens (2004, 2005).

Consider the VAR, equation (28), which we re-write as

$$z_t = A(L) z_{t-1} + e_t$$

where for convenience we set $A_0 = 0$, $A(L) = \sum_{i=1}^{p} \mathbf{A_i L}^i$ and L is the lag operator such that $z_{t-s} = L^s z_t$. Suppose that the policy change consists of determining one or more policy variables z_{2t} according to new rules and these rules can be expressed as linear functions of z_t, and its lagged values, where $z_t' = (z_{1t}', z_{2t}')$. We wish to form a new VAR based on replacing the old equations

for z_{2t}. But we cannot simply substitute the new equations for the old as this would also alter the correlation structure of the disturbances of the VAR model.

If we partition the original disturbances e_t conformably as $e_t' = (e_{1t}', e_{2t}')$, the problem can be reformulated as being due to e_{1t} and e_{2t} being contemporaneously correlated. This implies that if the VAR equations for the policy instruments are changed then the correlation structure of the VAR errors will change too. If the original errors were uncorrelated there would be no problem. We therefore seek a way of replacing the equation for the policy instruments so that the correlation structure of the VAR errors is unaffected. This can be accomplished if we transform the VAR equations for z_{1t} into a VAR that is conditional on the current value of the policy instrument.

To do this we define the linear function

$$e_{1t} = \varepsilon_t + Ge_{2t}$$

where $E(\varepsilon_t e_{2t}) = 0$, i.e. ε_t is the component of e_{1t} that is uncorrelated with e_{2t}. As result

$$e_t = \begin{bmatrix} e_{1t} \\ e_{2t} \end{bmatrix} = \begin{bmatrix} I & G \\ 0 & I \end{bmatrix} \begin{bmatrix} \varepsilon_t \\ e_{2t} \end{bmatrix}$$

In other words, we are applying a block Choleski decomposition to the the original VAR residuals.

We may derive G from Σ, the covariance matrix of the VAR errors as follows. G is defined such that

$$\begin{aligned} \Sigma &= E[e_t e_t'] \\ &= \begin{bmatrix} I & G \\ 0 & I \end{bmatrix} \begin{bmatrix} \Sigma_{\varepsilon\varepsilon} & 0 \\ 0 & \Sigma_{22} \end{bmatrix} \begin{bmatrix} I & 0 \\ G' & I \end{bmatrix} \\ &= \begin{bmatrix} \Sigma_{\varepsilon\varepsilon} + G\Sigma_{22}G' & G\Sigma_{22} \\ \Sigma_{22}G' & \Sigma_{22} \end{bmatrix} \end{aligned}$$

where $E[\varepsilon_t \varepsilon_t'] = \Sigma_{\varepsilon\varepsilon}$. Hence,

$$G = \Sigma_{12}\Sigma_{22}^{-1}$$

Thus G can easily be estimated from the covariance matrix of VAR residuals.

Denoting

$$H = \begin{bmatrix} I & G \\ 0 & I \end{bmatrix}$$

we pre-multiply the VAR by

$$H^{-1} \begin{bmatrix} I & -G \\ 0 & I \end{bmatrix}$$

with the result that the disturbances associated with z_{1t} are uncorrelated with those of z_{2t}

$$H^{-1} z_t = H^{-1} A(L) z_{t-1} + H^{-1} e_t$$

Partitioning $A(L)$ conformably,

$$\begin{aligned} z_{1t} &= [A_{11}(L) - GA_{21}(L)] z_{1,t-1} + G z_{2t} + [A_{12}(L) - GA_{22}(L)] z_{2,t-1} + \varepsilon_t \\ z_{2t} &= A_{21}(L) z_{1,t-1} + A_{22}(L) z_{2t} + e_{2t} \end{aligned}$$

The fact that z_{2t} appears in this z_{1t} equation is a reflection of the fact that the correlation between e_{1t} and e_{2t} implies that z_{2t} affects z_{1t} contemporaneously. The new equation for z_{1t} can be described as a conditional VAR as it is a VAR in which z_{2t} is exogenous.

Only at this stage do we replace the equation for z_{2t} by the new policy rule. Suppose this takes the general form

$$F z_{1t} + z_{2t} = A^*_{21}(L) z_{1,t-1} + A^*_{22}(L) z_{2t} + e^*_{2t}$$

A Taylor rule for inflation, for example, is non-stochastic and has no lagged dynamics, and so $A^*_{21}(L)$, $A^*_{22}(L)$ and e^*_{2t} would all be zero.

The complete model is now

$$\begin{bmatrix} I & -G \\ F & I \end{bmatrix} \begin{bmatrix} z_{1t} \\ z_{2t} \end{bmatrix} = \begin{bmatrix} I & -G \\ 0 & I \end{bmatrix} \begin{bmatrix} A_{11}(L) & A_{12}(L) \\ A^*_{21}(L) & A^*_{22}(L) \end{bmatrix} \begin{bmatrix} z_{1,t-1} \\ z_{2,t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_t \\ e^*_{2t} \end{bmatrix}$$

This can be written as the VAR

$$\begin{bmatrix} z_{1t} \\ z_{2t} \end{bmatrix} = \begin{bmatrix} I & -G \\ F & I \end{bmatrix}^{-1} \begin{bmatrix} I & -G \\ 0 & I \end{bmatrix} \begin{bmatrix} A_{11}(L) & A_{12}(L) \\ A_{21}^*(L) & A_{22}^*(L) \end{bmatrix} \begin{bmatrix} z_{1,t-1} \\ z_{2,t-1} \end{bmatrix}$$
$$+ \begin{bmatrix} I & -G \\ F & I \end{bmatrix}^{-1} \begin{bmatrix} \varepsilon_t \\ e_{2t}^* \end{bmatrix}$$

We have now constructed a new VAR that can be used for policy analysis. We can, for example, perform impulse response analysis on this VAR in the usual way. We can forecast under the policy change, and we can carry out counter-factual analysis examining how the economy would have behaved in the past under a change of policy, see Wickens (2005).

We note that the response of z_{1t} to ε_t in the new VAR must take account of the fact that we have carried out a transformation of the disturbances. Thus in the original VAR $\frac{\partial z_{1t}}{\partial \varepsilon_t} = I$ but in the new VAR $\frac{\partial z_{1t}}{\partial \varepsilon_t} = I - F(I + FG)^{-1}G$. Thus under the new policy rule the response of z_{1t} to ε_t is different. We also note that now z_{2t} will in general respond to ε_t.

We have not discussed whether the variables are stationary or non-stationary, and if difference stationary whether cointegrating relations exist. Such distinctions are not of much relevance in using a VAR purely for forecasting. If the data are I(1) and cointegrated then a VAR in levels will implicitly, and hence automatically, take account of any cointegration. Nonetheless, it would be straightforward to start by estimating a cointegrated VAR, re-write this in levels and then proceed as described above using a VAR in levels.

To summarize, if we wish to analyze the effect of a change in policy rule within a VAR framework, we construct an estimate of the VAR of the response variables that is conditional on the policy instrument and then combine this with the new policy rule to form a complete system. The conditional VAR can be constructed as a linear transformation of the original VAR. The transformation matrix is estimated from the covariance matrix of the original VAR. We can then derive a new VAR from the completed model. In Wickens (2005) it is shown how to derive the policy rule optimally from the VAR.

It is straightfoward to apply this to fiscal sustainability. In principle, we simply construct the new VAR as described and then calculate $FS(t,n)$ as before using the new VAR. In practice, there is one further problem. We propose to examine the effect on fiscal sustainability of using a monetary rule. The policy instrument for this is the short rate. R_t is not, however, the short rate, but is the effective rate on total government debt. Total debt consists of bonds of different maturities and so the effective rate of return is an implicit weighted average of rates on each maturity, weighted by the number of bonds issued at each maturity. We therefore include the nominal short rate rs_t as an additional variable in the VAR. Further, to help forecast R_t we also include a nominal long rate rl_t in the VAR. The long rate is, of course, affected by the short rate via the term structure of interest rates. As a result, we modify the definition of z_t to be

$$z_t = \left(ln\frac{b_t}{y_t}, ln\frac{g_t}{y_t}, ln\frac{v_t}{y_t}, ln\left(1+\rho_t\right), ln(1+\gamma_t), ln\left(1+\pi_t\right), ln\left(1+rl_t\right), ln\left(1+rs_t\right) \right)$$

6 Empirical results

6.1 The United states

The data for the US are for the period are quarterly for the period 1960.1 to 2005.4. The data sources and the construction of the variables are described in the Appendix. We note that debt is measured as net liabilities. This is different from the Maastricht definition of debt but, given the definitions of the other variables, is consistent with the government budget constraint. Figure 1 gives a plot of eight key variables: $\frac{b_t}{y_t}, \frac{g_t}{y_t}, \frac{v_t}{y_t}, \frac{g_t - v_t}{y_t}, R_t, \pi_t, \gamma_t, \rho_t$. The first four variables are expressed as percentages of GDP and the last four are annualised percentages.

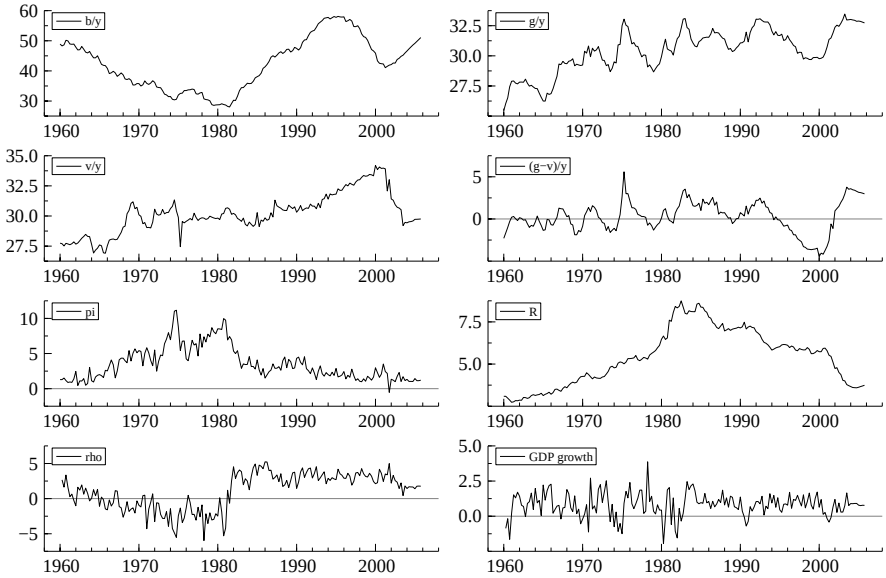

Figure 1: US data

In Table 1 we report Augmented Dickey-Fuller tests for these variables using up to 6 lags. We conclude that we cannot reject a unit root for any variable other than the real growth rate.

