Fiscal Policy as Stabilisation Device within EMU

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Abstract: Extending Gali and Monacelli (*2004*), we build an N-country open economy model, where each economy is subject to sticky wages and prices and, potentially, has access to sales and income taxes as well as government spending as fiscal instruments to deal with technology shocks once the monetary policy instrument as been given up as part of monetary union. We find that, with empirically plausible degrees of price and wage-stickiness, joining a monetary union imposes significant welfare costs on an economy's inhabitants when that economy is subject to technology shocks. These costs could be reduced by up to half by utilising fiscal instruments, with government spending being particularly effective in responding to idiosyncratic shocks due to its assumed home bias in the purchase of goods. However, even employing all three fiscal instruments could not reduce the costs of these shocks to the levels found when the economy stayed out of monetary union and retained use of a monetary policy (and therefore exchange rate) instrument. We also found that implementation lags could significantly affect the ability of fiscal instruments to deal with shocks, but that the need to ensure fiscal solvency when utilising tax and spending instruments had negligible welfare consequences.

JEL Codes:

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1 Overview

There has been a wealth of recent work deriving optimal monetary policy for both closed and open economies utilising New Classical Keynesian Synthesis models where the structural model of the economy and the description of policy makers' objectives are consistently microfounded (see for example, Woodford (2003) for a comprehensive treatment of the closed economy case, and Clarida *et al* (2001) for its extension to the open economy case.). The use of such a policy framework to evaluate fiscal policy in the context of monetary union has been less extensive, although there are some notable exceptions[1] which tend to examine such policies within 2 country models. One possible reason for this is the difficulty in deriving a tractable model of multiple fiscal actors acting under a single monetary authority without getting bogged down in the strategic interactions of policy makers. A recent model which tries to overcome these difficulties is due to Gali and Monacelli (2004) (henceforth GM), where they consider a model of a monetary union which is not made of simply 2 economies, but of a continuum of economies. This has a number of advantages aside from realism. Firstly, it means that it is reasonable to consider economies which set policy without regard to other union members (provided any jointly imposed restrictions on policy, such as the Stability and Growth Pact are satisfied). This seems reasonable in the context of a monetary union with 12 members, and more hoping to join. It also means that optimal policy can be described from the point of view of a small open economy operating within a monetary union without the need to model strategic interactions arising from policy spillovers. Furthermore, the analytical derivation of utility-based welfare measures is, under certain conditions, relatively tractable.

In GM, the only source of inertia is in price-setting (wages are flexible), there is no government debt and the only fiscal instrument available to policy makers is government spending. We extend GM in a number of significant ways. Firstly we assume that households supply differentiated labour to monopolistically competitive firms. This allows us to model nominal inertia in wage setting (as well as price setting) through the device of Calvo (1983) contracts. In our extensions, we also introduce income and sales taxes and allow for the possibility that there is no non-distortionary fiscal instrument available to balance the budget i.e. we allow for government debt. We derive a quadratic approximation to welfare for this model with which to evaluate alternative policies.

Within this framework we derive optimal commitment policies for both the small open economy and a member of monetary union. In a series of simulations we contrast these optimal commitment policies with optimal discretionary policy utilising alternative sets of fiscal policy instruments. We assess to what extent lags in implementing fiscal policy changes affect the ability of fiscal instruments to act as effective stabilisation tools. Finally we assess the extent to which debt sustainability issues impinge on the ability of national fiscal authorities to use

[1] Notable papers modelling fiscal policy within a monetary union using a New Neo-Classical Keynesian Synthesis model with a utility consistent measure of welfare include, Sutherland (2004) and Beetsma and Jensen (2004).

fiscal instruments as a stabilisation device.

Our next section derives the model. Section 3 outlines the social planner's problem such that we can write our model in 'gap' form. This representation of the model can also be used to derive a quadratic approximation to welfare. In section 4 we derive the optimal pre-commitment policies for the open economy and for a continuum of economies participating in monetary union. Section 5 simulates such economies to quantify the relative contribution of alternative fiscal instruments to macroeconomic stability. In this section we also consider the importance of implementation lags in relation to fiscal variables. Finally, in section 6, we introduce debt, derive the optimal policy under commitment and assess the extent to which fiscal sustainability issues affect a fiscal authority's ability to compensate for the lack of the monetary policy instrument under EMU.

2 The Model

This section outlines our model. As noted above this is similar in structure to GM, but we allow for the existence of sticky wages as well as prices and introduce distortionary sales and income taxes. The model is further extended by introducing government debt in section 6.

2.1 Households

There are a continuum of households of size one, who differ in that they provide differentiated labour services to firms in their economy. However, we shall assume full asset markets, such that, through risk sharing, they will face the same budget constraint and make the same consumption plans even if they face different wage rates due to stickiness in wage-setting. As a result the typical household will seek to maximise the following objective function,

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N(k)_t, G_t; \chi_t) \tag{1}$$

where C,G and N are a consumption aggregate, a public goods aggregate, and labour supply respectively, and χ is a shock. Here the only notation referring to the specific household, k, indexes the labour input, as full financial markets will imply that all other variables are constant across households.

The consumption aggregate is defined as

$$C = \frac{C_H^{1-\alpha} C_F^{\alpha}}{(1-\alpha)^{(1-\alpha)} \alpha^{\alpha}} \tag{2}$$

where, if we drop the time subscript, all variables are commensurate. C_H is a composite of domestically produced goods given by

$$C_H = (\int_0^1 C_H(j)^{\frac{\varepsilon-1}{\epsilon}} dj)^{\frac{\epsilon}{\epsilon-1}} \tag{3}$$

where j denotes the good's type or variety. The aggregate C_F is an aggregate across countries i

$$C_F = (\int_0^1 C_i^{\frac{\eta-1}{\eta}} di)^{\frac{\eta}{\eta-1}} \tag{4}$$

where C_i is an aggregate similiar to (3). Finally the public goods aggregate is given by

$$G = (\int_0^1 G(j)^{\frac{\varepsilon-1}{\epsilon}} dj)^{\frac{\epsilon}{\epsilon-1}} \tag{5}$$

which implies that public goods are all domestically produced. The elasticity of substitution between varieties $\epsilon > 1$ is common across countries. The parameter α is (inversely) related to the degree of home bias in preferences, and is a natural measure of openness.

The budget constraint at time t is given by

$$\begin{aligned} & \int_0^1 P_{H,t}(j)C_{H,t}(j)dj + \int_0^1 \int_0^1 P_{i,t}(j)C_{i,t}(j)djdi + E_t\{Q_{t,t+1}D_{t+1}\} \\ = & \ \Pi_t + D_t + W_tN(k)_t(1-\tau_t) - T_t \end{aligned} \tag{6}$$

where $P_{i,t}(j)$ is the price of variety j imported from country i expressed in home currency, D_{t+1} is the nominal payoff of the portfolio held at the end of period t, Π is the representative household's share of profits in the imperfectly competitive firms, W are wages, τ is an wage income tax rate, and T are lump sum taxes. $Q_{t,t+1}$ is the stochastic discount factor for one period ahead payoffs.

Households must first decide how to allocate a given level of expenditure across the various goods that are available. They do so by adjusting the share of a particular good in their consumption bundle to exploit any relative price differences - this minimises the costs of consumption. Optimisation of expenditure for any individual good implies the demand functions given below,

$$C_H(j) = (\frac{P_H(j)}{P_H})^{-\epsilon} C_H \tag{7}$$

$$C_i(j) = (\frac{P_i(j)}{P_i})^{-\epsilon} C_i \tag{8}$$

where we have price indices given by

$$P_H = (\int_0^1 P_H(j)^{1-\epsilon} dj)^{\frac{1}{1-\epsilon}} \tag{9}$$

$$P_i = (\int_0^1 P_i(j)^{1-\epsilon} dj)^{\frac{1}{1-\epsilon}} \tag{10}$$

It follows that

$$\int_0^1 P_H(j)C_H(j)dj = P_H C_H \tag{11}$$

$$\int_0^1 P_i(j)C_i(j)dj = P_i C_i \tag{12}$$

Optimisation across imported goods by country implies

$$C_i = (\frac{P_i}{P_F})^{-\eta} C_F \tag{13}$$

where

$$P_F = (\int_0^1 P_i^{1-\eta} di)^{\frac{1}{1-\eta}} \tag{14}$$

This implies

$$\int_0^1 P_i C_i di = P_F C_F \tag{15}$$

Optimisation between imported and domestically produced goods implies

$$P_H C_H = (1-\alpha) PC \tag{16}$$

$$P_F C_F = \alpha PC \tag{17}$$

where

$$P = P_H^{1-\alpha} P_F^{\alpha} \tag{18}$$

is the consumer price index (CPI). The budget constraint can therefore be rewritten as

$$P_t C_t + E_t\{Q_{t,t+1} D_{t+1}\} = D_t + W_t N(k)_t (1-\tau_t) - T_t \tag{19}$$

2.1.1 Households' Intertemporal Consumption Problem

The first of the households intertemporal problems involves allocating consumption expenditure across time. For tractability assume (following GM) that (1) takes the specific form

$$E_0 \sum_{t=0}^{\infty} \beta^t (\ln C_t + \chi_t \ln G_t - \frac{(N(k)_t)^{1+\varphi}}{1+\varphi}) \tag{20}$$

In addition, assume that the elasticity of substitution between the baskets of foreign goods produced in different countries is $\eta = 1$ (this is equivalent to adopting logarithmic utility in the aggregation of such baskets).

We can then maximise utility subject to the budget constraint (19) to obtain the optimal allocation of consumption across time,

$$\beta(\frac{C_t}{C_{t+1}})(\frac{P_t}{P_{t+1}}) = Q_{t,t+1} \tag{21}$$

Taking conditional expectations on both sides and rearranging gives

$$\beta R_t E_t\{(\frac{C_t}{C_{t+1}})(\frac{P_t}{P_{t+1}})\} = 1 \tag{22}$$

where $R_t = \frac{1}{E_t\{Q_{t,t+1}\}}$ is the gross return on a riskless one period bond paying off a unit of domestic currency in $t+1$. This is the familiar consumption Euler equation which implies that consumers are attempting to smooth consumption over time such that the marginal utility of consumption is equal across periods (after allowing for tilting due to interest rates differing from the households' rate of time preference).

A log-linearised version of (22) can be written as

$$c_t = E_t\{c_{t+1}\} - (r_t - E_t\{\pi_{t+1}\} - \rho) \tag{23}$$

where lowercase denotes logs (with an important exception for g noted below), $\rho = \frac{1}{\beta} - 1$, and $\pi_t = p_t - p_{t-1}$ is consumer price inflation.

2.1.2 Households' Wage-Setting Behaviour

We now need to consider the wage-setting behaviour of households. We assume that firms need to employ a CES aggregate of the labour of all households in the domestic production of consumer goods. This is provided by an 'aggregator' who aggregates the labour services of all households in the economy as,

$$N = \left[\int_0^1 N(k)^{\frac{\epsilon_w - 1}{\epsilon_w}} dk\right]^{\frac{\epsilon_w}{\epsilon_w - 1}} \tag{24}$$

where $N(k)$ is the labour provided by household k to the aggregator. Accordingly the demand curve facing each household is given by,

$$N(k) = \left(\frac{W(k)}{W}\right)^{-\epsilon_w} N \tag{25}$$

where N is the CES aggregate of labour services in the economy which also equals the total labour services employed by firms,

$$N = \int_0^1 N(j)dj \tag{26}$$

Where $N(j)$ is the labour employed by firm j. The price of this labour is given by the wage index,

$$W = \left[\int_0^1 W(k)^{1-\epsilon_w} dk\right]^{1-\epsilon_w} \tag{27}$$

The household's objective function for the setting on its nominal wage is given by,

$$E_t\left(\sum_{s=0}^{\infty}(\theta_w\beta)^s\left[\Lambda_{t+s}\frac{W(k)_t}{P_{t+s}}(1-\tau_{t+s})N(k)_{t+s} - \frac{(N(k)_{t+s})^{1+\varphi}}{1+\varphi}\right]\right) \tag{28}$$

where $\Lambda_{t+s} = C_{t+s}^{-1}$ is the marginal utility of real post-tax income and $N(k) = \left(\frac{W(k)}{W}\right)^{-\epsilon_w} N$ is the demand curve for the household's labour. The first-order condition is therefore given by,

$$E_t\left(\sum_{s=0}^{\infty}(\theta_w\beta)^s\left[\begin{array}{c}\Lambda_{t+s}P_{t+s}^{-1}\left(\frac{W(k)_t}{W_{t+s}}\right)^{-\epsilon_w}(1-\tau_{t+s})N_{t+s}(1-\epsilon_w)\\ +\epsilon_w\left(\frac{W(k)_t}{W_{t+s}}\right)^{-\epsilon_w(1+\varphi)}N_{t+s}^{(1+\varphi)}\end{array}\right]\right)=0 \quad (29)$$

Using the condition,

$$\beta^s\left(\frac{C_t}{C_{t+s}}\right)\left(\frac{P_t}{P_{t+s}}\right)=Q_{t,t+s} \quad (30)$$

this can be rewritten as,

$$E_t\left(\sum_{s=0}^{\infty}(\theta_w)^s\left[\begin{array}{c}Q_{t,t+s}W_{t+s}^{\epsilon_w}N_{t+s}(W(k)_t^{-\epsilon_w}(1-\tau_{t+s})\\ -\mu_w C_{t+s}P_{t+s}W(k)_t^{-\epsilon_w(1+\varphi)-1}N_{t+s}^{\varphi}W_{t+s}^{\varphi\epsilon_w}\end{array}\right]\right)=0 \quad (31)$$

where $\mu_w=\frac{\epsilon_w}{\epsilon_w-1}$ is the mark-up for wage-setting. Solving for the optimal wage,

$$\overline{W}_t^{-1-\varphi\epsilon_w}=\frac{E_t\left(\sum_{s=0}^{\infty}(\theta_w)^s\left[Q_{t,t+s}W_{t+s}^{\epsilon_w}N_{t+s}(1-\tau_{t+s})C_{t+s}^{-1}P_{t+s}^{-1}\right]\right)}{E_t\left(\sum_{s=0}^{\infty}(\theta_w)^s\left[Q_{t,t+s}\mu_w W_{t+s}^{\epsilon_w(1+\varphi)}N_{t+s}^{1+\varphi}\right]\right)} \quad (32)$$

where $\overline{W}$ denotes the wage chosen by all households that were able to renegotiate wages in period t. Note that when $\theta_w=0$ then wages are flexible and this condition reduces to,

$$(1-\tau)\left(\frac{W}{P}\right)=\mu_w N^{\varphi}C \quad (33)$$

which is the coventional labour supply decision (after allowing for the fact that households have market power in setting wages). The wage index evolves according to the following law of motion,

$$W_t=\left[(1-\theta_w)\overline{W}_t^{(1-\epsilon_w)}+\theta_w W_{t-1}^{1-\epsilon_w}\right]^{\frac{1}{1-\epsilon_w}} \quad (34)$$

where $\overline{W}_t$ is the optimal nominal wage set by those households that were able to do so in period t according to equation (32). These can be combined into a form of New Keynesian Phillips curve for wage inflation, as shown in Appendix 1 which yields a log-linearised expression for wage-inflation dynamics,

$$\pi_t^w=\beta E_t\pi_{t+1}^w+\frac{(1-\theta_w\beta)(1-\theta_w)}{(1+\varphi\epsilon_w)\theta_w}(\varphi n_t-w_t+c_t+p_t-\ln(1-\tau_t)+\ln(\mu^w)) \quad (35)$$

Note that the forcing variable in the NKPC is a log-linearsed measure of the extent to which wages are not at the level implied by the labour supply decision that would hold under flexible wages.