Table 1

Augmented Dickey-Fuller tests

D-Lag	Variable						
	$\ln \frac{b}{y}$	$\ln \frac{g}{y}$	$\ln \frac{v}{y}$	$\ln(1+R)$	$\ln(1+pi)$	ρ	$\Delta \ln y_t$
6	-1.280	-2.305	-2.127	-1.656	-1.984	-1.515	-5.347**
5	-1.734	-2.341	-2.089	-1.749	-2.031	-1.633	-5.415**
4	-2.156	-2.462	-2.076	-1.750	-1.927	-1.774	-5.947**
3	-1.197	-2.836	-2.062	-1.496	-1.704	-1.638	-5.481**
2	-0.7304	-2.511	-2.102	-1.324	-1.896	-2.082	-6.262**
1	-0.8553	-1.996	-1.952	-1.312	-2.375	-2.628	-6.882**
0	-0.4945	-1.824	-2.316	-1.270	-3.098*	-3.825**	-9.950**

Note: * denotes significance at the 5% level and ** denotes significance at the 10% level.

As we are using the VAR only for forecasting we estimate a VAR in levels of the variables and ignore any possible cointegration arising from the variables that have unit roots. Under sustainable fiscal policies we would expect to find cointegration, and the cointegrating vector would just be the long-run budget constraint. For space reasons we do not report the VAR estimates, but we note that a lag of 6 produces serially uncorrelated residuals.

In calculating the present values we require values for $\frac{v}{b}$, $\frac{g}{b}$ and ρ. We estimate b, g and ρ using their sample averages. Table 2 gives the average values for Germany, the UK and the US.

Table 2

	b	g	v	ρ
Germany	0.290	0.447	0.459	0.041
United Kingdom	0.352	0.405	0.435	0.086
United States	0.423	0.308	0.331	0.054

Note: b, g and ρ are sample averages, v is constructed from the steady-state equation $v = g + \rho b$

We examine fiscal sustainability based a constant target debt-GDP ratio for three horizons: one-year, two-years and five-years ahead. For each horizon we present four figures. Figures 2.1, 2.2 and 2.5 are plots of the fiscal sustainability index, $FSI(n)$. Figures 3-5 give various breakdowns of the index into its component parts. Thus, Figures 3.1, 3.2 and 3.3 are plots of $ln\frac{b_t}{y_t}$ and the forecast logarithm of the present value of current and future primary surpluses, $\ln K_{t,n}$, which we denote in the graph by $EPVGBC(n)$. There are three components to $FS(t,n)$: the desired change in discounted debt $PVdb(n)$, the present value of the primary surplus $PVs(n)$ and the term for the discount factor, $PVrho(n)$. These are plotted in Figures 4.1, 4.2 and 4.5. Finally, in Figures 5.1, 5.2 and 5.5 we plot the two components of $PVs(n)$. These are the present value of revenues $PVv(n)$ and of expenditures $PVg(n)$.

(i) One-year horizon

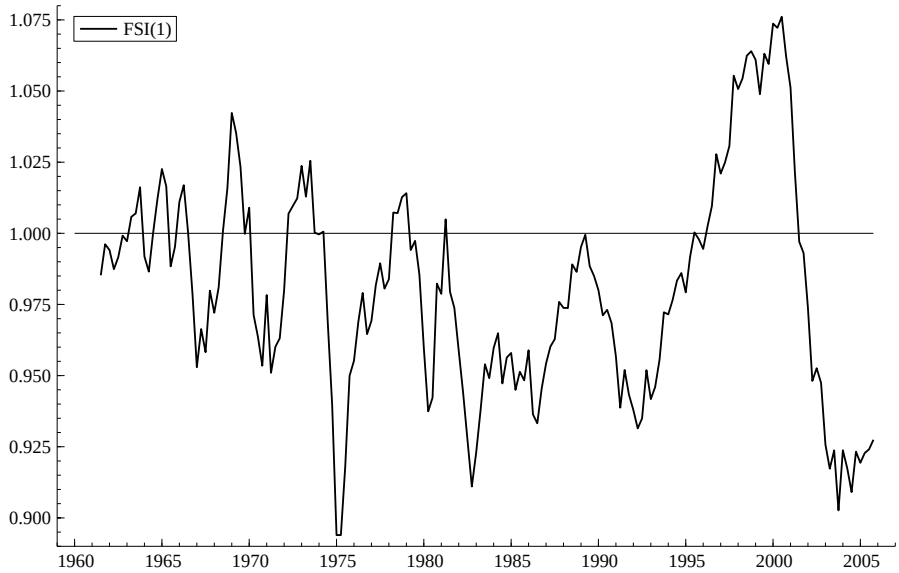

Figure 2.1: US FSI(1).

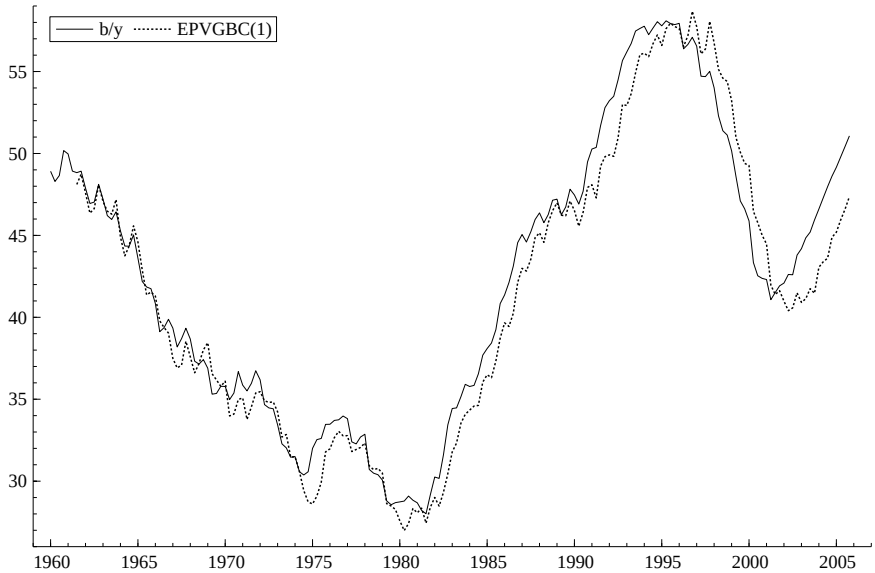

Figure 3.1: US b/y and exp[PVGBC(1)].

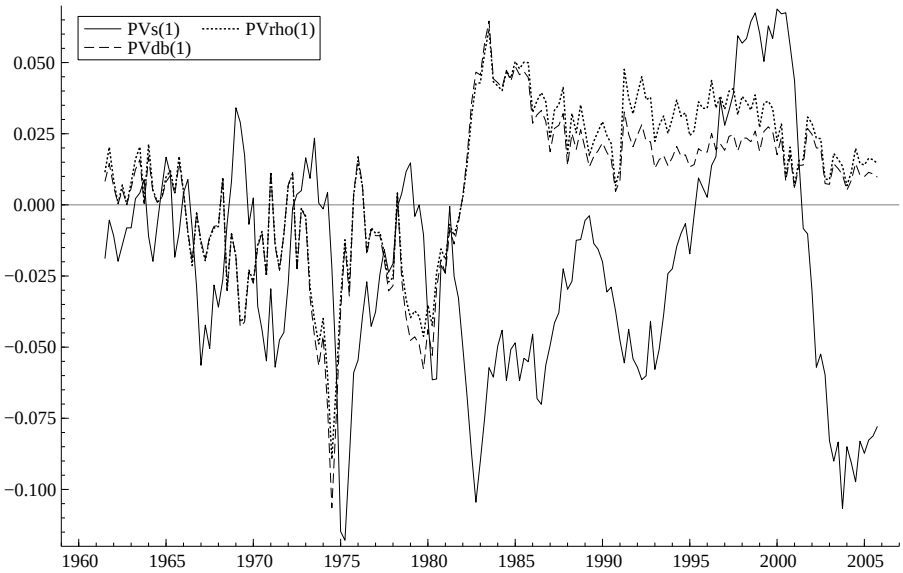

Figure 4.1: US PVs(1), PVdb(1) and PVrho(1).

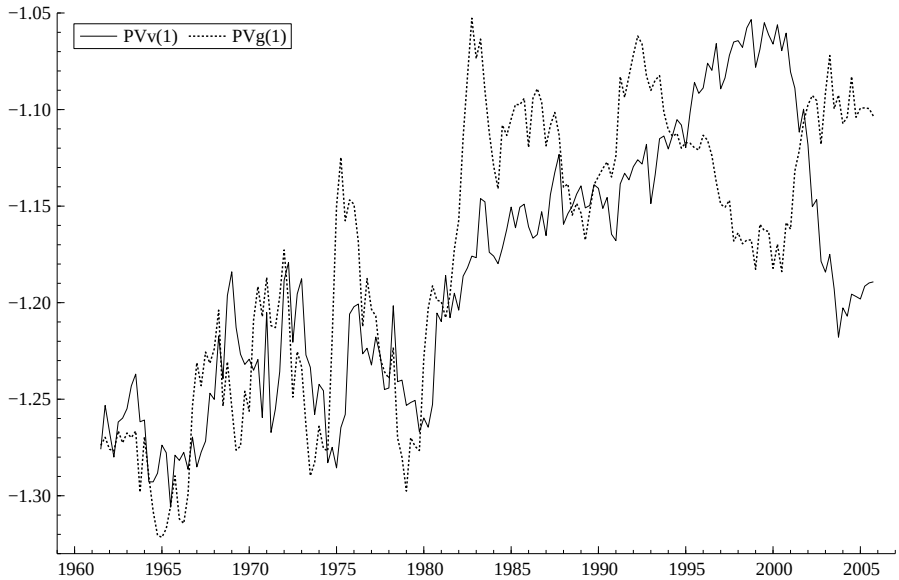

Figure 5.1: US PVv(1) and PVg(1).