2.2 Price and Exchange Rate Identities

The bilateral terms of trade are the price of country i's goods relative to home goods prices,

$$S_i = \frac{P_i}{P_H} \tag{36}$$

The effective terms of trade are given by

$$\begin{aligned} S &= \frac{P_F}{P_H} && (37) \\ &= \exp \int_0^1 (p_i - p_H) di && (38) \end{aligned}$$

Recall the definition of consumer prices,

$$P = P_H^{1-\alpha} P_F^{\alpha} \tag{39}$$

using the definition of the effective terms of trade this can be rewritten as,

$$P = P_H S^{\alpha} \tag{40}$$

or in logs as

$$p = p_H + \alpha s \tag{41}$$

where $s = p_F - p_H$ is the logged terms of trade. By taking first-differences it follows that,

$$\pi_t = \pi_{H,t} + \alpha(s_t - s_{t-1}) \tag{42}$$

There is assumed to be free-trade in goods, such that the law of one price holds for individual goods at all times. This implies,

$$P_i(j) = \varepsilon_i P_i^i(j) \tag{43}$$

where ε_i is the bilateral nominal exchange rate and $P_i^i(j)$ is price of county i's good j expresed in terms of country i's currency. Aggregating across goods this implies,

$$P_i = \varepsilon_i P_i^i \tag{44}$$

where $P_i^i = \left(\int_0^1 P_i^i(j)^{1-\epsilon} dj \right)^{\frac{1}{1-\epsilon}}$.

From the definition of P_F we have,

$$\begin{aligned} P_F &= (\int_0^1 P_i^{1-\eta} di)^{\frac{1}{1-\eta}} && (45) \\ &= (\int_0^1 \left(\varepsilon_i P_i^i\right)^{1-\eta} di)^{\frac{1}{1-\eta}} && (46) \end{aligned}$$

In log-linearised form,

$$\begin{aligned} p_F &= \int_0^1 (e_i + p_i^i) di && (47) \\ &= e + p^* && (48) \end{aligned}$$

where $e = \int_0^1 e_i di$ is the log of the nominal effective exchange rate, p_i^i is the logged domestic price index for country i, and $p^* = \int_0^1 p_i^i di$ is the log of the world price index. For the world as a whole there is no distinction between consumer prices and the domestic (world) price level.

Recall the definition of the terms of trade and using the result just obtained,

$$\begin{aligned} s &= p_F - p_H && (49) \\ &= e + p^* - p_H && (50) \end{aligned}$$

Now consider the link between the terms of trade and the real exchange rate. (Note that although we have free trade and the law of one price holds for individual goods, our economies do not exhibit PPP since there is a home bias in the consumption of home and foreign goods. PPP only holds if we eliminate this home bias and assume $\alpha = 1$ since this implies that the share of home goods in consumption is the same as any other country's i.e. infinitesimally small.) The bilateral real exchange rate is defined as,

$$Q_i = \frac{\varepsilon_i P_i}{P} \tag{51}$$

where P_i and P are the two countries respective CPI price levels. In logged form we can defined the real effective exchange rate as,

$$\begin{aligned} q_t &= \int_0^1 (e_i + p^i - p) di && (52) \\ &= e + p^* - p && (53) \\ &= s + p_H - p && (54) \\ &= (1 - \alpha) s && (55) \end{aligned}$$

2.3 International risk sharing

Assume symmetric initial conditions (e.g. zero net foreign assets etc) and recall the first-order condition for consumption,

$$\beta \frac{C_{t+1}^{-1}}{P_{t+1}} = \frac{C_t^{-1}}{P_t} Q_{t,t+1} \tag{56}$$

Since financial markets are complete, a similar condition must exist in the foreign economy, say country i,

$$\beta \left(\frac{C_{t+1}^i}{C_t^i} \right)^{-1} \left(\frac{P_t^i}{P_{t+1}^i} \right) \left(\frac{\varepsilon_{i,t}}{\varepsilon_{i,t+1}} \right) = Q_{t,t+1} \tag{57}$$

Equating the two yields,

$$\left(\frac{C_{t+1}^i}{C_t^i} \right)^{-1} \left(\frac{P_t^i}{P_{t+1}^i} \right) \left(\frac{\varepsilon_{i,t}}{\varepsilon_{i,t+1}} \right) = \left(\frac{C_{t+1}}{C_t} \right)^{-1} \left(\frac{P_t}{P_{t+1}} \right) \tag{58}$$

where ε_i is the nominal exchange rate between home and country i. Using the definition of the real exchange rate, $\mathcal{Q}_{i,t} = \frac{\varepsilon_{it} P_t^*}{P_t}$, this can be written as,

$$\mathcal{Q}_{i,t+1}\left(\frac{C_{t+1}^i}{C_{t+1}}\right) = \mathcal{Q}_{i,t}\left(\frac{C_t^i}{C_t}\right) \tag{59}$$

Which can be interated backwards, so that,

$$\mathcal{Q}_{i,t}\left(\frac{C_t^i}{C_t}\right) = \mathcal{Q}_{i,t-i}\left(\frac{C_0^i}{C_0}\right) \tag{60}$$

In other words risk sharing implies that the relationship between consumption at home and country i is given by the following expression,

$$C_t = z^i C_t^i \mathcal{Q}_{i,t} \tag{61}$$

where z^i is a constant which depends upon initial conditions. Loglinearising and integrating over all countries yields,

$$c = c^* + q \tag{62}$$

where $c^* = \int_0^1 c^i di$ while using the relationship between the terms of trade and the real exchange rate yields,

$$c = c^* + (1-\alpha)s \tag{63}$$

2.4 Allocation of Government Spending

The allocation of government spending across goods is determined by minimising total costs, $\int_0^1 P_H(j)G(j)dj$. Given the form of the basket of public goods this implies,

$$G(j) = (\frac{P_H(j)}{P_H})^{-\epsilon} G \tag{64}$$

2.5 Firms

The production function is linear, so for firm j

$$Y(j) = AN(j) \tag{65}$$

where $a = \ln(A)$ is time varying and stochastic. While the demand curve they face is given by,

$$Y(j) = (\frac{P_H(j)}{P_H})^{-\epsilon}[(1-\alpha)(\frac{PC}{P_H}) + \alpha\int_0^1 (\frac{\varepsilon_i P^i C^i}{P_H})di + G] \tag{66}$$

,which we rewrite as,

$$Y(j) = (\frac{P_H(j)}{P_H})^{-\epsilon} Y \tag{67}$$

where $Y = \left[\int_0^1 Y(j)^{\frac{\epsilon-1}{\epsilon}} dj\right]^{\frac{\epsilon}{\epsilon-1}}$. The objective function of the firm is given by,

$$\sum_{s=0}^{\infty}(\theta_p)^s Q_{t,t+s}\left[(1-\tau_{t+s}^v)\frac{P_H(j)_t}{P_{t+s}}Y(j)_{t+s} - \frac{W_{t+s}}{P_{t+s}}\frac{Y(j)_{t+s}(1-\varkappa)}{A}\right] \tag{68}$$

where $\varkappa$ is an employment subsidy which can be used to eliminate the steady-state distortion associated with monopolistic competition and distortionary sales and income taxes (assuming there is a lump-sum tax available to finance such a subsidy) and τ^v is a sales tax. Using the demand curve for the firm's product,

$$\sum_{s=0}^{\infty}(\theta_p)^s Q_{t,t+s}\left[\begin{array}{c}(1-\tau_{t+s}^v)\frac{P_H(j)_t}{P_{t+s}}(\frac{P_H(j)_t}{P_{H,t+s}})^{-\epsilon}Y_{t+s}\\ -\frac{W_{t+s}}{P_{t+s}}(\frac{P_H(j)_t}{P_{H,t+s}})^{-\epsilon}\frac{Y_{t+s}(1-\varkappa)}{A_{t+s}}\end{array}\right] \tag{69}$$

The solution to this problem is given by,

$$\sum_{s=0}^{\infty}(\theta_p)^s Q_{t,t+s}\left[\begin{array}{c}(1-\epsilon)(1-\tau_{t+s}^v)P_{t+s}^{-1}(\frac{P_H(j)_t}{P_{H,t+s}})^{-\epsilon}Y_{t+s}\\ +\epsilon\frac{W_{t+s}}{P_{t+s}}P_H(j)_t^{-\epsilon-1}P_{H,t+s}^{\epsilon}\frac{Y_{t+s}(1-\varkappa)}{A_{t+s}}\end{array}\right] \tag{70}$$

Solving for the optimal reset price, which is common across all firms able to reset prices in period t,

$$\overline{P}_{H,t} = \frac{\sum_{s=0}^{\infty}(\theta_p)^s Q_{t,t+s}\left[\epsilon\frac{W_{t+s}}{P_{t+s}}P_{H,t+s}^{\epsilon}\frac{Y_{t+s}}{A_{t+s}}\right]}{\sum_{s=0}^{\infty}(\theta_p)^s Q_{t,t+s}\left[(\epsilon-1)(1-\tau_{t+s}^v)P_{t+s}^{-1}P_{H,t+s}^{\epsilon}Y_{t+s}(1-\varkappa)\right]} \tag{71}$$

While domestic prices evolve according to,

$$P_{H,t} = \left[(1-\theta_w)P_t^{*(1-\epsilon_w)} + \theta_w P_{H,t-1}^{1-\epsilon_w}\right]^{\frac{1}{1-\epsilon_w}} \tag{72}$$

Appendix 2 then details the derivation of the New Keynesian Phillips curve for domestic price inflation which is given by,

$$\begin{aligned}\pi_{H,t} \;=\;& \beta E_t \pi_{H,t+1} \qquad (73)\\ &+\frac{(1-\theta_p\beta)(1-\theta_p)}{\theta_p}(-a_t + w_t - p_{H,t} - \ln(1-\tau_t^v) - v_t + \ln(\mu))\end{aligned}$$

where $mc = -a + w - p_H - \ln(1-\tau^v) - v$ are the real log-linearised marginal costs of production, and $v = -\ln(1-\varkappa)$. In the absence of sticky prices profit maximising behaviour implies, $mc = -\ln(\mu)$ where μ is the steady-state mark-up.

2.6 Equilibrium

Goods market clearing requires, for each good j,

$$Y(j) = C_H(j) + \int_0^1 C_H^i(j)di + G(j) \tag{74}$$

Symmetrical preferences imply,

$$C_H^i(j) = \alpha(\frac{P_H(j)}{P_H})^{-\epsilon}(\frac{P_H}{\varepsilon_i P^i})^{-1}C^i \tag{75}$$

which allows us to write,

$$Y(j) = (\frac{P_H(j)}{P_H})^{-\epsilon}[(1-\alpha)(\frac{PC}{P_H}) + \alpha\int_0^1(\frac{\varepsilon_i P^i C^i}{P_H})di + G] \tag{76}$$

Defining aggregate output as

$$Y = [\int_0^1 Y(j)^{\frac{\epsilon-1}{\epsilon}}dj]^{\frac{\epsilon}{\epsilon-1}} \tag{77}$$

allows us to write

$$\begin{aligned} Y &= (1-\alpha)\frac{PC}{P_H} + \alpha\int_0^1(\frac{\varepsilon_i P^i C^i}{P_H})di + G & (78)\\ &= S^\alpha[(1-\alpha)C + \alpha\int_0^1 \mathcal{Q}_i C_i di] + G & (79)\\ &= CS^\alpha + G & (80)\end{aligned}$$

Taking logs implies

$$\begin{aligned} \ln(Y-G) &= c + \alpha s & (81)\\ &= y + \ln(1-\frac{G}{Y}) & (82)\\ &= y - g & (83)\end{aligned}$$

where we define $g = -\ln(1-\frac{G}{Y})$. As this condition holds for all countries, we can write world (log) output as

$$y^* = \int_0^1(c^i + g^i + \alpha s^i)di \tag{84}$$

However $\int_0^1 s^i di = 0$, so we have

$$y^* = \int_0^1(c^i + g^i)di = c^* + g^* \tag{85}$$

We can use these relationships to rewrite (23) as

$$\begin{aligned} y_t &= E_t\{y_{t+1}\} - (r_t - E_t\{\pi_{t+1}\} - \rho) - E_t\{g_{t+1} - g_t\} - \alpha E_t\{s_{t+1} - s_t\}\\ &= E_t\{y_{t+1}\} - (r_t - E_t\{\pi_{H,t+1}\} - \rho) - E_t\{g_{t+1} - g_t\} & (86)\end{aligned}$$

Price inflation is determined by

$$\pi_{H,t} = \beta E_t\{\pi_{H,t+1}\} + \lambda(mc_t + \ln(\mu_t)) \tag{87}$$

where $\lambda = [(1-\beta\theta)(1-\theta)]/\theta$ and $mc = -a + w - p_H - \ln(1-\tau^v) - v$ such that the forcing variable is the difference between real marginal costs and the inverse of the steady-state mark-up. While wage inflation dynamics are determined by,

$$\pi^w_{H,t} = \beta E_t \pi^w_{H,t+1} + \frac{(1-\theta_w\beta)(1-\theta_w)}{(1+\varphi\epsilon_w)\theta_w}(\varphi n_t - w_t + c_t + p_t - \ln(1-\tau_t) + \ln(\mu^w)) \tag{88}$$

here the forcing variable captures the extent to which the consumer's labour supply decision is not the same as it would be under flexible prices. Define this variable as, $mc^w = \varphi n_t - w_t + c_t + p_t - \ln(1-\tau_t)$. This can be manipulated as follows,

$$mc^w = \varphi n - w + p_H + c + p - p_H - \ln(1-\tau) \tag{89}$$
$$= \varphi n - w + p_H + c + \alpha s - \ln(1-\tau) \tag{90}$$
$$= \varphi y - (w - p_H) + c^* + s - \ln(1-\tau) - \varphi a \tag{91}$$

From above we had

$$y = c^* + g + s \tag{92}$$

so we can also write marginal costs appropriate to wage inflation as

$$mc^w = (1+\varphi)y - (w - p_H) - \ln(1-\tau) - g - \varphi a \tag{93}$$

Real wages evolve according to,

$$w_t - p_{H,t} = \pi^w_{H,t} - \pi_{H,t} + w_{t-1} - p_{H,t-1} \tag{94}$$

While the marginal costs relevant for price inflation are given by,

$$mc = -v + w - p_H - a - \ln(1-\tau^v) \tag{95}$$

2.7 Summary of Model

We are now in a position to summarise our model. On the demand side we have an Euler equation for consumption,

$$y_t = E_t\{y_{t+1}\} - (r_t - E_t\{\pi_{H,t+1}\} - \rho) - E_t\{g_{t+1} - g_t\} \tag{96}$$

While on the supply side there are equations for price inflation,

$$\pi_{H,t} = \beta E_t\{\pi_{H,t+1}\} + \lambda(mc_t + \ln(\mu)) \tag{97}$$

where $\lambda = [(1-\beta\theta)(1-\theta)]/\theta$ and $mc = -a + w - p_H - \ln(1-\tau^v) - v$. There is a similar expression for wage inflation,

$$\pi^w_{H,t} = \beta E_t \pi^w_{H,t+1} \tag{98}$$
$$+ \frac{(1-\theta_w\beta)(1-\theta_w)}{(1+\varphi\epsilon_w)\theta_w}((1+\varphi)y_t - (w_t - p_{H,t}) - \ln(1-\tau_t) - g_t - \varphi a_t + \ln(\mu^w))$$

which together determine the evolution of real wages,

$$w_t - p_{H,t} = \pi^w_{H,t} - \pi_{H,t} + w_{t-1} - p_{H,t-1} \tag{99}$$

The model is then closed by the policy maker specifying the appropriate values of the fiscal and monetary policy variables. However, although this represents a fully specified model it is often recast in the form of 'gap' variables which are more consistent with utility-based measures of welfare.