(ii) Two year horizon

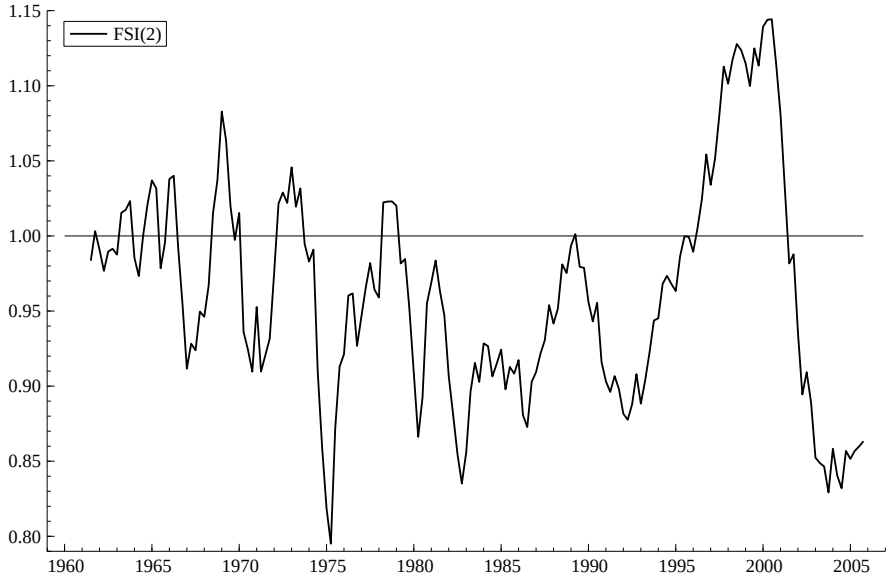

Figure 2.2: US FSI(2).

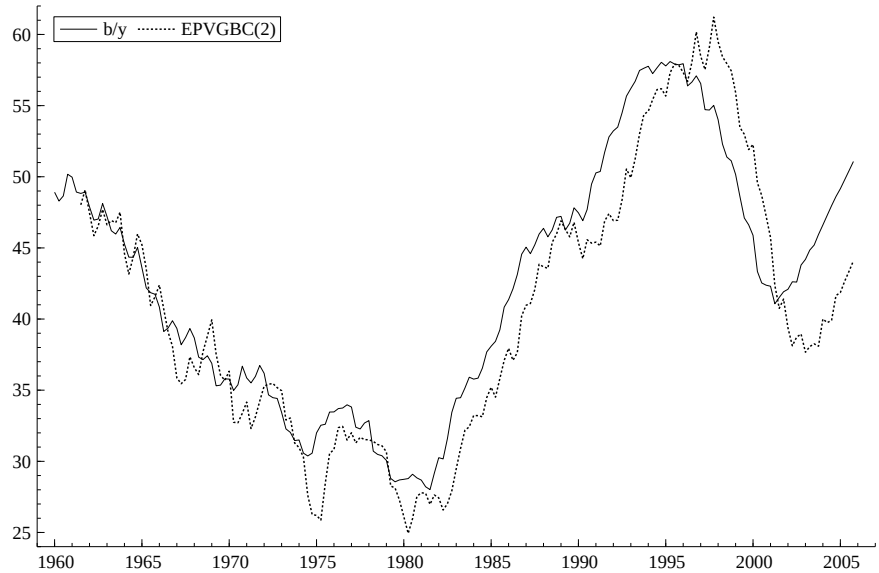

Figure 3.2: US b/y and exp[PVGBC(2)].

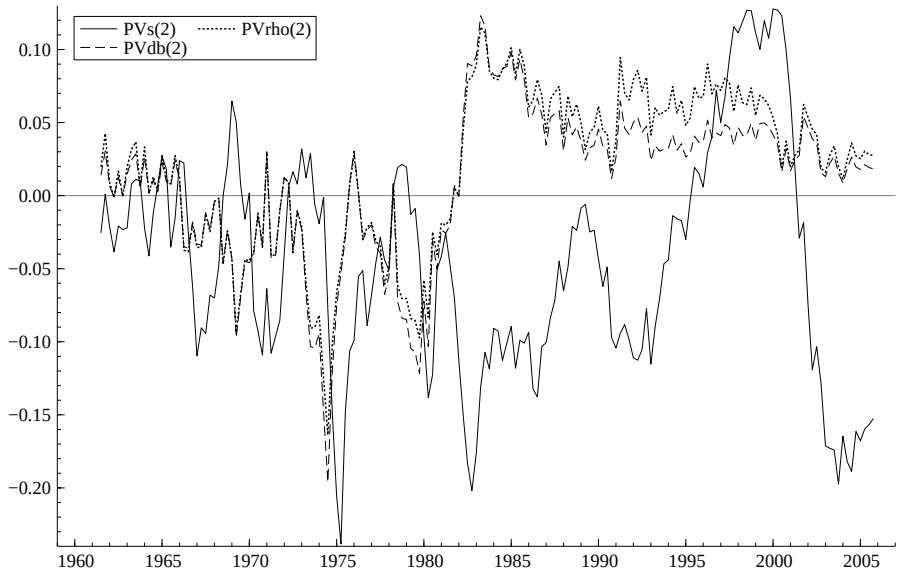

Figure 4.2: US PVs(2), PVdb(2) and PVrho(2).

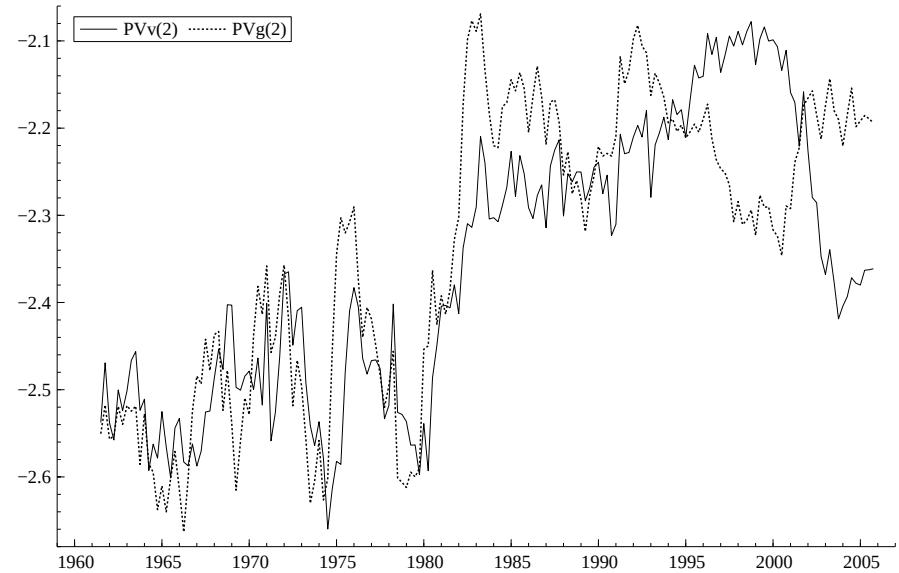

Figure 5.2: US PVv(2) and PVg(2).

(iii) Five-year horizon

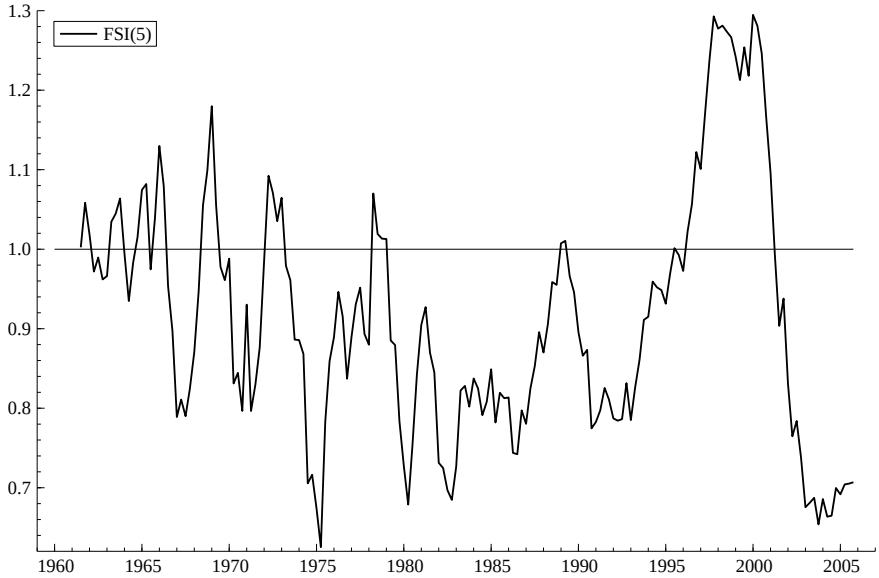

Figure 2.5: US FSI(5).

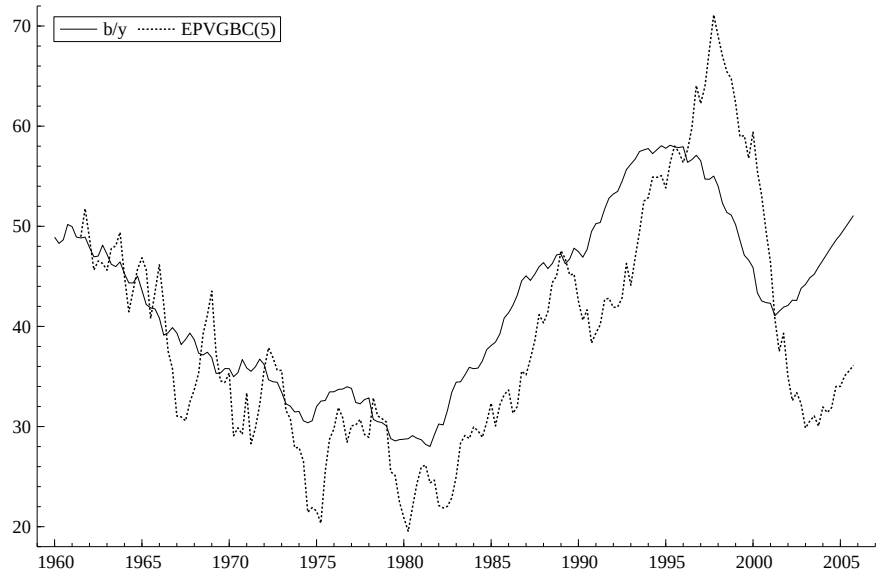

Figure 3.5: US b/y and exp[PVGBC(5)].

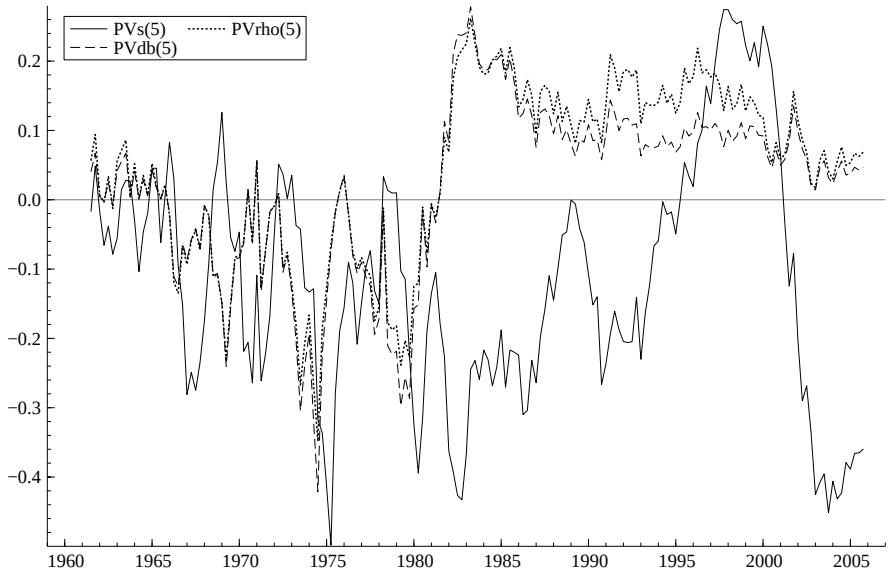

Figure 4.5: US PVs(5), PVdb(5) and PVrho(5).

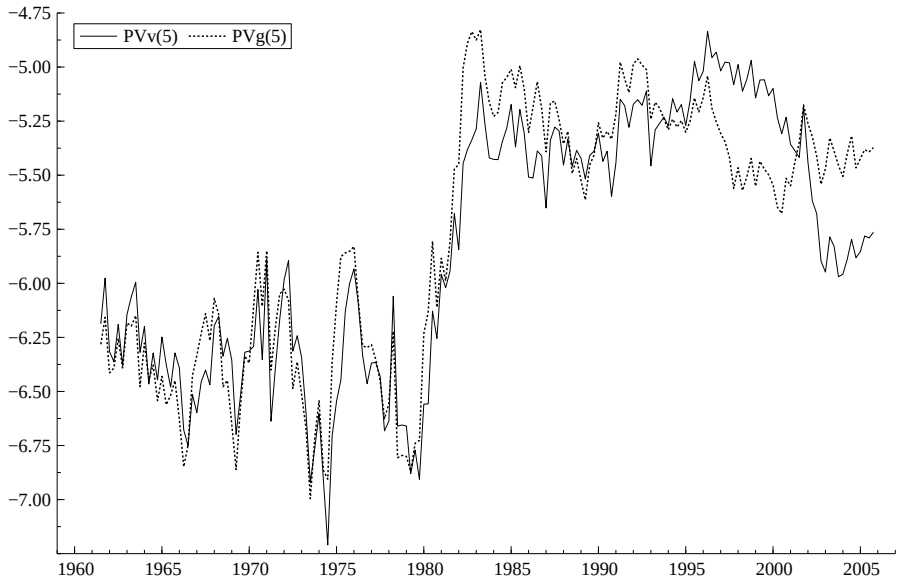

Figure 5.5: US PVv(5) and PVg(5).

We observe that $FSI(n)$, the fiscal sustainability index, exceeds unity for any length of time only during 1990's. In the other periods it is either roughly equal to unity (also implying that

the fiscal stance is sustainable) or less than unity (implying it is unsustainable). From 2001 the FSI strongly indicates non-sustainability at each horizon. The FSI is also less than unity for the period ending in 1989. The start date of this period depends on the time horizon. For one-year and two-year horizons it is similar, consisting of most of the 1980's, but for the five-year horizon it extends back through the 1970's, almost to 1965. Thus the 1990's marked a period of fiscal recovery which ended in around 2000.

Decomposing the index into its components, we find that $FSI < 1$ for the period 1979-1994 when the debt-GDP ratio rose substantially. We also find that variations in the present value of forecast primary surpluses are the main determinant of fluctuations in the index. The change in debt target and the discount factor nearly offset each other.

The present values for expenditures and revenues are similar before 1995 but are different thereafter. In the period 1995-2001 the present value of revenues exceed those of expenditures thereby producing a fiscal recovery. After 2001 the present value of expenditures exceed those of revenues. This fiscal deterioration was due to a combination of rising expenditures and sharply falling revenues. Fluctuations in the discount rate make an additional, but not large, contribution.

To summarize, there is clear evidence of a break in US fiscal policy from 2001 that has made the fiscal stance unsustainable no matter the horizon over which we look. This was due to a combination of a rising present value of expenditures and of sharply falling revenues. There have been previous periods when the fiscal stance was also unsustainable, most notably from 1979-1994. This was not fully corrected until the period 1995-2000 when the present value of expenditures was reduced and was much lower than that of revenues.

6.2 The United Kingdom

The data are annual for the period 1970 to 2005 and are plotted in Figure 6.

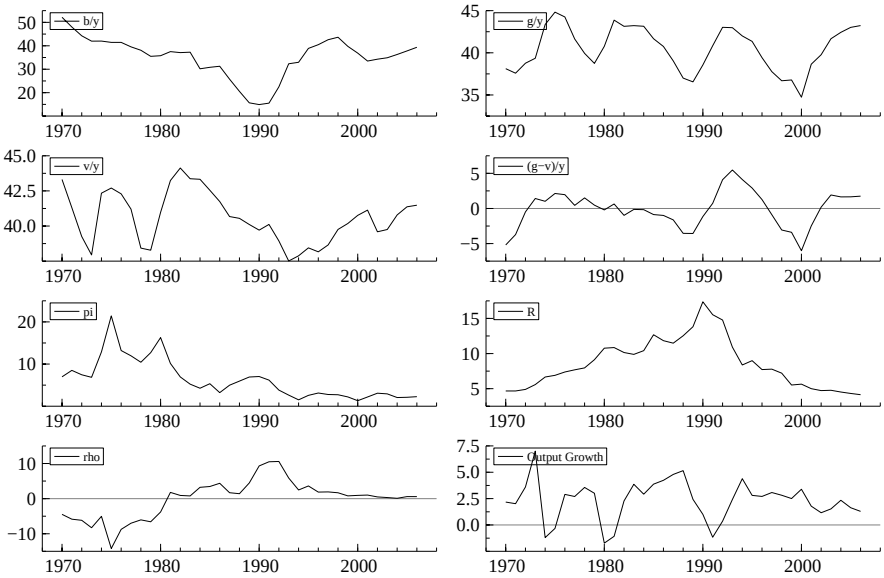

Figure 6: UK data

Augmented Dickey-Fuller tests are reported in Table 3. We conclude from these results that $\ln \frac{g}{y}$ and the real growth rate are stationary variables.

Table 3

Augmented Dickey-Fuller tests (sub-sample 1970-2005)

D-Lag	Variable						
	$\ln \frac{b}{y}$	$\ln \frac{g}{y}$	$\ln \frac{v}{y}$	$\ln(1+R)$	$\ln(1+pi)$	ρ	$\Delta \ln y_t$
2	-2.349	-4.184**	-2.416	-1.503	-1.267	-1.620	-3.600*
1	-2.432	-3.390*	-3.194*	-1.362	-1.768	-1.582	-4.595**
0	-1.400	-1.996	-2.250	-0.9936	-1.691	-1.757	-3.981**

Note: * denotes significance at the 5% level and ** denotes significance at the 10% level.

The results on fiscal sustainability are reported in Figures 7-10 for a one-year horizon.

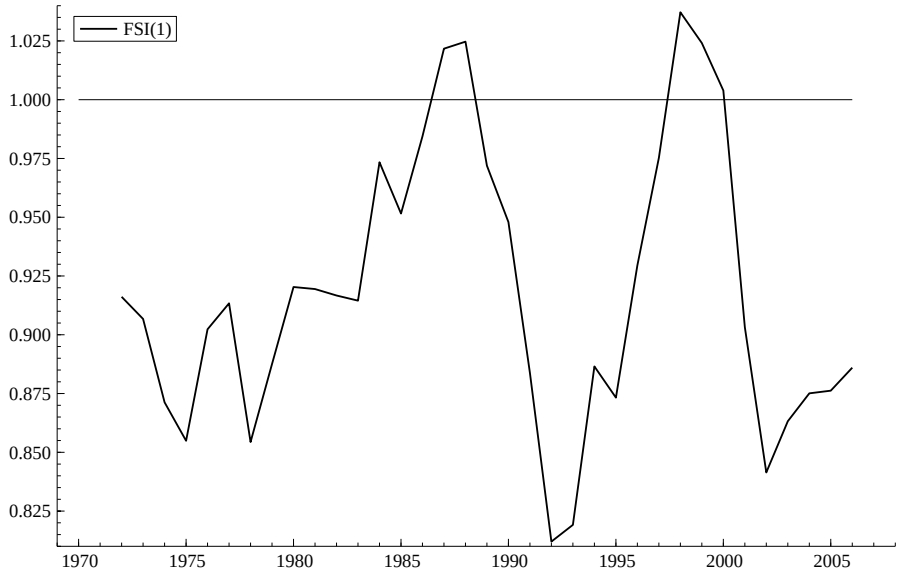

Figure 7: UK FSI(1).

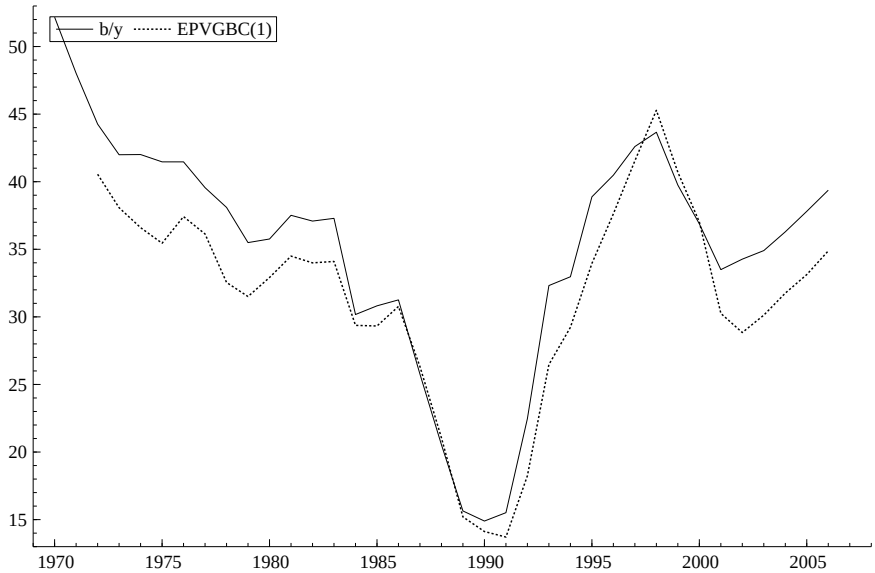

Figure 8: UK b/y and exp[PVGBC(1)].