2.8 Gap variables

Define the natural level of (log) output y^n as the level that would occur in the absence of nominal inertia and conditional on the optimal choice of government spending, the steady-state tax rates and the actual level of world output. Define the output gap as

$$y^g = y - y^n \tag{100}$$

With flexible prices and wages we have $mc^n = -\mu$ and $mc^{w,n} = -\mu^w$(see above). Substituting into the expressions for mc and mc^w implies,

$$-\ln(\mu) = -a + w^n - p^n_H - \ln(1 - \overline{\tau}^v) - v \tag{101}$$

where the consumption tax rate has been 'barred' to denote its steady-state value. Solving for equilibrium real wages,

$$w^n - p^n_H = -\ln(\mu) + a + \ln(1 - \overline{\tau}^v) + v \tag{102}$$

Similarly for the 'marginal costs' determining wage inflation,

$$\begin{aligned} -\ln(\mu^w) &= (1+\varphi)y^n - (w^n - p^n_H) - \ln(1-\overline{\tau}) - g^n + \varphi a \qquad (103) \\ -\ln(\mu^w) &= (\ln(\mu)) - \ln(1-\overline{\tau}^v) - v + (1+\varphi)(y^n - a) - \ln(1-\overline{\tau}) - g^n \\ y^n &= a + g^n/(1+\varphi) + (v + \ln(1-\overline{\tau}) - \ln(\mu) - \ln(\mu^w))/(1+\varphi) \end{aligned}$$

We can rearrange this as

$$-(v + \ln(1-\overline{\tau}) - \ln(\mu) - \ln(\mu^w)) = a(1+\varphi) + g^n - y^n(1+\varphi) \tag{104}$$

We can then write

$$\begin{aligned} mc^{w,g} &= mc^w + \ln(\mu^w) \qquad (105) \\ &= (1+\varphi)y - (w - p_H) - \ln(1-\tau) - g - \varphi a + \ln(\mu^w) \qquad (106) \\ &= (1+\varphi)y^g - g^g - (w^g - p^g_H) - \ln(1-\tau)^g \qquad (107) \end{aligned}$$

where $\ln(1-\tau)^g = \ln(1-\tau) - \ln(1-\overline{\tau})$. Substituting this into the Phillips curve for wage inflation gives,

$$\pi^w_{H,t} = \beta E_t \pi^w_{H,t+1} + \frac{(1-\theta_w\beta)(1-\theta_w)}{(1+\varphi\epsilon_w)\theta_w}((1+\varphi)y^g - g^g - (w^g - p^g_H) - \ln(1-\tau)^g) \tag{108}$$

while the similar expression for price inflation is given by,

$$\pi_{H,t} = \beta E_t\{\pi_{H,t+1}\} + \frac{(1-\theta_p\beta)(1-\theta_p)}{\theta_p}[(w_t^g - p_{H,t}^g) - \ln(1-\tau_t^v)^g] \quad (109)$$

where the 'gapped' real wage evolves according to,

$$w_t^g - p_{H,t}^g = \pi_{H,t}^w - \pi_{H,t} + w_{t-1}^g - p_{H,t-1}^g - \Delta a_t \quad (110)$$

We can also write (86) for natural variables as

$$y_t^n = E_t\{y_{t+1}^n\} - (r_t^n - \rho) - E_t\{g_{t+1}^n - g_t^n\} \quad (111)$$

so

$$r_t^n = \rho + E_t\{y_{t+1}^n - y_t^n\} - E_t\{g_{t+1}^n - g_t^n\} \quad (112)$$

This allows us to write (86) for gap variables as

$$y_t^g = y_t - y_t^n = E_t\{y_{t+1}^g\} - (r_t - E_t\{\pi_{H,t+1}\} - r_t^n) - E_t\{g_{t+1}^g - g_t^g\} \quad (113)$$

Note that, given (103), the real natural rate of interest depends - like natural output - only on the productivity shock, the steady-state levels of distortionary taxation and the optimal level of government spending.

3 Optimal policy

3.1 The Social Planner's Problem in a Small Open Economy.

The social planner simply decides how to allocate consumption and production of goods within the economy, subject to the various constraints implied by operating as part of a larger group of economies e.g. IRS. Since they are concerned with real allocations nominal inertia is not an issue.

The social planner will produce equal quantities of all goods, so we can write

$$Y = AN \quad (114)$$

Combining aggregate demand and international risk sharing implies

$$\begin{aligned} c &= c^* + (1-\alpha)s = c^* + (1-\alpha)(y - c^* - g) && (115) \\ &= \alpha c^* + (1-\alpha)(y-g) && (116) \end{aligned}$$

The social planner maximises

$$c + \chi g - \frac{N^{1+\psi}}{1+\psi} \quad (117)$$

subject to these two constraints, which implies (max wrt g and Y)

$$\begin{aligned} \frac{1-\alpha}{Y-G} - \frac{N^{1+\psi}}{Y} &= 0 && (118) \\ -\frac{1-\alpha}{Y-G} + \frac{\chi}{G} &= 0 && (119) \end{aligned}$$

so that

$$
\begin{aligned}
N &= (1-\alpha+\chi)^{\frac{1}{1+\psi}} && (120)\\
G &= \frac{Y\chi}{1-\alpha+\chi} && (121)
\end{aligned}
$$

which implies the optimal value for g,

$$
g = \ln(1+\frac{\chi}{1-\alpha}) \tag{122}
$$

3.2 Flexible Price Equilibrium

Profit-maximising behaviour implies that firms will operate at the point at which marginal costs equal marginal revenues,

$$
\begin{aligned}
mc^{w,n} &= -\ln(\mu^{w}) && (123)\\
&= -\ln(1-\tau)+a+(1+\varphi)n^{n}-g^{n}-(w^{n}-p_{H}^{n})\\
&= -\ln(\mu)+\ln(1-\varkappa)-\ln(1-\tau)-\ln(1-\tau^{v})\\
&\quad +(1+\varphi)n^{n}+\ln(1-\frac{G^{n}}{Y^{n}}) && (124)\\
\left(1-\frac{1}{\epsilon}\right)\left(1-\frac{1}{\epsilon_w}\right) &= \frac{(1-\varkappa)}{(1-\tau_v)(1-\tau)}(N^{n})^{(1+\varphi)}(1-\frac{G^{n}}{Y^{n}}) && (125)
\end{aligned}
$$

Now if G^n is given by the optimal rule (122), then

$$
1-\frac{G^{n}}{Y^{n}} = \frac{1-\alpha}{1-\alpha+\chi} \tag{126}
$$

If the subsidy $\varkappa$ is given by

$$
(1-\varkappa) = (1-\frac{1}{\epsilon})(1-\frac{1}{\epsilon_w})(1-\tau_v)(1-\tau)/(1-\alpha) \tag{127}
$$

then

$$
N^{n} = (1-\alpha+\chi)^{\frac{1}{1+\psi}} \tag{128}
$$

is identical to the optimal level of employment above. Here the subsidy has to overcome the distortions due to monopoly pricing in the goods and labour markets, as well as the distortionary income and sales taxes.

3.3 The Social Planner's Problem in a Monetary Union

Here the social planner maximises utility across all countries subject to

$$
\begin{aligned}
Y^{i} &= A^{i}N^{i} && (129)\\
Y^{i} &= C_{i}^{i}+\int_{0}^{1} C_{i}^{j}dj+G^{i} && (130)
\end{aligned}
$$

Recall that utility for country i at time t is

$$\ln C_t + \chi_t \ln G_t - \frac{N_t^{1+\varphi}}{1+\varphi} \tag{131}$$

and

$$C = (Y - G)^{1-\alpha}[\int_0^1 C^i di]^\alpha \tag{132}$$

Optimisation implies

$$(N^i)^\varphi = A^i \frac{1-\alpha}{C_i^i} = A^i \int_0^1 \frac{\alpha}{C_i^j} dj = A^i \frac{\chi^i}{G^i} \tag{133}$$

This implies

$$N^i = (1+\chi^i)^{\frac{1}{1+\varphi}} \tag{134}$$

$$C^i = (\frac{1-\alpha}{1+\chi^i})Y^i \tag{135}$$

$$C_i^j = (\frac{\alpha}{1+\chi^i})Y^i \qquad j \neq i \tag{136}$$

$$G^i = \frac{\chi^i}{1+\chi^i} Y^i = \frac{\chi^i A^i}{(1+\chi^i)^{\frac{\varphi}{1+\varphi}}} \tag{137}$$

The latter implies $g = \ln(1+\chi^i)$ which is a different fiscal rule than in the case of the small open economy. Why? In the small open economy case governments have an incentive to increase government spending (which is devoted solely to domestically produced goods) to induce an appreciation in the terms of trade. In aggregate this cannot happen, but it leaves government spending inefficiently high. The government spending rule under monetary union eliminates this externality. This also has implications for the derivation of union and national welfare which is discussed below.

3.4 Social Welfare in a Monetary Union

Appendix 3 derives the quadratic approximation to utility across member states to obtain a union-wide objective function.

$$\Gamma = -\frac{(1+\chi)}{2} \sum_{t=0}^{\infty} \beta^t \int_0^1 [\frac{\epsilon}{\lambda}\pi_{i,t}^2 + \frac{\epsilon_w(1+\varphi\epsilon_w)}{\lambda_w}(\pi_{i,t}^w)^2 + (y_t^{i,g})^2(1+\varphi) + \frac{1}{\chi}(g_t^{i,g})^2]di + tip + O[3] \tag{138}$$

It contains quadratic terms in price and wage inflation reflecting the costs of price and wage dispersion induced by inflation in the presence of nominal inertia, as well as terms in the output gap and government spending gap. The weights attached to each element are a function of deep model parameters. The key to obtaining this quadratic specification is in adopting an employment subsidy which eliminates the distortions caused by imperfect competition in labour and product markets as well as the impact of distortionary sales and income taxes. It is also important to note that it is assumed that national fiscal authorities have internalised the externality caused by their desire to appreciate the terms of trade through excessive government expenditure.

In deriving national welfare for an economy outside of monetary union this externality has not been corrected for and, it can be shown that the objective function becomes,

$$\Gamma = -\frac{(1-\alpha+\chi)}{2}\sum_{t=0}^{\infty}\beta^t[\frac{\epsilon}{\lambda}\pi_{i,t}^2+\frac{\epsilon_w(1+\varphi\epsilon_w)}{\lambda_w}(\pi_{i,t}^w)^2+(y_t^{i,g})^2(1+\varphi)+\frac{1}{\chi}(g_t^{i,g})^2]+tip+O[3] \quad (139)$$

However, in the simulations that follow below we correct for this renormalisation to allow welfare to be compared across EMU and open economy cases.

4 Precommitment Policy

In this section we shall consider the precommitment policies for the various variants of our model.

4.1 Precommitment in the Small Open Economy

We shall initially consider policy in an economy not participating in monetary union. This will serve as a benchmark against which to evaluate policy within the union, and is also informative as union-wide policy will be of the same form as national monetary policy in the open economy. In the small open economy case, our 'gapped' model of country i consists of the following equations. Firstly, the Phillips curve for wage inflation,

$$\pi_{i,t}^w = \beta E_t\pi_{i,t+1}^w+\frac{(1-\theta_w\beta)(1-\theta_w)}{(1+\varphi\epsilon_w)\theta_w}((1+\varphi)y_t^{i,g}-g_t^{i,g}-(w_t^{i,g}-p_{i,t}^g)-\ln(1-\tau_t^i)^g) \quad (140)$$

while the similar expression for price inflation is given by,

$$\pi_{i,t} = \beta E_t\{\pi_{i,t+1}\}+\frac{(1-\theta_p\beta)(1-\theta_p)}{\theta_p}[(w_t^{i,g}-p_{i,t}^g)-\ln(1-\tau_t^{i,s})^g] \quad (141)$$

where the 'gapped' real wage evolves according to,

$$w_t^{t,g}-p_{i,t}^g = \pi_{i,t}^w-\pi_{i,t}+w_{t-1}^{i,g}-p_{i,t-1}^g-\Delta a_t^i \quad (142)$$

Finally there is the euler equation for consumption,

$$y_t^{i,g} = g_t^{i,g} + E_t\{y_{t+1}^i - g_{t+1}^i + \pi_{i,t+1}\} - (r_t^i - r_t^{i,n}) \tag{143}$$

Define, $\lambda = \frac{(1-\theta_p\beta)(1-\theta_p)}{\theta_p}$, $\lambda_w = \frac{(1-\theta_w\beta)(1-\theta_w)}{\theta_w}$ and $\widetilde{\lambda}_w = \frac{\lambda_w}{(1+\varphi\epsilon_w)}$.