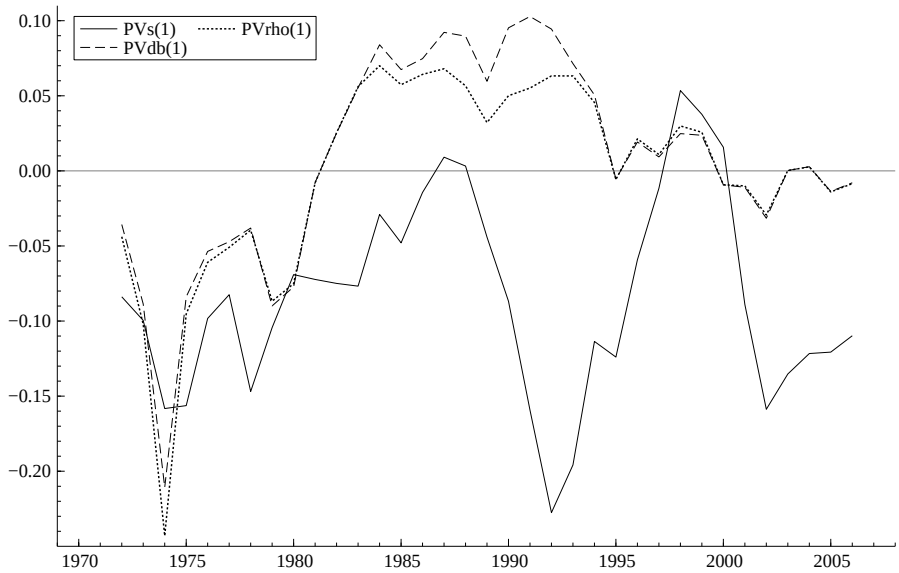

Figure 9: UK PVs(1), PVdb(1) and PVrho(1).

Figure 10: UK PVv(1) and PVg(1).

We observe only two brief periods where $FSI > 1$. These are 1986-1988 and 1997-2000. From 1971-1984 and after 2000 the index was considerably less than unity. The period 1984-2005 has

four clear episodes. From 1984-1989 there were falls in the debt-GDP ratio and in both revenues and expenditures in present value terms resulting in an improving fiscal position. This was a period where privatization receipts were used to pay off debt, even though the assets were not included in our measure of debt, namely, net government liabilities. From 1989-1992, when sterling was ejected from the ERM, the fiscal position deteriorated sharply due to rising expenditures. After 1992 the debt-GDP rose steadily as it did in the US, but expenditures after continuing to rise turned down which caused an improvement in the fiscal stance. From 1996-2001 there was a marked improvement in the fiscal position mainly due to rising revenues from the upturn in eceonomic activity. From 2001 the fiscal stance deteriorated again due to expenditures, which started to increase in 1998, rising much more than revenues. The Chancellor of the Exchequer has said throughout his tenure that the UK is meeting its fiscal targets, but this evidence indicates that this has not precluded an obvious decline in UK fiscal sustainability.

6.3 Germany

The data are annual for the period 1960 to 2005 and are plotted in Figure 11.

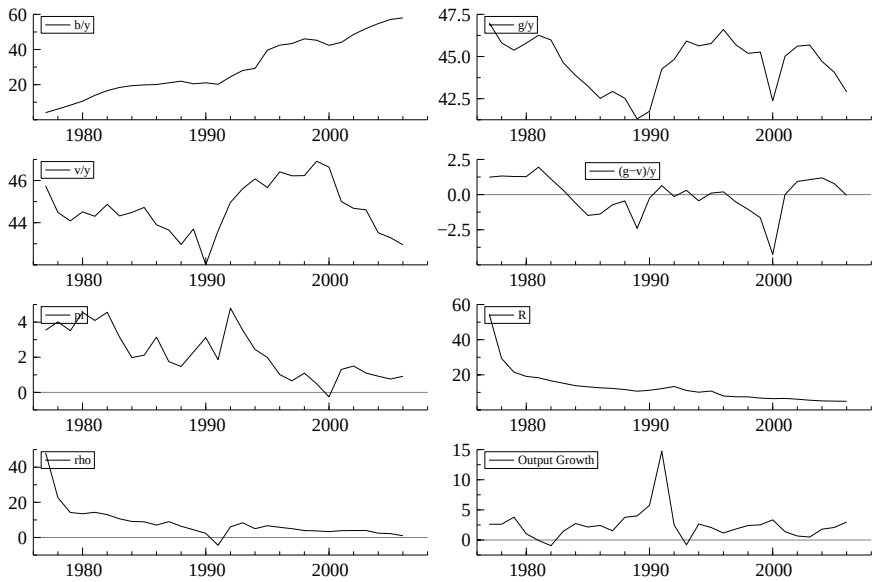

Figure 11: Germany data

The augmented Dickey-Fuller tests reported in Table 4 do not allow us to reject a unit root for any of the variables

Table 4

Augmented Dickey-Fuller tests (sub-sample 1976-2005)

D-Lag	Variable						
	$\ln \frac{b}{y}$	$\ln \frac{g}{y}$	$\ln \frac{v}{y}$	$\ln(1+R)$	$\ln(1+pi)$	ρ	$\Delta \ln y_t$
2	-1.315	-2.102	-1.653	-2.016	-1.355	-2.176	-2.515
1	-1.918	-2.080	-1.382	-1.645	-1.635	-2.850	-3.472*
0	-3.582*	-2.017	-1.422	-5.303**	-2.125	-3.431*	-3.680*

Note: * denotes significance at the 5% level and ** denotes significance at the 10% level.

The results on fiscal sustainability for the period from 1977 are reported in Figures 11-15 for a one-year horizon. The reason for starting in 1977 is that prior to this the debt-GDP ratio was negative.

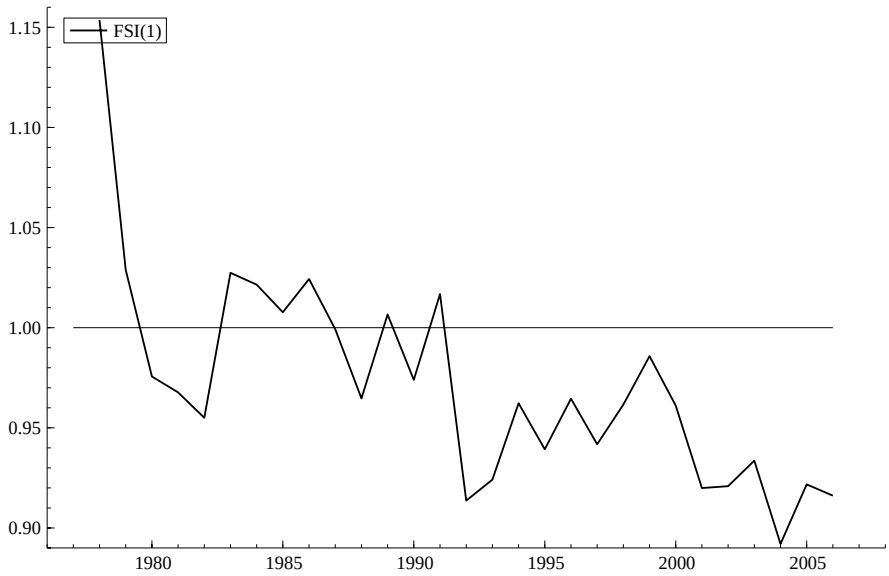

Figure 12: Germany FSI (1).

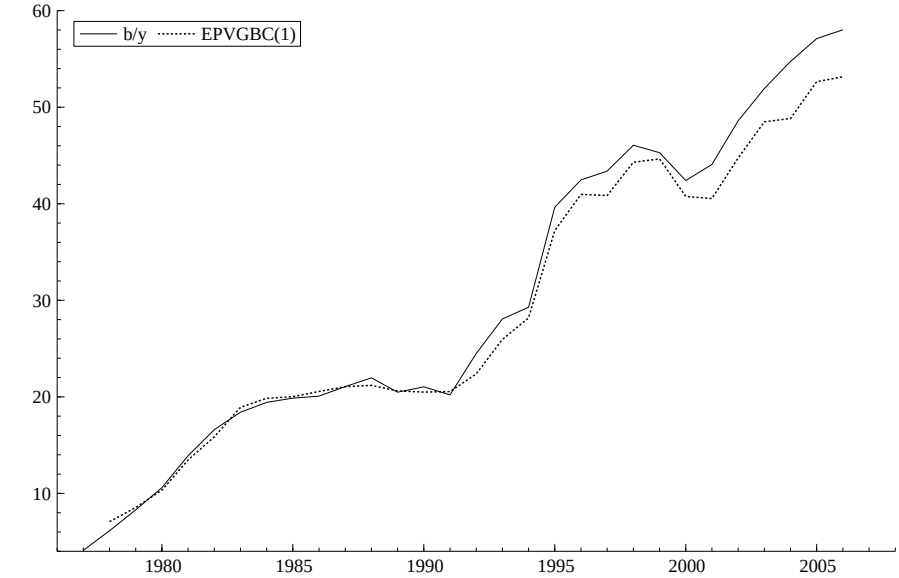

Figure 13: Germany b/y and exp[PVGBC(1)].

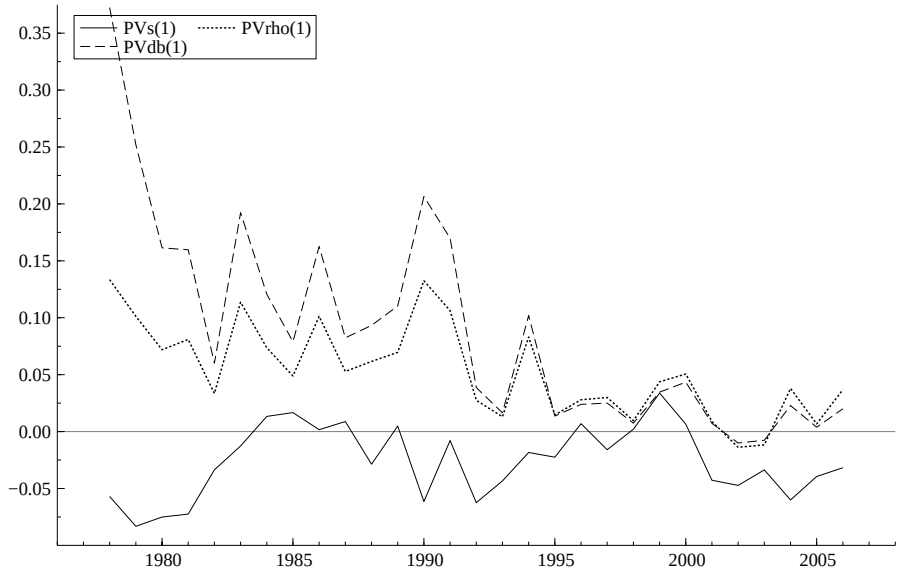

Figure 14: Germany PVs(1), PVdb(1) and PVrho(1).

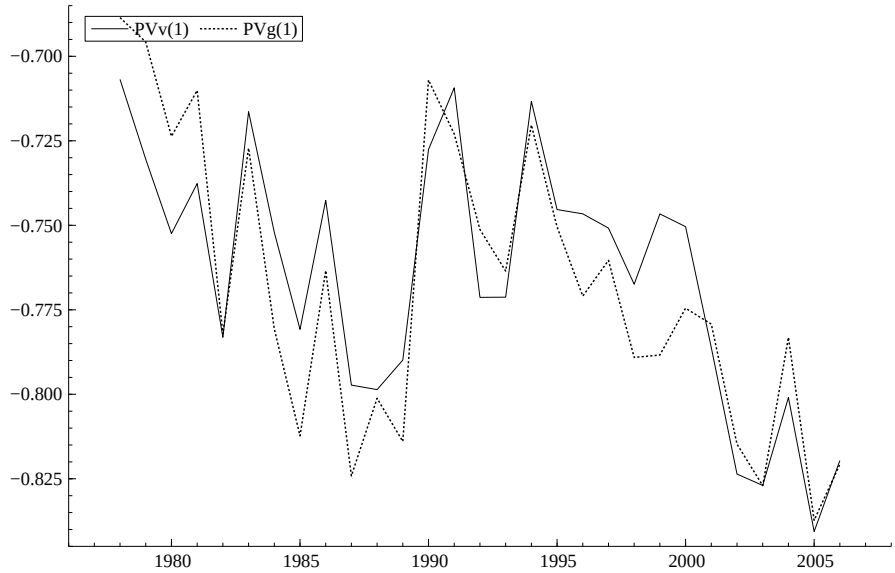

Figure 15: Germany PVv(1) and PVg(1).

There has been a steady deterioration in the FSI over the whole period since 1977. There were two occasions when the index worsened sharply. They are in 1989 on German unification, and again in 1999 shortly after EMU began. Both events seems to have been very harmful to the fiscal stance. Throughout the period the debt-GDP ratio has risen and, with the exception of the period 1992-1999, the fiscal position has gradually deteriorated. The improvement during the period 1992-1999 coincides with improvements in the US and UK and is due to sustained economic growth raising tax revenues. Since expenditures also increased the improvement in the German fiscal stance was less marked that those of the US and UK. Since 1999 the fiscal stance has continued to worsen as expenditures, although falling over the period, have exceeeded revenues which have also decreased. The observed secular decline in German fiscal sustainability reflects and supports the widespread perception that Germany is in need of structural reform.

7 Fiscal sustainability under changed monetary policy

We consider the effect on fiscal sustainability of a counter-factual change of monetary policy so that it follows the Taylor rule

$$rs_t = 3 + 0.5(\ln y_t - \ln y_t^T) + 1.5(\pi_t - 2)$$

where $\ln y_t - \ln y_t^T$ is the output gap and $\ln y_t^T$ is taken to be a cubic function of time. We prefer this measure of the output gap to the popular HP filter because the HP filter is two-sided and hence causes the output gap to be a function of future output. This would cause a time distortion to the VAR that is inappropriate for forecasting. Due to the presence of lagged variables, iIncluding the output gap in the VAR together with $\Delta \ln y_t$ results in near perfect collinearity. Consequently we use only the output gap and its lags.

The new equation for rs_t, which includes an intercept, is

$$Fz_{1t} + z_{2t} = A_{20}^* + A_{21}^*(L)z_{1,t-1} + A_{22}^*(L)z_{2t} + e_{2t}^*$$

where the coefficients are constrained to be

$$\begin{aligned} F &= \begin{bmatrix} 0 & 0 & 0 & 0 & -0.5 & -1.5 & 0 \end{bmatrix} \\ A_{20}^* &= 0.03,\ A_{21}^*(L) = A_{22}^*(L) = e_{2,t}^* = 0 \end{aligned}$$

We carry out the analysis of the effects on fiscal sustainability of a change in monetary on annual data for the US, the UK and Germany over a one-year horizon.

Figure 16 plots the observed nominal short-term interest rate together with the value given by the Taylor rule for each of the three countries. For the US we find that with the exception of the period 1981-1986 the Taylor rule results in a higher interest rate. The difference is particularly marked throughout the 1970's and again from 2001. For the UK, Taylor rule interest rate is also higher until 1980 but similar to the actual rate thereafter. For Germany, the Taylor rule gives a lower interest rate during the 1980's, but a similar rate thereafter.

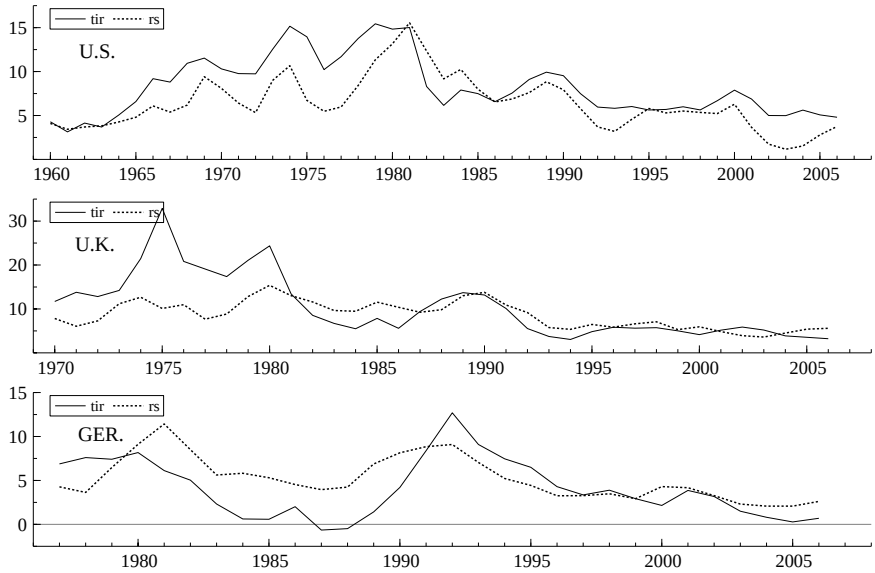

Figure 16: Taylor interest rate and IRS in the US, the UK and Germany.

7.1 The United States

Figures 17-20 plot the index and its components. Results based on the original VAR and the policy modified VAR (PVAR) are depicted.

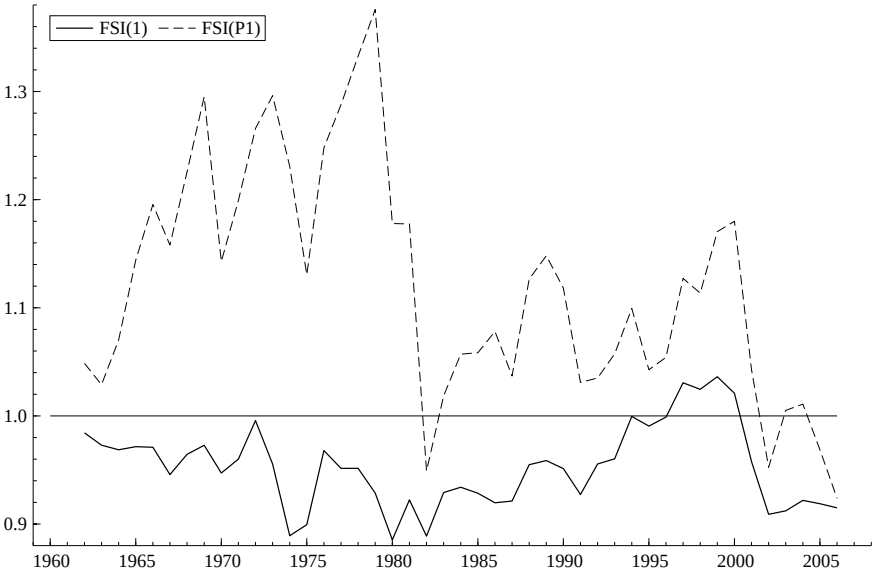

Figure 17: US FSI(1) for VAR and PVAR.

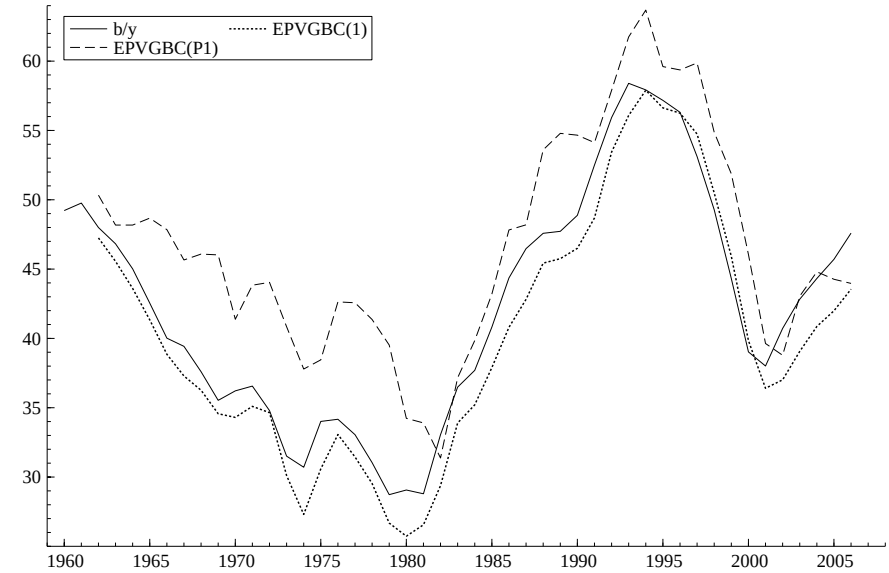

Figure 18: US b/y and exp[PVGBC(1)] for VAR and PVAR.