The objective function for the national government is given by,

$$\sum_{t=0}^{\infty}\beta^t\left[\frac{\epsilon}{\lambda}\pi_{i,t}^2 + \frac{\epsilon_w}{\widetilde{\lambda}_w}(\pi_{i,t}^w)^2 + (y_t^{i,g})^2(1+\varphi) + \frac{1}{\chi}(g_t^{i,g})^2\right] \tag{144}$$

Forming the Lagrangian,

$$\begin{aligned} L_t \quad = \quad & \sum_{t=0}^{\infty}\beta^t[\frac{\epsilon}{\lambda}\pi_{i,t}^2 + \frac{\epsilon_w}{\widetilde{\lambda}_w}(\pi_{i,t}^w)^2 + (y_t^{i,g})^2(1+\varphi) + \frac{1}{\chi}(g_t^{i,g})^2 \\ & +\lambda_t^{\pi^w,i}(\pi_{i,t}^w - \beta E_t\pi_{i,t+1}^w - \widetilde{\lambda}_w((1+\varphi)y_t^{i,g} - g_t^{i,g} - (w_t^{i,g} - p_{i,t}^g) - \ln(1-\tau_t^i)^g) \\ & +\lambda_t^{\pi,i}(\pi_{i,t} - \beta E_t\{\pi_{i,t+1}\} - \lambda[rw^{i,g} - \ln(1-\tau_t^{i,s})^g]) \\ & +\lambda_t^{y,i}(y_t^{i,g} - g_t^{i,g} - E_t\{y_{t+1}^{i,g} - g_{t+1}^{i,g} + \pi_{i,t+1}\} + (r_t^i - r_t^{i,n})) \\ & +\lambda_t^{rw,i}(rw_t^{i,g} - \pi_{i,t}^w + \pi_{i,t} - rw_{t-1}^{i,g} + \Delta a_t)] \end{aligned}$$

The first first-order conditions is for the interest rate,

$$\lambda_t^{y,i} = 0 \tag{145}$$

i.e. when there is a national monetary policy it is as if the monetary authorities have control over consumption such that the consumption Euler equation ceases to be a constraint. The foc for the sales tax gap, $\ln(1-\tau^v)^g$,

$$\lambda\lambda_t^{\pi,i} = 0 \tag{146}$$

i.e. the price Phillips curve ceases to be a constraint on maximising welfare -VAT tax changes can offset the impact on any other variables driving price inflation. Similarly, the condition for income taxes is given by,

$$\widetilde{\lambda}_w\lambda_t^{\pi^w,i} = 0 \tag{147}$$

The remaining focs are for real wages,

$$-\lambda\lambda_t^{\pi,i} + \widetilde{\lambda}_w\lambda_t^{\pi^w,i} + \lambda_t^{rw,i} - \beta E_t\lambda_{t+1}^{rw,i} = 0 \tag{148}$$

inflation,

$$\frac{2\epsilon}{\lambda}\pi_{i,t} + \lambda_t^{\pi,i} - \lambda_{t-1}^{\pi,i} - \beta^{-1}\lambda_{t-1}^{y,i} + \lambda_t^{rw,i} = 0 \tag{149}$$

wage inflation,

$$\frac{2\epsilon_w}{\widetilde{\lambda}_w}\pi_{i,t}^w + \lambda_t^{\pi^w,i} - \lambda_{t-1}^{\pi^w,i} - \lambda_t^{rw,i} = 0 \tag{150}$$

the government spending gap,

$$\frac{2}{\chi}g_t^{i,g} + \widetilde{\lambda}_w \lambda_t^{\pi^w,i} - \lambda_t^{y,i} + \beta^{-1}\lambda_{t-1}^{y,i} = 0 \tag{151}$$

and the output gap,

$$2(1+\varphi)y_t^{i,g} - \widetilde{\lambda}_w(1+\varphi)\lambda_t^{\pi^w,i} + \lambda_t^{y,i} - \beta^{-1}\lambda_{t-1}^{y,i} = 0 \tag{152}$$

Combinations of these first order conditions define the target criteria for a variety of cases, such that alternative fiscal regimes are modelled by retaining or dropping the focs associated with a specific fiscal instrument. In deriving precommiment policy we consider the general solution to the system of focs after the initial time period, which gives us a set of target criteria which policy must achieve. In the initial period we have two ways of solving the system of focs. We can derive a set of initial values for lagrange multipliers dated at time t=-1, such that the target criteria are also followed in the initial period - this constitutes what is known as the policy from a 'timeless perspective' (see Woodford 2003). Alternatively we can allow policy makers to exploit the fact that expectations are fixed in the initial period and utilise the discretionary solution for the initial period only. This amounts to setting the time t=-1 dated lagrange multipliers to zero (see Currie and Levine (1993)). Although we adopt the latter approach in simulations, we do not report the focs associated with it in order to conserve space since these do not provide any additional economic intuition.

4.1.1 Small Open Economy - All Fiscal Instruments

Let us consider the case where the fiscal authorities have access to government spending and both tax instruments in order to stabilise their economy, when operating alongside the national monetary authorities. Appendix 5 details the derivation of target criteria in this case which are, for government spending,

$$g_t^{i,g} = 0 \tag{153}$$

the output gap,

$$y_t^{i,g} = 0 \tag{154}$$

price inflation,

$$\pi_{i,t} = 0 \tag{155}$$

and wage inflation,

$$\pi_{i,t}^w = 0 \tag{156}$$

In other words the effects of shocks on these gap variables are completely offset and do not have any welfare implications. Since these target criteria are all static, it will also be the case that the optimal discretionary policy will be the same as this precommitment policy. In terms of policy assignments, monetary policy ensures the output gap is zero. Wage inflation is eliminated by following the following rule for income taxes,

$$\ln(1-\tau_t^i)^g = -rw_t^{i,g} \tag{157}$$

while a similar form of rule (but of the opposite sign) for sales taxes eliminates price inflation,

$$\ln(1-\tau_t^{i,s})^g = rw_t^{i,g} \tag{158}$$

4.1.2 Small Open Economy - VAT and Government Spending

Now suppose we only have access to the sales tax and government spending as fiscal instruments. In this case our government spending rule becomes,

$$y_t^{i,g} + \frac{1}{\chi} g_t^{i,g} = 0 \tag{159}$$

while monetary policy achieves the following trade-off between output and inflation under commitment,

$$\frac{\epsilon_w}{\widetilde{\lambda}_w}\pi_{i,t}^w + \frac{\epsilon}{\lambda}\pi_{i,t} + \frac{1}{\widetilde{\lambda}_w}\Delta y_t^{i,g} = 0 \tag{160}$$

this is similar to the form of target criteria that emerges when only prices are sticky and the only policy instrument is interest rates. Essentially the presence of the sales tax instrument simplifies the target criteria that emerges when both prices and wages are sticky (See Woodford (2003), Chapter 7). The sales tax rule that simplfies the output-inflation trade-off facing the national monetary authorities is given by,

$$y_t^{i,g} - \epsilon r w_t^{i,g} + \epsilon \ln(1-\tau_t^{i,s})^g = 0 \tag{161}$$

4.1.3 Small Open Economy - Income Tax and Government Spending

Now suppose we have the income tax instrument and government spending. Appendix 5 shows that our policy assignment contains the usual goverment spending rule,

$$y_t^{i,g} + \frac{1}{\chi} g_t^{i,g} = 0 \tag{162}$$

which is our first target criterion.

The optimal mix of inflation and output to be achieved through the monetary policy instrument gives us our second target crtiterion,

$$\frac{\epsilon_w}{\widetilde{\lambda}_w}\pi_{i,t}^w + \frac{\epsilon}{\lambda}\pi_{i,t} + \frac{1}{\lambda\widetilde{\lambda}_w}(\Delta y_t^{i,g}) = 0 \tag{163}$$

and the income tax rule is,

$$\frac{2}{\chi} g_t^{i,g} + 2\epsilon_w((1+\varphi)y_t^{i,g} - g_t^{i,g} - rw_t^{i,g} - \ln(1-\tau_t^i)^g) = 0 \tag{164}$$

4.1.4 Small Open Economy - No Tax Instruments, Only Government Spending

With only a single instrument the target criteria under commitment becomes more complex generating a target criterion for monetary policy with a mixture of backward and forward-looking elements.

$$\begin{aligned} 0 &= \frac{\epsilon}{\lambda}\pi_{i,t} + \frac{\epsilon_w}{\widetilde{\lambda}_w}\pi^w_{i,t} + \frac{1}{\widetilde{\lambda}_w}\Delta y^{i,g}_t \qquad (165) \\ &+ \frac{1}{\lambda}\left(\Delta y^{i,g}_t + \epsilon_w((1+\varphi)y^{i,g}_t - g^{i,g}_t - rw^{i,g}_t) + \frac{1}{\widetilde{\lambda}_w}(\Delta^2 y^{i,g}_t - \beta\Delta^2 E_t y^{i,g}_{t+1})\right) \end{aligned}$$

while government spending follows the usual rule,

$$y^{i,g}_t + \frac{1}{\chi}g^{i,g}_t = 0 \qquad (166)$$

This describes pre-commitment policy for all cases in the small open economy.

4.2 Optimal Precommitment Under EMU:

The Lagrangian associated with the EMU case is given by,

$$\begin{aligned} L_t &= \int_0^1 \sum_{t=0}^{\infty} \beta^t [\frac{\epsilon}{\lambda}\pi^2_{i,t} + \frac{\epsilon_w}{\widetilde{\lambda}_w}(\pi^w_{i,t})^2 + (y^{i,g}_t)^2(1+\varphi) + \frac{1}{\chi}(g^{i,g}_t)^2 \\ &+ \lambda^{\pi^w,i}_t(\pi^w_{i,t} - \beta E_t\pi^w_{i,t+1} - \widetilde{\lambda}_w((1+\varphi)y^{i,g}_t - g^{i,g}_t - (w^{i,g}_t - p^g_{i,t}) - \ln(1-\tau^i_t)^g) \\ &+ \lambda^{\pi,i}_t(\pi_{i,t} - \beta E_t\{\pi_{i,t+1}\} - \lambda[rw^{i,g}_t - \ln(1-\tau^{i,s}_t)^g]) \\ &+ \lambda^{y,i}_t(y^{i,g}_t - g^{i,g}_t - E_t\{y^{i,g}_{t+1} - g^{i,g}_{t+1} + \pi_{i,t+1}\} + (r_t - r^{i,n}_t)) \\ &+ \lambda^{rw,i}_t(rw^{i,g}_t - \pi^w_{i,t} + \pi_{i,t} - rw^{i,g}_{t-1} + \Delta a_t)]di \end{aligned}$$

The key differences between this and the previous problem is that we now have a national interest rate and welfare is integrated across all member states. As a result, we no longer have an foc for the national interest rate, but the foc for the union-wide interest rate is given by,

$$\int_0^1 \lambda^{y,i}_t di = 0 \qquad (167)$$

However, since all economies in our model are symmetrical in structure, we can aggregate focs across our economies which delivers, in terms of union-wide aggregates, an identical set of focs as we find in the small open economy case above. Therefore, the target criterion for the ECB will take the same form as that attributed to the national monetary authority, but re-specified in terms of union wide aggregates.

In terms of national focs, we begin with the foc for the sales tax gap, $\ln(1 - \tau^v)^g$,

$$\lambda\lambda_t^{\pi,i} = 0 \tag{168}$$

i.e. the price Phillips curve ceases to be a constraint on maximising welfare -VAT tax changes can offset the impact on any other variables driving price inflation. Similarly, the condition for income taxes is given by,

$$\widetilde{\lambda}_w \lambda_t^{\pi^w,i} = 0 \tag{169}$$

implying that income taxes can control wage inflation, and for real wages,

$$-\lambda\lambda_t^{\pi,i} + \widetilde{\lambda}_w \lambda_t^{\pi^w,i} + \lambda_t^{rw,i} - \beta E_t \lambda_{t+1}^{rw,i} = 0 \tag{170}$$

The remaining first-order conditions are for inflation,

$$\frac{2\epsilon}{\lambda}\pi_{i,t} + \lambda_t^{\pi,i} - \lambda_{t-1}^{\pi,i} - \beta^{-1}\lambda_{t-1}^{y,i} + \lambda_t^{rw,i} = 0 \tag{171}$$

The foc for wage inflation is given by,

$$\frac{2\epsilon_w}{\widetilde{\lambda}_w}\pi_{i,t}^w + \lambda_t^{\pi^w,i} - \lambda_{t-1}^{\pi^w,i} - \lambda_t^{rw,i} = 0 \tag{172}$$

All that remains is the foc for the government spending gap,

$$\frac{2}{\chi}g_t^{i,g} + \widetilde{\lambda}_w \lambda_t^{\pi^w,i} - \lambda_t^{y,i} + \beta^{-1}\lambda_{t-1}^{y,i} = 0 \tag{173}$$

and the output gap,

$$2(1+\varphi)y_t^{i,g} - \widetilde{\lambda}_w(1+\varphi)\lambda_t^{\pi^w,i} + \lambda_t^{y,i} - \beta^{-1}\lambda_{t-1}^{y,i} = 0 \tag{174}$$

Combinations of these first order conditions define the national target criteria for a variety of cases. Alternative fiscal regimes are modelled by retaining or dropping the focs associated with a specific fiscal instrument. The details of these manipulations are in Appendix 6.

4.2.1 EMU Case - All Fiscal Instruments

With all fiscal instruments, but the loss of the monetary policy instrument we can no-longer eliminate the welfare effects of shocks. Therefore our policy configuration is no longer trivial. It involves the following government spending rule,

$$(1+\varphi)y_t^{i,g} + \frac{1}{\chi}g_t^{i,g} = 0 \tag{175}$$

which ensures the optimal composition of output. There is an income tax rule,

$$(1+\varphi)y^{i,g} - g^{i,g} - rw^{i,g} - \ln(1-\tau_i^i)^g) = 0 \tag{176}$$

which eliminates wage inflation, and VAT tax rule,

$$(1+\varphi)y_t^{i,g} + \epsilon(\ln(1-\tau_t^{i,s})^g - rw_t^{i,g}) = 0 \tag{177}$$

which achieves the appropriate balance between output and inflation while recognising that competitiveness will need to be restored once any shock has passed.

With these fiscal rules in place in each member state, the ECB will act to ensure the average output gap within the union is zero,

$$\int_0^1 y_t^{i,g} di = y_t^g = 0 \tag{178}$$

which will imply that the average government spending gap and rates of price and wage inflation will all be zero in the union.

4.2.2 EMU Case - VAT and Government Spending

Our rule for the sales tax is given by,

$$\frac{1}{\chi} g_t^{i,g} = \epsilon(\ln(1-\tau_t^{i,s})^g - rw_t^{i,g}) \tag{179}$$

while the government spending rule is more dynamic, implying,

$$\begin{aligned} -\frac{2}{\varphi\chi} g_t^{i,g} &= 2\frac{(1+\varphi)}{\varphi} y_t^{i,g} + 2\epsilon_w((1+\varphi)y_t^{i,g} - g_t^{i,g} - rw_t^{i,g}) \\ &+ \frac{2}{\varphi\chi\widetilde{\lambda}_w}(\Delta g_t^{i,g} - \beta\Delta E_t g_{t+1}^{i,g}) + 2\frac{(1+\varphi)}{\varphi\widetilde{\lambda}_w}(\Delta y_t^{i,g} - \beta\Delta E_t y_{t+1}^{i,g}) \end{aligned} \tag{180}$$

With only two instruments and four constraints, the precommitment policy implies a degree of both inertial and forward-looking behaviour typical of analysis of monetary policy in such settings (see Woodford (2003), Chapter 7). With these national fiscal rules in place, the ECB's monetary policy will seek to achieve the following balance between inflation and output for the union as a whole,

$$\frac{\epsilon_w}{\widetilde{\lambda}_w}\pi_t^w + \frac{\epsilon}{\lambda}\pi_t + \frac{1}{\widetilde{\lambda}_w}\Delta y_t^g = 0 \tag{181}$$

4.2.3 EMU Case - Income Tax and Government Spending

Now suppose now income tax the only tax instrument, we have a rule for this instrument of the form,

$$
\begin{aligned}
0 \quad = \quad & (1+\varphi)y_t^{i,g} - \epsilon(rw_t^{i,g}) - \epsilon_w((1+\varphi)y_t^{i,g} - g_t^{i,g} - rw_t^{i,g} - \ln(1-\tau_t^i)^g) \\
& + \frac{\epsilon_w}{\lambda}((1+\varphi)(\beta E_t y_{t+1}^{i,g} - y_t^{i,g}) - (\beta E_t g_{t+1}^{i,g} - g_t^{i,g}) \qquad (182) \\
& -(\beta E_t rw_{t+1}^{i,g} - rw_t^{i,g}) - (\beta E_t \ln(1-\tau_{t+1}^i)^g - \ln(1-\tau_t^i)^g)
\end{aligned}
$$

, which is complemented by our government spending rule,

$$
\frac{1}{\chi}g_t^{i,g} + (1+\varphi)y_t^{i,g} = 0 \qquad (183)
$$

Assuming the national fiscal authorities implement these rules, then the ECB will seek to achieve the following balance between output and inflation across the union as a whole,

$$
\frac{\epsilon_w}{\widetilde{\lambda}_w}\pi_t^w + \frac{\epsilon}{\lambda}\pi_t + \frac{1}{\lambda\widetilde{\lambda}_w}(\Delta y_t^g) = 0 \qquad (184)
$$

4.2.4 EMU Case - Government Spending the Only Instrument

Appendix 6 details the solution in this case, which is too complex to afford any real intuition. Numerical analysis of this and the other cases is considered in the next section. As well as a comparison with policy under the assumption of discretion.