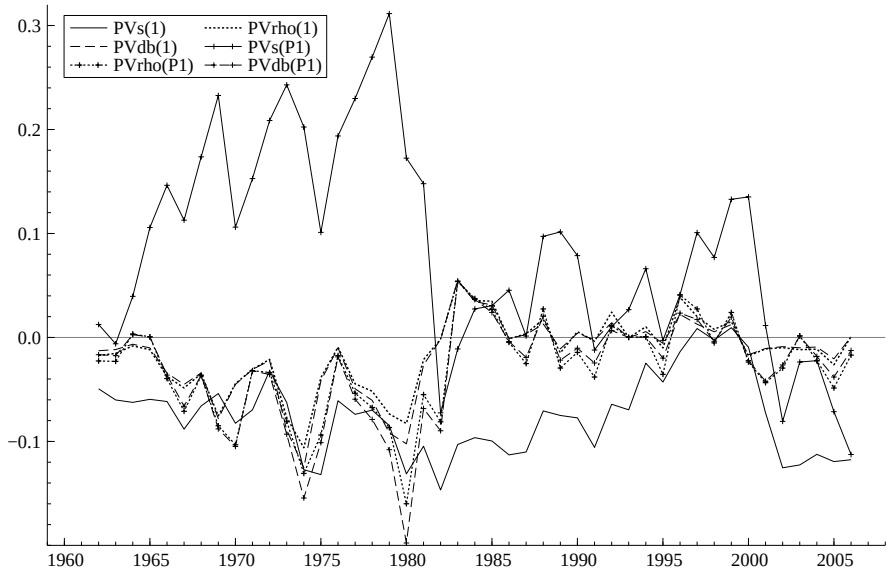

Figure 19: US PVs(1), PVdb(1) and PVrho(1) for VAR and PVAR.

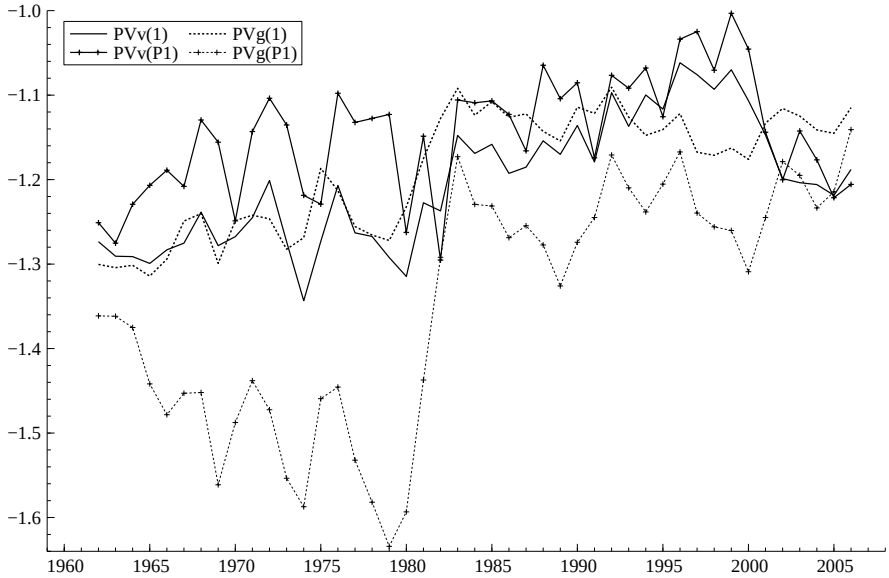

Figure 20: US PVv(1) and PVg(1) for VAR and PVAR.

There is an improvement in the US fiscal stance throughout, except from 2001. The most dramatic improvement occurs in the 1970's when the Taylor rule gives much higher interest rates. The cause of the improvement is lower expenditures and slightly higher revenues.

7.2 The United Kingdom

It is assumed that the real interest rate is 1% for the UK and not 3% as for the US. The results are reported in Figures 21-24.

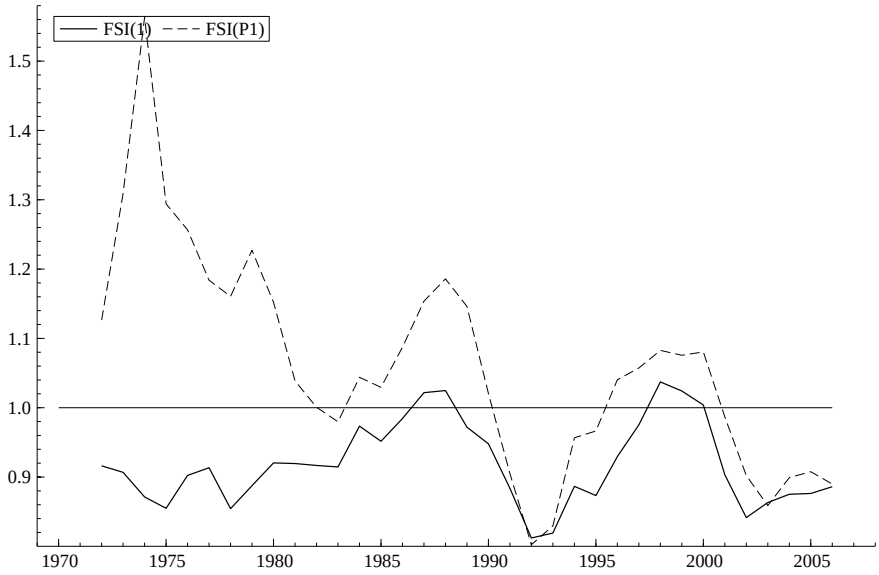

Figure 21: UK FSI(1) for VAR and PVAR.

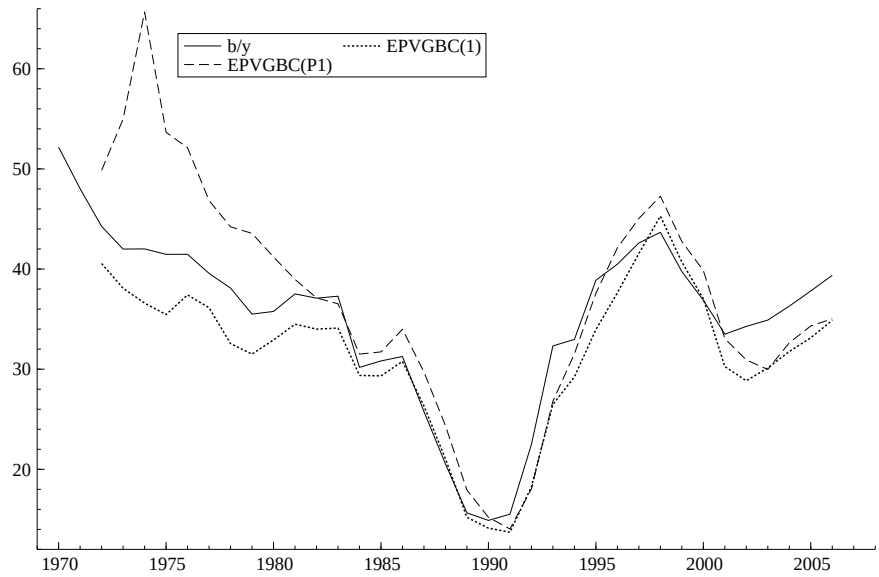

Figure 22: UK b/y and exp[PVGBC(1)] for VAR and PVAR.

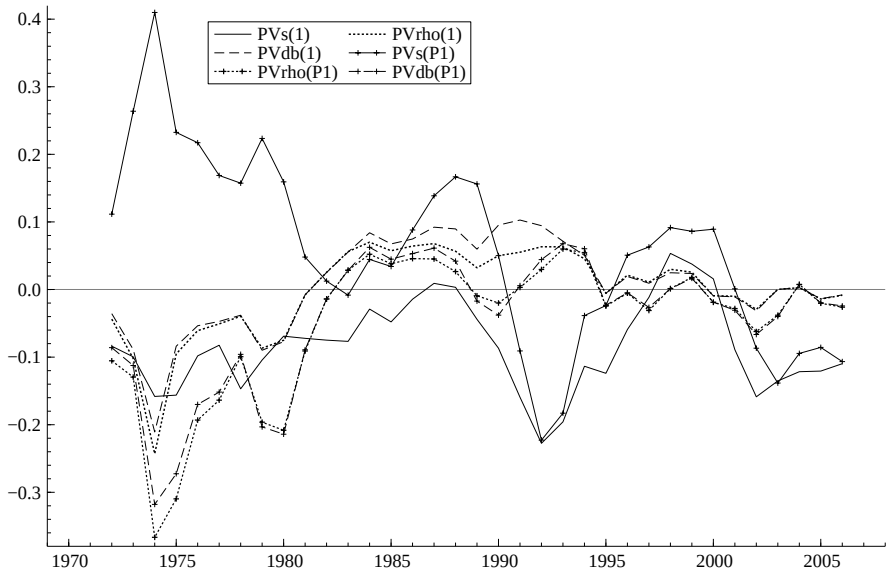

Figure 23: UK PVs(1), PVdb(1) and PVrho(1) - for VAR and PVAR.

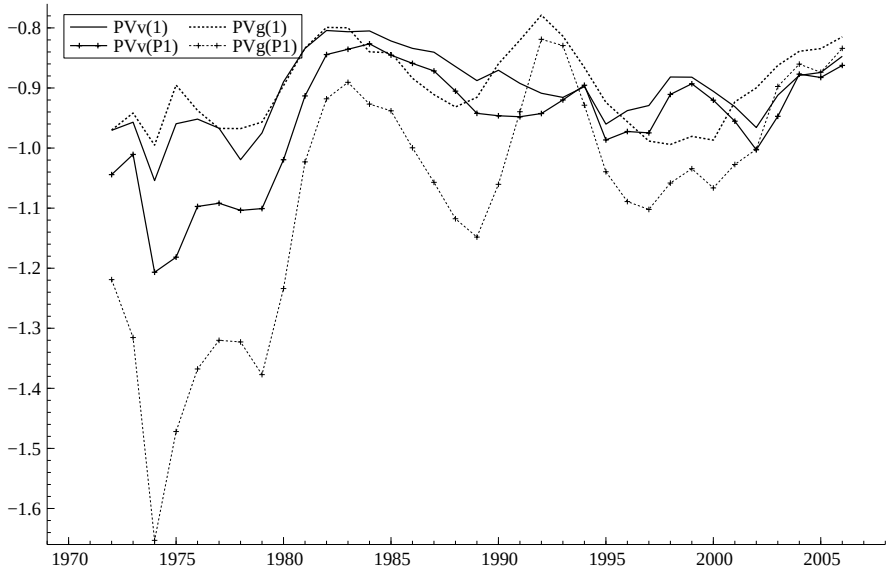

Figure 24: UK PVv(1) and PVg(1) for VAR and PVAR.

Like the US, using a Taylor rule to determine UK monetary policy would have improved the fiscal position in each year, but especially in the 1970's, primarily by reducing expenditures in present value terms.

7.3 Germany

Like the UK, the real interest rate was chosen as 1%. Figures 25-28 present the findings.