5 Optimal Policy Simulations

In this section we examine the optimal policy response to a technology shock both within and outside monetary union. We consider discretionary and commitment policies and compuete the welfare benefits of employing our various fiscal instruments as stabilisation devices. In this section we outline the response of the model to a series of shocks. Following GM we adopt the following parameter set, $\varphi = 1$, $\mu = 1.2$, $\epsilon = 6$, $\theta_p = 0.75$, $\theta_w = 0.75, \beta = 0.99$, $\alpha = 0.4$, and $\gamma = 0.25$. The ratio of government spending to gdp of 0.25 implies that $\chi = \frac{\gamma}{1-\gamma} = 1/3$ in the EMU case[2]. Additionally, since we have sticky wages we need to adopt a measure of the steady-state mark-up in the labour market, $\mu^w = 1.2$ which implies,$\epsilon_w = 6$. While the degree of wage stickiness is given by $\theta_w = 0.75$ which means that wage contracts last for, on average, one year. This

[2]In the small open economy case, $\gamma = \frac{\chi}{1-\alpha+\chi}$ such that fixing the share of government spending requires a rescaling of χ to take account of the incentive to excessive government spending which is assumed to be eliminated within the union. In the simulations, to facilitate comparisons, we fix χ at the value described above in both the open economy and EMU cases. We also rescale welfare to eliminate this difference in welfare functions.

is consistent with the evidence in Leith and Malley (2005). The productivity shock follows the following pattern,

$$a_t = \rho_a a_{t-1} + \xi_t \tag{185}$$

where we adopting a degree of persistence in the productivity shock of $\rho_a = 0.6$, although we consider the implications of alternative degrees of persistence below.

5.1 Small Open Economy Simulations

We begin by considering the response of a small open economy to a 1% technology shock with the degree of persistence described above, when no use is made of fiscal policy for stabilisation purposes i.e. only monetary policy is used to stabilise the economy in the face of shocks. Figure 1 details the responses of key endogenous variables to the technology shock, under both commitment and discretion[3]. It is important to note that, in the absence of sticky wages, monetary policy could completely offset the welfare consequences of this shock by reducing interest rates in line with the increase in productivity. This would ensure that domestic and foreign demand rises for the additional products and that the full effects of the producivity gain are captured in real wages. However, when nominal wages are also sticky it is not possible for monetary policy alone to offset the effects of the shock. As a result of the wage stickiness, real wages are slow to rise following the positive productivity shock and, as a result, marginal costs fall initially and this means that the initial jump in inflation is negative. This leads to a cut in nominal interest rates (greater than that implied by the productivity shock's affect on the natural interest rate) and a jump depreciation of the nominal exchange rate, although interest rates will be relatively lower after this initial jump as rising marginal costs increase inflation. The terms of trade depreciate initially, but this is far more modest than in the flex wage case. As a result consumption rises in the home country relative to abroad, but not by as much as output since the depreciation of the terms of trade makes domestic goods attractive to foreign consumers. Implicitly IRS and the positive productivity shock imply that resources are being sent abroad to support foreign consumption, although this is not as pronounced as in the flexible wage case.

We know from our derivation of optimal policy above that when we utilise all fiscal instruments we can completely offset the impact of this shock on all welfare-relevant gap variables, implying that there is no welfare cost to the shock. Essentially, the monetary instrument eliminates the impact on the output gap of the shock by cutting interest rates. This creates demand for domestically produced goods by encouraging domestic consumption, which has a bias towards domestically produced goods, and depreciating the exchange rate leading to an increase in foreign demand. Income taxes are reduced to eliminate wage inflation, but simultaneously achieve the required increase in the post tax real

[3] The numerical solution of optimal policy under commitment and discretion is based on Soderlind (2003).

wage. The sales tax is increased to eliminate the deflation that would otherwise emerge as a result of the reduction in marginal costs (due to falling income taxes and rising productivity). There is no need to adjust government spending when the government has access to the tax instruments without constraint.

We can also consider a number of intermediate cases where not all fiscal instruments are employed. The welfare benefits of various combinations of fiscal instrument are given in Table 1. These suggest that the greatest gains to stabilisation in the open economy case, come from the tax instruments with only relatively minor benefits from varying government spending. Either tax instrument is highly effective in reducing the welfare costs of the technology shock.

Table 1 - Costs of Technology Shock in Small Open Economy with Alternative Fiscal Instruments.

Discretionary Policy	No Taxes	Income Tax	Sales Tax	Both Taxes
Govt Spending	1.2479	0.2252	0.2906	0
No Govt Spending	1.2503	0.2319	0.3025	0

Commitment Policy	No Taxes	Income Tax	Sales Tax	Both Taxes
Govt Spending	1.2414	0.1443	0.1850	0
No Govt Spending	1.2438	0.1517	0.1961	0

5.2 EMU Simulations

We now consider the response to an idiosyncratic technology shock for a country operating under EMU (see Figure 2). We begin by considering the case where there is no fiscal response to the shock. In this case the equilibriating mechanism is the need to restore competitiveness following the shock. Relative to the small open economy case, there is now no monetary policy response to either the local productivity shock or its inflationary repercussions. As a result there is no attempt to boost consumption and output with a fall in interest rates in response to the shock (in an attempt to replicate the flex price outcome).There is an initial fall in marginal costs and inflation which induces a depreciation in the terms of trade, although this is far smaller than in the open economy case above. This shifts demand towards domestic goods such that eventually prices and wages rise until the competitiveness gain has been reversed. In the presence of nominal inertia and with no monetary policy/exchange rate instrument, it is difficult to induce the necessary movements in the terms of trade/real exchange rate to create a market for the extra goods that could be produced as a result of the productivity shock. This failure is reflected the large negative output gap and real wage gap.

We then contrast this with the case where country i employs all the fiscal instruments at its disposal (see Figure 3). We find that optimal policy attempts to reduce the impact of the shock on competitiveness. Therefore following the productivity shock the sales and income taxes are increased. The latter completely offsets the impact of the shock on wage inflation, while the former allows for only a very limited reduction in prices following the productivity shock. As a result of this attempt to avoid price adjustment, there is a substantial negative output gap although this is partially offset by a rise in government spending. This has the advantage of creating a market for the additional goods, which given the complete home bias in government spending, boosts real wages and moderates the fall in inflation. There is now a smaller depreciation of the terms of trade due to the changes in taxation and since there is less need to encourage foreign consumption of the increased domestic production of goods due to the home bias in government consumption. The welfare gain from fiscal stabilisation to this degree is an approximate halving of the costs of a technology shock when part of a monetary union.

We again consider a number of intermediate cases where not all fiscal instruments are employed. The welfare benefits of various combinations of fiscal instrument are given in Table 2. This suggest that the greatest gains to stabilisation, when part of monetary union, come from utilising government spending as a stabilisation instrument. This is due to the assumed home-bias in government spending which allows policy makers to purchase the additional goods produced as a result of the productivity shock without requiring any competiveness changes which subsequently have to be undone once the shock has passed. It is also interesting to note that even with all fiscal instruments in place the costs of the shock under EMU are still greater than in the small open economy case with monetary policy as the only available policy instrument.

Table 2 - Costs of Technology Shock Under EMU with Alternative Fiscal Instruments - Discretion.

Discretionary Policy	No Taxes	Income Tax	Sales Tax	Both Taxes
Govt Spending	2.5132	2.4173	1.8196	1.7229
No Govt Spending	3.4681	3.2306	3.0110	2.7730

Commitment Policy	No Taxes	Income Tax	Sales Tax	Both Taxes
Govt Spending	2.5060	2.4075	1.8133	1.7229
No Govt Spending	3.4681	3.2242	2.9982	2.7730

5.3 Implementation Lags

A frequently cited argument against employing fiscal instruments in a stabilisation role is that it often takes long periods to implement the tax changes and government spending changes suggested by optimal policy. In this subsection we assess the extent to which implementation lags affect the welfare gains from fiscal stabilisation. We assume that it takes n-periods to change policy instruments following a change in the information set. This can be modelled by conditioning policy instruments on information sets of n-periods ago, such that our structural model can be written as follows, with our NKPC for wage inflation,

$$\pi_{i,t}^{w}=\beta E_t\pi_{i,t+1}^{w}+\frac{(1-\theta_w\beta)(1-\theta_w)}{(1+\varphi\epsilon_w)\theta_w}((1+\varphi)y_t^{i,g}-E_{t-n}g_t^{i,g}-(w_t^{i,g}-p_{i,t}^{g})-E_{t-n}\ln(1-\tau_t^{i})^{g}) \tag{186}$$

the similar expression for price inflation,

$$\pi_{i,t}=\beta E_t\{\pi_{i,t+1}\}+\frac{(1-\theta_p\beta)(1-\theta_p)}{\theta_p}[(w_t^{i,g}-p_{i,t}^{g})-E_{t-n}\ln(1-\tau_t^{i,s})^{g}] \tag{187}$$

and the euler equation for consumption,

$$y_t^{i,g}=E_{t-n}g_t^{i,g}+E_t\{y_{t+1}^{i}-E_{t-n}g_{t+1}^{i}+\pi_{i,t+1}\}-(r_t-r_t^{i,n}) \tag{188}$$

the equation describing the evolution of the 'gapped' real wage is unaffected. This implies that it will take n-periods following the shock for the fiscal authorities to be able to implement a fiscal policy plan. Details of how implementation lags affect the welfare affects of policy are given below in Table 3. Here it is clear that, in the context of a monetary union, implementation lags serious affect the ability of the fiscal variables to deal with the shock, with lags of one year almost completely eliminating any benefits of fiscal stabilisation through the use of a single instrument.

Table 3 - Costs of Technology Shock Under EMU with Alternative Fiscal Instruments - Various Implementation Lags.

Commitment Policy	Government Spending	Income Tax	Sales Tax
No Lags	2.5060	3.2242	2.9982
1-Period Lag	3.1754	3.2564	3.2173
2-Period Lag	3.3674	3.3023	3.3181
3-Period Lag	3.4173	3.3457	3.3804
4-Period Lag	3.4279	3.3795	3.4112

Of course these results are highly dependent upon the amount of inertia in the economy. For example, increasing the degree of presistence in the shock from 0.6 to 0.9 implies the following set of results where the proportional increase in the welfare costs of the shock arising from implementation lags are smaller (for example a year's delay in implementing government spending changes implies an increase in costs of 37% (19%) with a degree of persistence of 0.6 (0.9)).

Table 4 - Costs of Technology Shock Under EMU with Alternative Fiscal Instruments - Various Implementation Lags.

Commitment Policy	Government Spending	Income Tax	Sales Tax
No Lags	6.5893	7.2093	6.8543
1-Period Lag	7.1708	7.2377	7.1074
2-Period Lag	7.5185	7.2994	7.3198
3-Period Lag	7.7266	7.3870	7.5286
4-Period Lag	7.8500	7.4892	7.7019

5.4 Adding in Debt

In this subsection we consider the impact of introducing government debt to our analysis. Up until now we have assumed that there was a lump-sum tax instrument which was utilised to balance the budget whenever other fiscal instruments were used in a stabilisation role. In this section we assume that any variations in government spending or our sales or income tax instruments are not automatically adjusted for in this way. Instead, any inconsistency between government tax revenues and spending will affect government debt. Policy must then ensure that any relevant government budget constraint is satisfied. Appendix 7 derives the intertemporal budget constraint for the union as a whole,

$$\int D_t^i di = R_{t-1}B_{t-1} = -\sum_{T=t}^{\infty} E_t[Q_{t,T}(\int_0^1 (P_{i,T}G_{i,T} - W_{i,T}N_{i,T}(\tau_{i,T} - \varkappa_i) - \tau_{i,T}^v P_{i,T}Y_T^i - T_{i,T})di)] \tag{189}$$

where B_t is the aggregate level of the national debt stocks. With global market clearing in asset markets the series of national budget constraints imply that the only public-sector intertemporal budget constraint in our model is a union-wide constraint. What is the intuition for this? Given complete capital markets and our assumed initial conditions (zero net foreign assets and identical *ex ante* structures in each economy) this means that initially consumers expect similar fiscal policy regimes in their respective economies. To the extent that *ex post* this is not the case, there will be state contingent payments under IRS that ensure marginal utilities are equated throughout the union (after controlling for real exchange rate differences)[4]. This would seem to suggest that fiscal sustainability questions within this framework are a union-wide rather than a national concern. Given that a national government's contribution to union-wide finances is negligible then this could be taken to imply that debt is not an issue in utilising fiscal instruments at the national level.

However, given the fiscal institutions which have been constructed as part of EMU, it seems unlikely that without such constraints each member state would expect to operate under *ex ante* similar fiscal regimes. Therefore it may be reasonable to assume that each member state operates a budget constraint of this form at the national level, such that there is no need for the only institution with a union-wide instrument, the ECB, to be concerned with issues of fiscal solvency. Therefore we impose, as an external constraint created within the institutions of EMU, a national government budget constraint of the form,

$$D_t^i = R_{t-1}B_{t-1}^i = -\sum_{T=t}^{\infty} E_t[Q_{t,T}(P_{i,T}G_T^i - W_T^i N_T^i(\tau_T^i - \varkappa_i) - \tau_T^{i,s} P_{i,T}Y_T^i - T_T^i)] \tag{190}$$

We need to transform this budget constraint into a loglinearised 'gap' equation to allow it to be integrated into our policy problem. Additionally, in order

[4] For the purposes of illustration, suppose taxes were lump-sum and one economy unexpectedly cut all taxes to zero. There would be transfers from this economy to the other economies to ensure that the consumers in the other economies were not disadvantaged by the higher taxes they had to pay to ensure union-wide solvency.

to keep the welfare functions we have just developed an obvious assumption to make is that lump-sum taxation is used to finance the steady-state subsidy (which offsets, in steady-state, the distortions caused by distortionary taxation and imperfect competition in wage and price setting). We shall then assume that lump-sum taxation cannot be used to alter this subsidy or to finance any other government activities, including the kind of spending and distortionary tax adjustments as stabilisation measures we are interested in. This implies that $W_T^i N_T^i \varkappa_i = T_T^i$ in all our economies at all points in time, allowing us to simplify the budget constraint to,

$$R_{t-1}B_{t-1}^i = -\sum_{T=t}^{\infty} E_t[Q_{t,T}(P_{i,T}G_T^i - W_T^i N_T^i \tau_T^i - \tau_T^{i,s} P_{i,T} Y_T^i)] \tag{191}$$

i.e. distortionary taxation and spending adjustments are required to service government debt as well as stabilise the economy. This defines the basic trade-off facing policy makers in utilising these instruments. In real terms this can be written as,

$$\frac{D_t^i}{P_{i,t}} = -\sum_{T=t}^{\infty} E_t[R_{t,T}(P_{i,T}G_T^i - \frac{W_T^i}{P_{i,T}} N_T^i \tau_T^i - \tau_T^{i,s} P_{i,T} Y_T^i)] \tag{192}$$

where $R_{t,T} \equiv Q_{t,T} P_{i,T}/P_{i,t}$ is the stochastic discount factor used to discount a real income stream, which implies the flow budget constraint,

$$B_t^i = R_{t-1}B_{t-1}^i + P_{i,t}G_t^i - P_{i,t}Y_t^i \tau_t^{i,s} - W_t^i N_t^i \tau_t^i \tag{193}$$

Rewriting in real terms,

$$\frac{B_t^i}{P_{i,t}} = R_{t-1}\frac{P_{i,t-1}}{P_{i,t}}\frac{B_{t-1}^i}{P_{i,t-1}} + G_t^i - Y_{i,t}^i \tau_t^{i,s} - \frac{W_t^i}{P_{i,t}} N_t^i \tau_t^i \tag{194}$$

Rewriting to get the budget constraint in a form consistent with the gapped definitions of the tax rates,

$$\frac{B_t^i}{P_{i,t}} = R_{t-1}\frac{P_{i,t-1}}{P_{i,t}}\frac{B_{t-1}^i}{P_{i,t-1}} - Y_{i,t}^i + G_t^i - Y_{i,t}^i(1-\tau_t^{i,s}) - \frac{W_t^i}{P_{i,t}} N_t^i (1-\tau_t^i) + \frac{W_t^i}{P_{i,t}} N_t^i \tag{195}$$

This can be log-linearised as,

$$\begin{aligned}
b_t^i \quad = \quad & \overline{R} b_{t-1}^i + \overline{R}(r_{t-1} - \pi_{i,t}) + \frac{\overline{G}^i}{\overline{B}^i} \ln G_t^i + \frac{(1-\overline{\tau}^{i,s})\overline{Y}^i}{\overline{B}^i} \ln(1-\tau_t^{i,s}) \qquad (196) \\
& -\frac{\overline{\tau}^v \overline{Y}^i}{\overline{B}^i} y_t^i + \frac{(1-\overline{\tau}^i)\overline{rw}^i \overline{N}^i}{\overline{B}^i} \ln(1-\tau_t^i) - \frac{\overline{\tau rw}^i \overline{N}^i}{\overline{B/P}^i}(rw_t^i + n_t^i) \\
& -\overline{R} \ln \overline{B}^i - \overline{R}(\overline{r}) - \frac{\overline{G}^i}{\overline{B}^i} \ln \overline{G}^i - \frac{(1-\overline{\tau}^{i,s})\overline{Y}^i}{\overline{b}^i} \ln(1-\overline{\tau}_t^i) \\
& -\frac{\overline{\tau}^{i,s}\overline{Y}^i}{\overline{B}^i}\overline{Y}^i + \frac{(1-\overline{\tau}^i)\overline{rw}^i \overline{N}^i}{\overline{B}^i} \ln(1-\overline{\tau}^i) - \frac{\overline{\tau}^i \overline{rw}^i \overline{N}^i}{\overline{B}^i}(\overline{rw}^i + \overline{n}^i)
\end{aligned}$$

where $b^i_t = \ln(\frac{B^i_t}{P_{i,t}})$ and $\overline{B}^i = (\overline{B^i/P_i})$. Re-writing in gap form,

$$\begin{aligned} b^g_t &= \overline{R}b^g_{t-1} + \overline{R}(r^g_{t-1} - \pi_{i,t}) + \frac{\overline{G}^i}{\overline{B}^i}\ln G^{i,g}_t + \frac{(1-\overline{\tau}^{i,s})\overline{Y}^i}{\overline{B}^i}\ln(1-\tau^{i,s}_t)^g \qquad (197) \\ &- \frac{\overline{\tau}^{i,s}\overline{Y}^i}{\overline{B}^i}y^{i,g}_t + \frac{(1-\overline{\tau}^i)\overline{rw}^i\overline{N}^i}{\overline{B}^i}\ln(1-\tau^i_t)^g - \frac{\overline{\tau rw}^i\overline{N}^i}{\overline{B}^i}(rw^{i,g}_t + n^{i,g}_t) \end{aligned}$$

From the production function, to the first order, $y^{i,g}_t = n^{i,g}_t$ so this can be rewritten as,

$$\begin{aligned} b^{i,g}_t &= \overline{R}b^{i,g}_{t-1} + \overline{R}(r^g_{t-1} - \pi_{i,t}) + \frac{\overline{G}^i}{\overline{b}}\ln G^{i,g}_t + \frac{(1-\overline{\tau}^{i,s})\overline{y}}{\overline{b}}\ln(1-\tau^{i,s}_t)^g \qquad (198) \\ &-(\frac{\overline{\tau}^{i,s}\overline{Y}^i}{\overline{B}^i} + \frac{\overline{\tau rw}^i\overline{N}^i}{\overline{B}^i})y^{i,g}_t + \frac{(1-\overline{\tau}^i)\overline{rw}^i\overline{N}^i}{\overline{B}^i}\ln(1-\tau^i_t)^g - \frac{\overline{\tau rw}^i\overline{N}^i}{\overline{B}^i}(rw^{i,g}_t) \end{aligned}$$

Note, however that g_t in model is defined as, $\ln(1-\frac{G}{Y})$. This implies, to a first order, that,

$$\begin{aligned} \ln G^i &= \ln(\frac{G^i}{Y^i}) + \ln(Y^i) \qquad (199) \\ &= \ln(1-\exp(-g^i)) + y^i \qquad (200) \\ &= \frac{1-\gamma^{i,n}}{\gamma^{i,n}}g^i + y^i \qquad (201) \end{aligned}$$

where $\gamma^{i,n} = G^i/Y^i$. In gap form this becomes,

$$\ln G^{i,g} = \frac{1-\gamma^{i,n}}{\gamma^{i,n}}g^{i,g} + y^{i,g} \qquad (202)$$

Introducing this to the budget constraint,

$$\begin{aligned} b^{i,g}_t &= \overline{R}b^{i,g}_{t-1} + \overline{R}(r^g_{t-1} - \pi_{i,t}) + \frac{\overline{G}^i}{\overline{B}^i}\frac{1-\gamma^{i,n}}{\gamma^{i,n}}g^{i,g}_t + \frac{(1-\overline{\tau}^{i,s})\overline{Y}^i}{\overline{B}^i}\ln(1-\tau^{i,s}_t)^g \qquad (203) \\ &-(\frac{\overline{\tau}^{i,s}\overline{Y}^i}{\overline{B}^i} + \frac{\overline{\tau rw}^i\overline{N}^i}{\overline{B}^i} - \frac{\overline{G}^i}{\overline{B}^i})y^{i,g}_t + \frac{(1-\overline{\tau}^i)\overline{rw}^i\overline{N}^i}{\overline{B}^i}\ln(1-\tau^i_t)^g - \frac{\overline{\tau rw}^i\overline{N}^i}{\overline{B}^i}(rw^{i,g}_t) \end{aligned}$$

This then constitutes our national government budget constraint which must remain stationary as an additional constraint on policy makers.

5.5 Optimal Precommitment Policy with Government Debt

5.5.1 Open Economy Case

The Lagrangian associated with the open economy case in the presence of a national government budget constraint is given by,

$$\begin{aligned} L_t &= \sum_{t=0}^{\infty}\beta^t[\frac{\epsilon}{\lambda}\pi_{i,t}^2 + \frac{\epsilon_w}{\widetilde{\lambda}_w}(\pi_{i,t}^w)^2 + (y_t^{i,g})^2(1+\varphi) + \frac{1}{\chi}(g_t^{i,g})^2 \\ &+\lambda_t^{\pi^w,i}(\pi_{i,t}^w - \beta E_t\pi_{i,t+1}^w - \widetilde{\lambda}_w((1+\varphi)y_t^{i,g} - g_t^{i,g} - (w_t^{i,g} - p_{i,t}^g) - \ln(1-\tau_t^i)^g)) \\ &+\lambda_t^{\pi,i}(\pi_{i,t} - \beta E_t\{\pi_{i,t+1}\} - \lambda[rw_t^{i,g} - \ln(1-\tau_t^{i,s})^g]) \\ &+\lambda_t^{y,i}(y_t^{i,g} - g_t^{i,g} - E_t\{y_{t+1}^{i,g} - g_{t+1}^{i,g} + \pi_{i,t+1}\} + (r_t^i - r_t^{i,n})) \\ &+\lambda_t^{rw,i}(rw_t^{i,g} - \pi_{i,t}^w + \pi_{i,t} - rw_{t-1}^{i,g} + \Delta a_t) \\ &+\lambda_t^{b,i}(b_t^{i,g} - \overline{R}b_{t-1}^{i,g} - \overline{R}(r_{t-1}^{i,g} - \pi_{i,t}) - b_g g_t^{i,g} - b_{\tau^v}\ln(1-\tau_t^{i,s})^g \\ &+b_y y_t^{i,g} - b_\tau\ln(1-\tau_t^i)^g + b_{rw}rw_t^{i,g})] \end{aligned}$$

where $b_g = \frac{\overline{G}^i}{\overline{B}^i}\frac{1-\gamma^{i,n}}{\gamma^{i,n}}$, $b_{\tau^v} = \frac{(1-\overline{\tau}^{i,s})\overline{Y}^i}{\overline{B}^i}$,$b_y = \frac{\overline{\tau}^{i,s}\overline{Y}^i}{\overline{B}^i} + \frac{\overline{\tau}\overline{rw}^i\overline{N}^i}{\overline{B}^i} - \frac{\overline{G}^i}{\overline{B}^i}$,$b_\tau = \frac{(1-\overline{\tau}^i)\overline{rw}^i\overline{N}^i}{\overline{B}^i}$, and $b_{rw} = \frac{\overline{\tau}\overline{rw}^i\overline{N}^i}{\overline{B}^i}$. The foc for the national interest rate is given by,

$$\lambda_t^{y,i} - E_t\lambda_{t+1}^{b,i} = 0 \tag{204}$$

Here monetary policy must now take account of its impact on the government's finances.

In terms of national focs, we begin with the foc for the sales tax gap, $\ln(1-\tau^{i,s})^g$,

$$\lambda\lambda_t^{\pi,i} - b_{\tau^v}\lambda_t^{b,i} = 0 \tag{205}$$

Similarly, the condition for income taxes is given by,

$$\widetilde{\lambda}_w\lambda_t^{\pi^w,i} - b_\tau\lambda_t^{b,i} = 0 \tag{206}$$

and for real wages,

$$-\lambda\lambda_t^{\pi,i} + \widetilde{\lambda}_w\lambda_t^{\pi^w,i} + \lambda_t^{rw,i} - \beta\lambda_{t+1}^{rw,i} + b_{rw}\lambda_t^{b,i} = 0 \tag{207}$$

The remaining first-order conditions are for debt,

$$\lambda_t^{b,i} - \beta\overline{R}\lambda_{t+1}^{b,i} = 0 \tag{208}$$

which implies that, $E_0\lambda_t^{b,i} = \lambda^{b,i}\ \forall t$. In other words policy must ensure that the 'cost' of the government's budget constraint is constant following a shock which is the basis of the random walk result of Schmitt-Grohe and Uribe (2004). This also implies that the lagrange multipliers for the wage and price phillips curves are constant over time too. The remaining focs are for inflation,

$$\frac{2\epsilon}{\lambda}\pi_{i,t} + \lambda_t^{\pi,i} - \lambda_{t-1}^{\pi,i} - \beta^{-1}\lambda_{t-1}^{y,i} + \lambda_t^{rw,i} + \overline{R}\lambda_t^{b,i} = 0 \tag{209}$$

wage inflation,

$$\frac{2\epsilon_w}{\widetilde{\lambda}_w}\pi_{i,t}^w + \lambda_t^{\pi^w,i} - \lambda_{t-1}^{\pi^w,i} - \lambda_t^{rw,i} = 0 \tag{210}$$

the government spending gap,

$$\frac{2}{\chi}g_t^{i,g} + \widetilde{\lambda}_w \lambda_t^{\pi^w,i} - \lambda_t^{y,i} + \beta^{-1}\lambda_{t-1}^{y,i} - b_g \lambda_t^{b,i} = 0 \tag{211}$$

and the output gap,

$$2(1+\varphi)y_t^{i,g} - \widetilde{\lambda}_w(1+\varphi)\lambda_t^{\pi^w,i} + \lambda_t^{y,i} - \beta^{-1}\lambda_{t-1}^{y,i} + b_y \lambda_t^{b,i} = 0 \tag{212}$$

Combinations of these first order conditions define the national target criteria for a variety of cases. In the open economy case the optimal combination of wage and price inflation is given by,

$$\frac{2\epsilon}{\lambda}\pi_{i,t} + \frac{2\epsilon_w}{\widetilde{\lambda}_w}\pi_{i,t}^w = 0 \tag{213}$$

This essentially describes the balance between wage and price adjustment in achieving the new steady-state real wage consistent with the new steady-state tax rates required to stabilise the debt stock following the shock. Taking the foc for the output gap, we have,

$$2(1+\varphi)y_t^{i,g} + \lambda^{b,i}(-b_\tau(1+\varphi) + (1-\beta^{-1}) + b_y) = 0 \tag{214}$$

which defines the value of the Lagrange multiplier associated with the government's budget constraint which implies that the output gap is constant, but non-zero. The sales and income tax rules for the open economy case are given by, respectively,

$$-2\epsilon(rw_t^{i,g} - \ln(1-\tau_t^v)^g) + (b_{rw} + b_\tau - b_{\tau^v})\lambda^{b,i} = 0 \tag{215}$$

and,

$$2\epsilon_w((1+\varphi)y_t^{i,g} - g_t^{i,g} - rw_t^{i,g} - \ln(1-\tau_t)^g)) + (b_{rw} + b_\tau - b_{\tau^v})\lambda^{b,i} = 0 \tag{216}$$

Finally the government spending rule is given by,

$$\frac{2}{\chi}g_t^{i,g} + (b_\tau - (1-\beta^{-1}) - b_g)\lambda^{b,i} = 0 \tag{217}$$

which is again constant given the definition of $\lambda^{b,i}$ above.

Taken together these target criteria imply that optimal policy ensures that output and government spending adjust instantaneously to their new steady-state levels, while gradual price and wage adjustment implies that we eventually reach the new steady-state tax rates consistent with debt sustainability.

5.5.2 EMU Case

If we formulate the corresponding problem for the EMU case it is given by,

$$L_t = \int_0^1 \sum_{t=0}^{\infty} \beta^t [\frac{\epsilon}{\lambda}\pi_{i,t}^2 + \frac{\epsilon_w}{\widetilde{\lambda}_w}(\pi_{i,t}^w)^2 + (y_t^{i,g})^2(1+\varphi) + \frac{1}{\chi}(g_t^{i,g})^2$$

$$
\begin{aligned}
&+\lambda_t^{\pi^w,i}(\pi_{i,t}^w - \beta E_t \pi_{i,t+1}^w - \widetilde{\lambda}_w((1+\varphi)y_t^{i,g} - g_t^{i,g} - (w_t^{i,g} - p_{i,t}^g) - \ln(1-\tau_t^i)^g) \\
&+\lambda_t^{\pi,i}(\pi_{i,t} - \beta E_t\{\pi_{i,t+1}\} - \lambda[rw_t^{i,g} - \ln(1-\tau_t^{i,s})^g]) \\
&+\lambda_t^{y,i}(y_t^{i,g} - g_t^{i,g} - E_t\{y_{t+1}^{i,g} - g_{t+1}^{i,g} + \pi_{i,t+1}\} + (r_t - r_t^{i,n})) \\
&+\lambda_t^{rw,i}(rw_t^{i,g} - \pi_{i,t}^w + \pi_{i,t} - rw_{t-1}^{i,g} + \Delta a_t) \\
&+\lambda_t^{b,i}(b_t^{i,g} - \overline{R}b_{t-1}^{i,g} - \overline{R}(r_{t-1}^g - \pi_{i,t}) - b_g g_t^{i,g} - b_{\tau^v}\ln(1-\tau_t^{i,s})^g \\
&+b_y y_t^{i,g} - b_\tau \ln(1-\tau_t^i)^g + b_{rw} rw_t^{i,g})]di
\end{aligned}
$$

In order to obtain intuition for optimal policy in this case it is helpful to relate the (constant) value of the lagrange multiplier associated with the national government budget constraint to national output and government spending gaps,

$$2(1+\varphi)y_t^{i,g} + \frac{2}{\chi}g_t^{i,g} + (b_y - \varphi b_\tau - b_g)\lambda_t^{b,i} = 0 \tag{218}$$

which also implies a constant relationship between the output and government spending gaps following a shock.