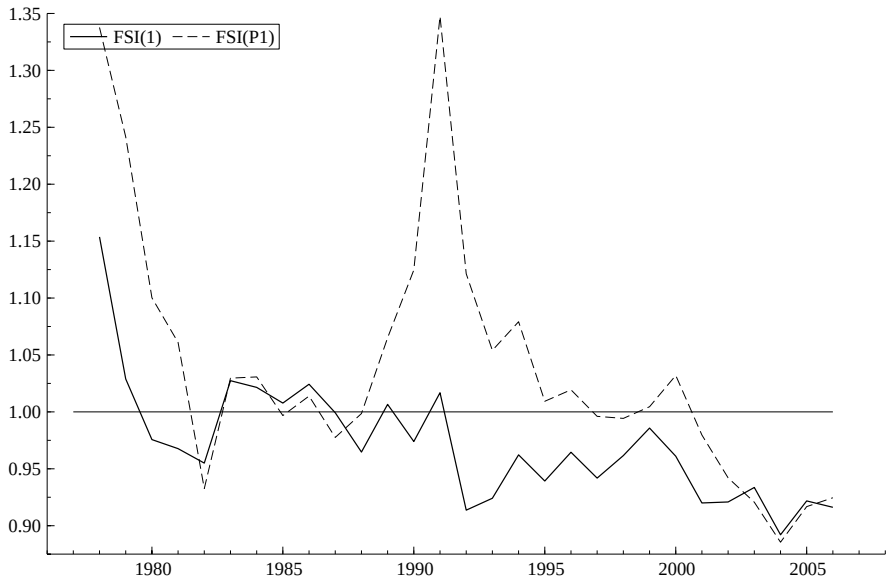

Figure 25: Germany FSI(1) for VAR and PVAR.

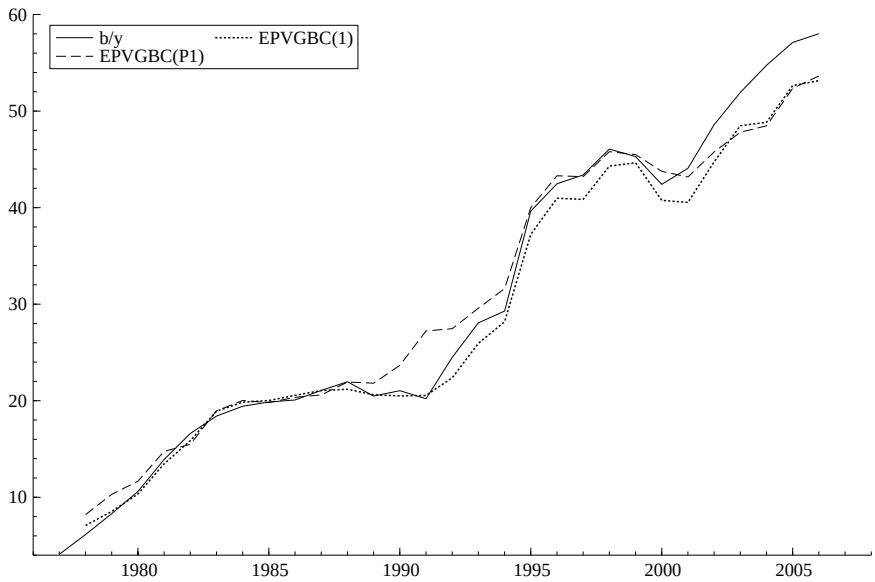

Figure 26: Germany b/y and exp[PVGBC] for VAR and PVAR.

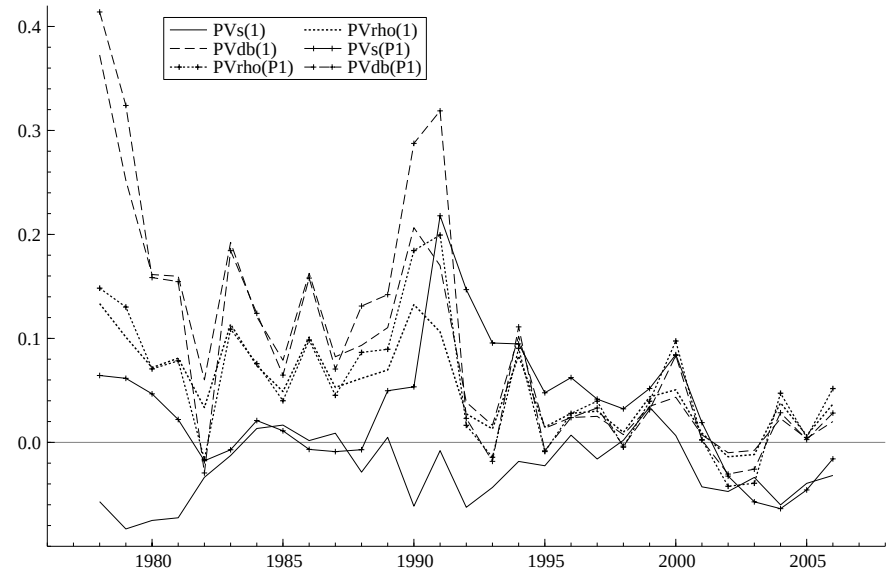

Figure 27: Germany PVs(1), PVdb(1) and PVrho(1) for VAR and PVAR.

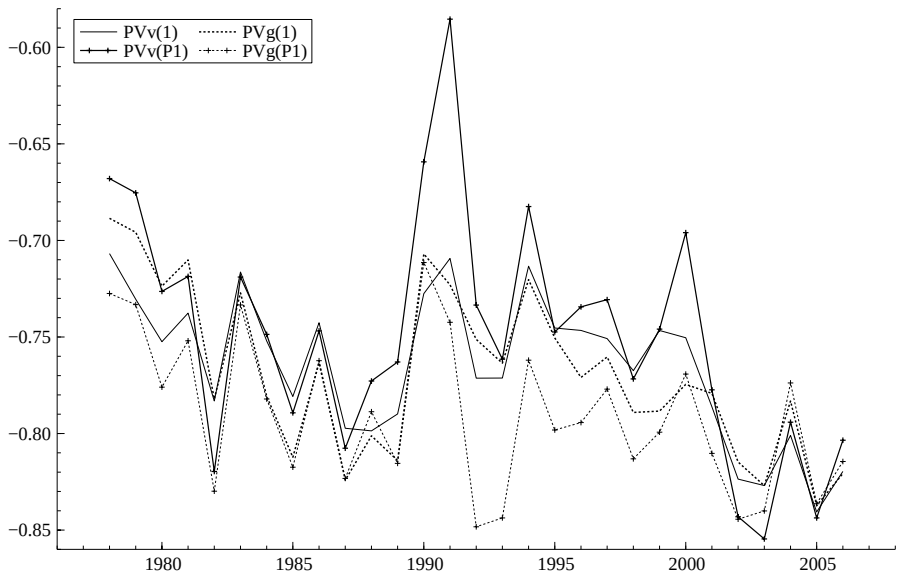

Figure 28: Germany PVv(1) and PVg(1) for VAR and PVAR.

There is a moderate fiscal improvement for Germany that is much less than that for the US and the UK. The greatest improvement occurs over the period 1988-1995 when the fiscal stance becomes sustainable. After 2000 there is little difference between the two sets of results. The

probable reasons for these findings are that German monetary policy was already tight and close to following a Taylor rule prior to Germany joining EMU, and that this continued under ECB interest rate policy.

8 Conclusions

We state our conclusions briefly. We have shown that existing fiscal sustainability measures indicate whether the current policy stance is sustainable in the long run. We have argued that for practical purposes this is not sufficiently helpful and what is needed is a short-term indicator. We have proposed the use of an index of fiscal sustainability. It can apply to any time horizon, including a short time horizon and it is easy to compute automatically using a VAR. We have shown how to identify individual components of the index that may be causing fiscal sustainability. We have proposed a method that enables a VAR to be used after some of its equations have been altered, in this case due to a change in policy. We have employed this procedure to analyse the possible effect on fiscal sustainability of a change in monetary policy so that it adheres to a Taylor rule.

We have applied this methodology to three countries: the US, the UK and Germany. In the UK and US the index of fiscal sustainability has fluctuated considerably with periods of non-sustainability followed by periods of sustainability. During the period of strong economic growth in the 1990's the fiscal positions of all three countries improved considerably, but in recent years the fiscal stance in all three countries has been steadily deteriorating. Our index indicates that a continuation of the present fiscal stances is leading to fiscal unsustainability in the three countries. We have shown that the German fiscal position has worsened steadily over the last thirty years with only a brief respite in the mid 1990's. A sharp deterioration occurred after unification and again on joining EMU.

A finding of non-sustainability should not be interpreted as implying that the fiscal stance will remain unsustainable; it indicates that, in the absence of a more benign economic climate, a

change of fiscal policy is required. We have shown that tighter monetary policy, particularly in the 1970's, would have improved the fiscal position and taken some of the pressure off fiscal policy, but would not have much effect on the current fiscal stances.

Finally, we note that this approach could also be applied to current account sustainablility.

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Data appendix

The US data are quarterly for the period 1960.1 to 2005.4 and are taken from the $OECD$ Economic Outlook database and are described in the $OECD$ Economic Outlook Database Inventory and on the Annex Tables session of the Sources and Methods.

GDP, Value, at market prices, of gross domestic product;

$GNFL$, Value of government net financial liabilities[4] ;

$PGDP$, deflator of GDP at market prices;

$GGINTP$, Value of gross government interest payments;

$GGINTR$, Value of gross government interest receipts;

$GNINTP$, Value of net government interest payments[5] ;

$YPGT$, Value of government total disbursement;

$YRGT$, Value of government total receipts;

IRS, Short-term nominal interest rate (in percentages)[6] ;

IRL, Long-term interest rate (in percentages)[7] .

The variables used in this study are then calculated as follows:

1. $\frac{b_t}{y_t}$ is $GNFL$ deflated by GDP.

2. $\frac{v_t}{y_t}$ is $YRGT$ minus $GGINTR$ and deflated by GDP.

3. $\frac{g_t}{y_t}$ is $YPGT$ minus $GGINTP$ deflated by GDP.

4. R_t is $GNINTP$ deflated by the $GNFL$ in the previous period value

5. π_t is the quarterly rate of change in the natural logarithm of $PGDP$.

[4] This variable refers to the consolidated gross financial liabilities of the government sector net of short-term financial assets, such as cash, bank deposits, loans to the private sector etc.

[5] $GGINTP = GNINTP - GNINTR$

[6] U.S. rates refer to interest rates on United States dollar three-month deposits in London, UK interest rates are 3-month rates on interbank loans, while Germany interest rates refer to the 3-month FIBOR rate.

[7] Rates refer to the ten-year government bond yield for the US and the UK, while they refer to the federal bond yield in the case of Germany.

6. rs_t is IRS divided by 100

7. rl_l is IRL divided by 100