There is an income tax rule,

$$2\epsilon_w((1+\varphi)y_t^{i,g} - g_t^{i,g} - rw_t^{i,g} - \ln(1-\tau_t)^g)) + (b_{rw} + b_\tau - b_{\tau^v})\lambda^{b,i} = 0 \tag{219}$$

and a sales-tax rule,

$$
\begin{aligned}
0 &= 2(1+\varphi)y_t^{i,g} + (b_y - \varphi b_\tau + 1 - \beta^{-1} + b_{rw} - b_{\tau^v})\lambda^{b,i} && (220)\\
&\quad -2\epsilon(rw_t^{i,g} - \ln(1-\tau_t^v)^g) && (221)
\end{aligned}
$$

and a government spending rule,

$$
\begin{aligned}
0 &= \frac{2}{\chi}g_t^{i,g} - 2(1+\varphi)\frac{(b_\tau - b_g - 1 + \beta^{-1})}{(-b_\tau(1+\varphi) + (1-\beta^{-1}) + b_y)}y_t^{i,g} \\
&\quad +2\epsilon(1 + \frac{(b_\tau - b_g - 1 + \beta^{-1})}{(-b_\tau(1+\varphi) + (1-\beta^{-1}) + b_y)})(rw_t^{i,g} - \ln(1-\tau_t^v)^g) \\
&\quad +2\epsilon_w(1 + \frac{(b_\tau - b_g - 1 + \beta^{-1})}{(-b_\tau(1+\varphi) + (1-\beta^{-1}) + b_y)})((1+\varphi)y_t^{i,g} - g_t^{i,g} - rw_t^{i,g} - \ln(1-\tau_t)^g)
\end{aligned} \tag{222}
$$

which in conjuction with the tax rules, will achieve the constant relationship between government spending and the output gap given above. Here we can see that the presence of the national government budget constraint essentially introduces a constant wedge into the target criteria outlined above for the EMU case without debt which reflects the needs to adjust fiscal instruments and steady-state output and real wages to be consistent with the new steady-state level of government debt which follows a random walk.

While the ECB will set the union-wide interest rate consistently with the following first-order condition,

$$\int_0^1 (\lambda_t^{y,i} - E_t\lambda_{t+1}^{b,i})di = 0$$

Assuming that the national fiscal authorities will follow these fiscal rules, this will ensure that union-wide monetary policy achieves the following balance between wage and price inflation,

$$\frac{\epsilon}{\lambda}\pi_t + \frac{\epsilon_w}{\widetilde{\lambda}_w}\pi_t^w = 0 \tag{223}$$

with other union wide variables following paths consistent with the target criteria outlined for the small open economy case above.

5.5.3 Simulations

We then consider the ability of an economy operating under EMU to stabilise the economy following a productivity shock through the use of fiscal instruments when it must also ensure sustainability of the government's finances. Figure 4 details the paths of key endogenous variables following the same shock considered above. In the case of commitment policy, the results are very similar to the case where there was a lump-sum tax instrument balancing the national fiscal budget. The main difference is that there is a gradual reduction in government debt in response to the higher tax revenues generated by the positive productivity shock, until it reaches its new lower steady-state with reduced sales and income taxes and higher government spending to satisfy the national fiscal constraint. This is essentially a generalisation of the random walk result of Benigno and Woodford (2003) and Schmitt-Grohe and Uribe (2004), which also has echoes of tax smoothing (Barro (1979)), but with additional inertia caused by the various sources of inertia in the model. Essenitally, following the shock we have a random-walk in the steady-state debt and tax levels. However, these differences have little welfare implications with the costs of the shock rising from 1.7251 to 1.7309.

A more substantial difference occurs when we consider the discretionary solution. Under discretion the national fiscal authorities taking future inflationary expectations as given, are tempted to use inflation rather than their fiscal instruments to stabilise national government debt. As a result, the larger initial fall in inflation and the initial fall in income taxes serves to increase rather than reduce debt initially. This temptation, which is a form of inflationary bias, remains unless the debt stock returns close to its initial value. Therefore, even although there is no explicit debt target, optimal discretionary policy comes close to eliminating the effects of the productivity shock on the debt stock. In this particular case, the welfare consequences of the shock are not dramatically affected by the introduction of government debt and welfare costs rise from 1.7251 to 1.7889.

6 Conclusions

The paper detailed a microfounded model of an economy trading with several trading partners which can serve as a basis for analysing issues relating to the optimal implementation of a stabilising fiscal policy for economies that have joined together as part of a monetary union. The model presented followed Gali and Monacelli (*op. cit.*), but introduced some additional features - namely distortionary income and sales taxes, as well as sticky-wages (in addition to sticky prices). Despite the fact that there are several economies being modelled simultaneously the absence of strategic interactions between their policy makers, made the analysis of policy relatively tractable. As a result it was possible to analyse optimal policy for a single country within the union. We found that, with empirically plausible degrees of price and wage-stickiness, joining a monetary union imposed significant welfare costs on an economy's inhabitants when that economy was subject to technology shocks. These costs could be reduced by up to half by utilising fiscal instruments, with government spending being particularly effective in responding to idiosyncratic shocks due to its assumed home bias in the purchase of goods. However, even employing all three fiscal instruments could not reduce the costs of these shocks to the levels found when the economy stayed out of monetary union and retained use of a monetary policy (and therefore exchange rate) instrument. We also found that implementation lags could significantly affect the ability of fiscal instruments to deal with shocks, but that the need to ensure fiscal solvency when utilising tax instruments had negligible welfare consequences.

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Appendix 1 - Wage Setting

Recall the optimal wage set by those households that are able to re-set wages in period t,

$$W(k)_t^{-1-\varphi\epsilon_w} = \frac{E_t\left(\sum_{s=0}^{\infty}(\theta_w)^s\left[Q_{t,t+s}W_{t+s}^{\epsilon_w}N_{t+s}(1-\tau)\right]\right)}{E_t\left(\sum_{s=0}^{\infty}(\theta_w)^s\left[Q_{t,t+s}\mu_w W_{t+s}^{\epsilon_w(1+\varphi)}N_{t+s}^{1+\varphi}C_{t+s}P_{t+s}\right]\right)} \quad (224)$$

Note that in equilibrium,

$$\beta^s\left(\frac{C_t}{C_{t+s}}\right)\left(\frac{P_t}{P_{t+s}}\right) = Q_{t,t+s} \quad (225)$$

Accordingly the expression for the optimal re-set wage is given by,

$$\overline{W}_t^{-1-\varphi\epsilon_w} = \frac{E_t\left(\sum_{s=0}^{\infty}(\theta_w\beta)^s\left[W_{t+s}^{\epsilon_w}N_{t+s}(1-\tau)C_{t+s}^{-1}P_{t+s}^{-1}\right]\right)}{E_t\left(\sum_{s=0}^{\infty}(\theta_w\beta)^s\left[\mu_w W_{t+s}^{\epsilon_w(1+\varphi)}N_{t+s}^{1+\varphi}\right]\right)} \quad (226)$$

This expression can be log-linearised as,

$$\frac{1+\varphi\epsilon_w}{1-\theta_w\beta}\overline{w}_t - \frac{1}{1-\theta_w\beta}\ln(\mu^w) = E_t\left(\sum_{s=0}^{\infty}(\theta_w\beta)^s\left[\varphi n_{t+s} + \epsilon_w\varphi w_{t+s} + c_{t+s} + p_{t+s} - \ln(1-\tau_{t+s})\right]\right) \quad (227)$$

Quasi-differencing this expression yields,

$$\frac{1+\varphi\epsilon_w}{1-\theta_w\beta}\overline{w}_t = \frac{1+\varphi\epsilon_w}{1-\theta_w\beta}a_w\beta E_t\overline{w}_{t+1} + \varphi n_{t+s} + \epsilon_w\varphi w_{t+s} + c_{t+s} + p_{t+s} - \ln(1-\tau_{t+s}) - \ln(\mu^w) \quad (228)$$

The wage index evolves according to the following law of motion,

$$W_t = \left[(1-\theta_w)\overline{W}_t^{(1-\epsilon_w)} + \theta_w W_{t-1}^{1-\epsilon_w}\right]^{\frac{1}{1-\epsilon_w}} \quad (229)$$

Log-linearising this expression gives,

$$w_t = (1-\theta_w)\overline{w}_t + \theta_w w_{t-1} \quad (230)$$

These two expressions can be solved for wage inflation to obtain the New Keynesian Phillips curve for wage inflation,

$$\pi_t^w = \beta E_t\pi_{t+1}^w + \frac{(1-\theta_w\beta)(1-\theta_w)}{(1+\varphi\epsilon_w)\theta_w}(\varphi n_t - w_t + c_t + p_t - \ln(1-\tau_t) + \ln(\mu^w)) \quad (231)$$

here the forcing variable captures the extent to which the consumer's labour supply decision is not the same as it would be under flexible prices.

Appendix 2 - Price Setting

Recall the optimal price set by firms that are able to reset prices in period t,

$$P_t^* = \frac{\sum_{s=0}^{\infty}(\theta_p)^s Q_{t,t+s}\left[\epsilon \frac{W_{t+s}}{P_{t+s}} P_{H,t+s}^{\epsilon} \frac{Y_{t+s}}{A_{t+s}}\right]}{\sum_{s=0}^{\infty}(\theta_p)^s Q_{t,t+s}\left[(\epsilon-1)(1-\tau_{t+s}^v)P_{t+s}^{-1}P_{H,t+s}^{\epsilon}Y_{t+s}(1+\chi)\right]} \tag{232}$$

Note that in equilibrium,

$$\beta^s\left(\frac{C_t}{C_{t+s}}\right)\left(\frac{P_t}{P_{t+s}}\right) = Q_{t,t+s} \tag{233}$$

Accordingly, the expression for the optimal price can be re-written as,

$$P_t^* = \frac{\sum_{s=0}^{\infty}(\theta_p\beta)^s \frac{C_tP_t}{C_{t+s}Pt+s}\left[\epsilon \frac{W_{t+s}}{P_{t+s}} P_{H,t+s}^{\epsilon} \frac{Y_{t+s}}{A_{t+s}}\right]}{\sum_{s=0}^{\infty}(\theta_p\beta)^s \frac{C_tP_t}{C_{t+s}Pt+s}\left[(\epsilon-1)(1-\tau_{t+s}^v)P_{t+s}^{-1}P_{H,t+s}^{\epsilon}Y_{t+s}(1+\chi)\right]} \tag{234}$$

This can be loglinearised as,

$$\overline{p}_{H,t} = \ln(\mu) + (1-\theta_p\beta)E_t\left(\sum_{s=0}^{\infty}(\theta_w\beta)^s\left[-a_{t+s}+w_{t+s}-\ln(1-\tau_{t+s}^v)-v_t\right]\right) \tag{235}$$

where $\overline{p}_{H,t}$ is the log of the optimal price set by those firms that were able to set price in period t, and $v = -\ln(1+\chi)$.Quasi-differencing this expression yields,

$$\frac{1}{1-\theta_p\beta}\overline{p}_{H,t} = \frac{1}{1-\theta_p\beta}\theta_p\beta E_t\overline{p}_{H,t+1} - a_t + w_t - \ln(1-\tau_t^v) - v_t + \ln(\mu) \tag{236}$$

While domestic prices evolve according to,

$$P_{H,t} = \left[(1-\theta_w)P_t^{*(1-\epsilon_w)} + \theta_w P_{H,t-1}^{1-\epsilon_w}\right]^{\frac{1}{1-\epsilon_w}} \tag{237}$$

This can be log-linearised as,

$$p_{H,t} = (1-\theta_p)\overline{p}_{H,t} + \theta_p p_{H,t-1} \tag{238}$$

Solving for $\overline{p}_{H,t}$ and substituting into the expression for quasi-differenced optimal price yields,

$$\begin{aligned}\frac{1}{1-\theta_p\beta}\left(\frac{p_{H,t}}{1-\theta_p} - \frac{\theta_p p_{H,t-1}}{1-\theta_p}\right) &= \frac{1}{1-\theta_p\beta}\theta_p\beta\left(\frac{E_tp_{H,t+1}}{1-\theta_p} - \frac{\theta_p p_{H,t}}{1-\theta_p}\right) \\ &\quad -a_t + w_t - \ln(1-\tau_t^v) - v_t + \ln(\mu)\end{aligned} \tag{239}$$

This can be solved as,

$$\pi_t^P = \beta E_t\pi_{t+1}^P + \frac{(1-\theta_p\beta)(1-\theta_p)}{\theta_p}(-a_t+w_t-p_{H,t}-\ln(1-\tau_t^v)-v_t+\ln(\mu)) \tag{240}$$

where $mc = -a_t + w_t - p_{H,t} - \ln(1-\tau_t^v) - v_t$ are the real log-linearised marginal costs of production. In the absence of sticky prices profit maximising behaviour implies, $mc = -\ln(\mu)$.

Appendix 3 - Derivation of Union Welfare

The measure of welfare which we shall seek to approximate is based on an aggregate of household utility,

$$\ln C_t + \chi_t \ln G_t - \int_0^1 \frac{(N(k)_t)^{1+\varphi}}{1+\varphi} dk \tag{241}$$

The first term can be expanded as

$$\begin{aligned} c &= c^n + c^g && (242) \\ &= c^n + \alpha \int_0^1 c^{g,j} dj + (1-\alpha)(y^g - g^g) && (243) \end{aligned}$$

using (115). Before considering the second term we need to note the following general result relating to second order approximations,

$$\frac{Y_t - Y}{Y_t} = y_t + \frac{1}{2} y_t^2 + o\left(\|a\|^3\right) \tag{244}$$

where $o\left(\|a\|^3\right)$represents terms that are of order higher than 3 in the bound $\|a\|$on the amplitude of the relevant shocks.

Suppose we take the Taylor series expansion of $\ln(\frac{Y_t}{Y})$, we obtain,

$$\ln(\frac{Y_t}{Y}) = \ln(1) + \frac{1}{Y}(Y_t - Y) - \frac{1}{2}\frac{1}{Y^2}(Y_t - Y)^2 + o\left(\|a\|^3\right) \tag{245}$$

Soving for the precentage deviation we have,

$$\frac{1}{Y}(Y_t - Y) = \ln(\frac{Y_t}{Y}) + \frac{1}{2}\frac{1}{Y^2}(Y_t - Y)^2 + o\left(\|a\|^3\right) \tag{246}$$

Since we are ignoring all terms of order higher than 2 we can rewrite the second order term as follows,

$$\frac{1}{Y}(Y_t - Y) = \ln(\frac{Y_t}{Y}) + \frac{1}{2}\ln(\frac{Y_t}{Y})^2 + o\left(\|a\|^3\right) \tag{247}$$

This will be used in various places in the derivation of welfare. Now consider the second order approximation to the second term for an individual household k,

$$\begin{aligned} \frac{N(k)^{1+\varphi}}{1+\varphi} &= \frac{(N(k)^n)^{1+\varphi}}{1+\varphi} + (N(k)^n)^{\varphi}(N(k)_t - N(k)^n) && (248) \\ &+ \frac{1}{2}\varphi(N(k)^n)^{\varphi-1}((N(k)_t - N(k)^n))^2\} + o\left(\|a\|^3\right) \end{aligned}$$

which can be re-written as,

$$\begin{aligned} \frac{N(k)^{1+\varphi}}{1+\varphi} &= \frac{(N(k)^n)^{1+\varphi}}{1+\varphi} + (N(k)^n)^{\varphi+1}(\frac{N(k)_t - N(k)^n}{N(k)^n}) && (249) \\ &+ \frac{1}{2}\varphi(N(k)^n)^{\varphi+1}((\frac{N(k)_t - N(k)^n}{N(k)^n}))^2\} + o\left(\|a\|^3\right) \end{aligned}$$

using the above relationship this can be rewritten in terms of gap variables,

$$\frac{N(k)^{1+\varphi}}{1+\varphi} = \frac{(N(k)^n)^{1+\varphi}}{1+\varphi} + (N(k)^n)^{1+\varphi}\{n(k)^g + \frac{1}{2}(n(k)^g)^2(1+\varphi)\} + o\left(\|a\|^3\right) \quad (250)$$

We now need to aggregate this over households and relate to aggregate variables.

$$\begin{aligned}\int_0^1 \frac{N(k)^{1+\varphi}}{1+\varphi}dk &= \frac{(N^n)^{1+\varphi}}{1+\varphi} \quad (251)\\ &+(N^n)^{1+\varphi}\{\int_0^1 n(k)^g dk + \frac{1}{2}(1+\varphi)\int_0^1 (n(k)^g)^2 dk\} + o\left(\|a\|^3\right)\end{aligned}$$

The demand for an individual household's labour is given by,

$$N(k) = \left(\frac{W(k)}{W}\right)^{-\epsilon_w} N \quad (252)$$

Taking logs and integrating over households,

$$\int_0^1 n^{k,g} dk = n^g + \int_0^1 \ln\left(\frac{W(k)}{W}\right)^{-\epsilon_w} dk \quad (253)$$

Consider the relative price, $\left(\frac{W(k)}{W}\right)^{-\epsilon_w}$.

Let $\widehat{w}(k) = w(k) - w$ notice that, $\left(\frac{W(k)}{W}\right)^{1-\epsilon_w} = \exp[(1-\epsilon_w)\widehat{w}(k)]$
$= 1 + (1-\epsilon_w)\widehat{w}_{H,t}(i) + \frac{(1-\epsilon_w)^2}{2}\left(\widehat{w}_{H,t}(k)\right)^2 + o\left(\|a\|^3\right)$

From the definition of W we have $1 = \int_0^1 \left(\frac{W(k)}{W}\right)^{1-\epsilon_w} dk$. Therefore integrating the above expression across k the $LHS = 1$ and the expression simplifies to,

$$E_k\{\widehat{w}(k)\} = \frac{\epsilon_w - 1}{2} E_k\{\widehat{w}(k)^2\} \quad (254)$$

which is of second order.

Therefore we can rewrite the relationship between the sum of household labour inputs and the CES aggregate of these inputs as,

$$\begin{aligned}\int_0^1 n(k)^g dk &= n^g + \epsilon_w \frac{1-\epsilon_w}{2} E_k\{\widehat{w}(k)^2\}\\ &= n^g + \epsilon_w \frac{1-\epsilon_w}{2} var_k\{w(k)^2\} \quad (255)\end{aligned}$$

From the definition of the variance it is also the case that,

$$\int_0^1 (n(k)^g)^2 dk = var_k\{n(k)^g\} + (\int_0^1 (n(k)^g) dk)^2 \quad (256)$$

where $var_k\{n(k)^g\} = (\epsilon_w)^2 var_k\{w(k)\}$.Using this expression and (255) the second order approximation to the disutility of labour supply can be written as,

$$\begin{aligned}\frac{N^{1+\varphi}}{1+\varphi}dk &= \frac{(N^n)^{1+\varphi}}{1+\varphi} + \\ & (N^n)^{1+\varphi}(n^g + \frac{1}{2}(1+\varphi)(n^g)^2 + (\epsilon_w \frac{1-\epsilon_w}{2} + \frac{(\epsilon_w)^2(1+\varphi)}{2})var_k\{w(k)^2\}) + o\left(\|a\|^3\right) \\ &= \frac{(N^n)^{1+\varphi}}{1+\varphi} + (N^n)^{1+\varphi}\{n^g + \frac{1}{2}(1+\varphi)(n^g)^2 + \frac{\epsilon_w(1+\varphi\epsilon_w)}{2}var_k\{w(k)^2\}) + o\left(\|a\|^3\right)\end{aligned} \tag{257}$$

Now we need to relate the labour input gap to the output gap and a measure of price dispersion. Aggregating the individual firms' demand for labour yields,

$$N = (\frac{Y}{A})\int_0^1 (\frac{P_H(i)}{P_H})^{-\epsilon} di \tag{258}$$

It can be shown that

$$n^g = y^g + \ln[\int_0^1 (\frac{P_H(i)}{P_H})^{-\epsilon} di] \tag{259}$$

$$= y^g + \frac{\epsilon}{2} var_i\{p_H(i)\} + o\left(\|a\|^3\right) \tag{260}$$

While the first line is fairly straight-forward, there are several steps behind the next line. The first things to note is that $z_t = \ln[\int_0^1 (\frac{P_H(i)}{P_H})^{-\epsilon} di]$ is of second order.

$$z_t = \frac{\epsilon}{2} var_i\{p_H(i)\} + o\left(\|a\|^3\right) \tag{261}$$

Let $\widehat{p}_H(i) = p_{H,t}(i) - p_{H,t}$

$\left(\frac{P_H(i)}{P_H}\right)^{1-\epsilon} = \exp[(1-\epsilon)\widehat{p}_H(i)]$

$= 1 + (1-\epsilon)\widehat{p}_H(i) + \frac{(1-\epsilon)^2}{2}\left(\widehat{p}_H(i)\right)^2 + o\left(\|a\|^3\right)$

From the definition of P_H, we have $1 = \int_0^1 \left(\frac{P_H(i)}{P_H}\right)^{1-\epsilon} di$. Therefore integrating the above expression across i the $LHS = 1$ and the expression simplifies to,

$$E_i\{\widehat{p}_H(i)\} = \frac{\epsilon-1}{2} E_i\{\widehat{p}_H(i)^2\} \tag{262}$$

Additionally, a second order approximation to $\left(\frac{P_H(i)}{P_H}\right)^{-\epsilon}$ yields,

$$\left(\frac{P_H(i)}{P_H}\right)^{-\epsilon} = 1 - \epsilon(\widehat{p}_H(i)) + \frac{1}{2}\epsilon^2(\widehat{p}_H(i))^2 + o\left(\|a\|^3\right) \tag{263}$$

Integrating over i we can rewrite this as,

$$\int_0^1 \left(\frac{P_H(i)}{P_H}\right)^{-\epsilon} di = 1 - \epsilon E_i\{\widehat{p}_H(i)\} + \frac{1}{2}\epsilon^2 E_i\{(\widehat{p}_H(i)^2)\} + o\left(\|a\|^3\right) \tag{264}$$

But we have already shown that,

$$E_i\{\widehat{p}_H(i)\} = \frac{\epsilon - 1}{2} E_i\{\widehat{p}_H(i)^2\} \tag{265}$$

Therefore, this can be rewritten as,

$$\begin{aligned} \int_0^1 \left(\frac{P_H(i)}{P_H}\right)^{-\epsilon} di &= 1 + \frac{\epsilon}{2} E_i\{\widehat{p}_H(i)^2\} + o\left(\|a\|^3\right) \\ &= 1 + \frac{\epsilon}{2} var_i\{p_H(i)\} + o\left(\|a\|^3\right) \end{aligned} \tag{266}$$

This can be related to the variance in relative prices as follows,

$$\begin{aligned} var_i\{p_H(i)\} &= E_i\{(p_H(i) - E_i p_H(i))^2\} \\ &= E_i\{\widehat{p}_H(i)^2\} \end{aligned} \tag{267}$$

From which is follows that

$$z = \ln \int_0^1 \left(\frac{P_H(i)}{P_H}\right)^{-\epsilon} di = \frac{\epsilon}{2} var_i\{p_H(i)\} + o\left(\|a\|^3\right) \tag{268}$$

so we can write

$$\begin{aligned} \frac{N^{1+\varphi}}{1+\varphi} &= \frac{(N^n)^{1+\varphi}}{1+\varphi} \\ &+ (N^n)^{1+\varphi}\{y^g + \frac{1}{2}(y^g)^2(1+\varphi) + \frac{\epsilon}{2} var_i\{p_H(i)\} + \frac{\epsilon_w(1+\varphi\epsilon_w)}{2} var_k\{w(k)^2\}\} + o\left(\|a\|^3\right) \end{aligned} \tag{269}$$

The term in G can be expanded as

$$\ln G = \ln(\frac{G}{Y}) + y^g + tip \tag{270}$$

$$= \ln(1 - \exp(-g)) + y^g + tip \tag{271}$$

$$= \frac{1-\gamma^n}{\gamma^n} g^g - \frac{1}{2}\frac{1-\gamma^n}{(\gamma^n)^2}(g^g)^2 + y^g + tip + o\left(\|a\|^3\right) \tag{272}$$

where γ^n=G^n/Y^n. We can then write

$$\chi_t \ln G_t = \frac{1-\gamma^n}{\gamma^n} \ln G_t \tag{273}$$

$$= g_t^g - \frac{1}{2\gamma^n}(g_t^g)^2 + \chi y^g + tip + o\left(\|a\|^3\right) \tag{274}$$

Using these expansions, individual utility can be written as

$$\ln C_t + \chi_t \ln G_t - \frac{N_t^{1+\varphi}}{1+\varphi} = c_t^n + \alpha \int_0^1 c_t^{g,j} dj + (1-\alpha)(y_t^g - g_t^g) + \tag{275}$$

$$g_t^g - \frac{1}{2\gamma^n}(g_t^g)^2 + \chi y^g \tag{276}$$

$$-[\frac{(N^n)^{1+\varphi}}{1+\varphi} + (N^n)^{1+\varphi}\{y_t^g + \frac{1}{2}(y_t^g)^2(1+\varphi)$$
$$+\frac{\epsilon}{2}var_i\{p_{H,t}(i)\} + \frac{\epsilon_w(1+\varphi\epsilon_w)}{2}var_k\{w_t(k)^2\}\} \quad (277)$$
$$+tip + O[3]$$

Now, adding natural terms to tip and if we have an optimal subsidy, then

$$N^n = (1+\chi)^{\frac{1}{1+\varphi}} \quad (278)$$

, so we can simplify this as

$$\begin{aligned}\ln C_t + \chi_t \ln G_t - \frac{N_t^{1+\varphi}}{1+\varphi} &= \alpha \int_0^1 c_t^{g,j} dj - \alpha(y_t^g - g_t^g) + \\ &-\frac{1}{2\gamma^n}(g_t^g)^2 + \\ &-(1+\chi)\{\frac{1}{2}(y_t^g)^2(1+\varphi) + \frac{\epsilon}{2}var_i\{p_H, t(i)\} + \frac{\epsilon_w(1+\varphi\epsilon_w)}{2}var_k\{w_t(k)^2\}\} \\ &+tip + O[3]\end{aligned} \quad (279)$$

Total individual welfare in country i is therefore given by

$$\begin{aligned}\Gamma^i &= \sum_{t=0} \beta^t[-\alpha(y_t^g - g_t^g) + \alpha \int_0^1 c_t^{g,j} dj \\ &-\frac{(1+\chi)}{2}((y_t^g)^2(1+\varphi) + \frac{1}{\chi}(g_t^g)^2 + \epsilon var_i\{p_H, t(i)\} + \frac{\epsilon_w(1+\varphi\epsilon_w)}{2}var_k\{w_t(k)^2\})] \\ &+tip + O[3]\end{aligned} \quad (280)$$

utilising the fact that $1 - \frac{G^n}{Y^n} = 1 - \gamma^n = \frac{1}{1+\chi}$.

Woodford shows that

$$\sum \beta^t var_i\{p_H, t(i)\} = \frac{1}{\lambda}\sum \beta^t \pi_{H,t}^2 \quad (281)$$

where $\lambda = (1-\theta_p)(1-\beta\theta_p)/\theta_p$. (This is in fact the same λ given in the New Keynesian Phillips curve).

Lemma 1 *$\sum_{t=0}^{\infty} \beta^t var_i\{p_{H,t}(i)\} = \frac{1}{\lambda}\sum \beta^t \pi_{H,t}^2$ where $\lambda = \frac{(1-\theta)(1-\beta\theta)}{\theta}$*

The Calvo price-setting rules imply that the distribution of prices elvolve as,

$$\begin{aligned}E_i p_{H,t}(i) - E_i p_{H,t-1}(i) &= \theta_p E_i\{p_{H,t-1}(i) - E_i\{p_{H,t-1}(i)\}\} + (1-\theta_p)(\overline{p}_{H,t} - E_i\{p_{H,t-1}(i)\}) \\ &= (1-\theta_p)(\overline{p}_{H,t} - E_i\{p_{H,t-1}(i)\})\end{aligned} \quad (282)$$

where $\overline{p}_{H,t}$ is the optimal price set by all firms who are able to reset their price in period t.(Note that this is an exact relationship which will later be related to the usual Dixit-Stiglitz price indices through a second order approximation)

Now consider the dispersion measure, $var_i\{p_{H,t}(i)\}$,

$$\begin{aligned} var_i\{p_{H,t}(i)\} &= var_i\{p_{H,t}(i) - E_i\{p_{H,t-1}(i)\}\} && (283)\\ &= E_i\{(p_{H,t}(i) - E_i\{p_{H,t-1}(i)\})^2\} - (E_i\{p_{H,t}(i) - E_i\{p_{H,t-1}(i)\}\})^2 \end{aligned}$$

Now by the same logic that applies to price levels, the distribution of prices also elvolves as a result of the Calvo assumptions as,

$$\begin{aligned} var_i\{p_{H,t}(i)\} &= \theta E_i\{(p_{H,t-1}(i) - E_i\{p_{H,t-1}(i)\})^2\} + (1-\theta)(\overline{p}_{H,t} - E_i\{p_{H,t-1}(i)\})^2 \\ &\quad - (E_i\{p_{H,t}(i) - E_i\{p_{H,t-1}(i)\}\})^2 && (284) \end{aligned}$$

From the evolution of prices above, and using the definition of the variance, we can rewrite this

$$\begin{aligned} &= \theta var_i\{p_{H,t-1}(i)\} + \frac{1}{1-\theta}\left(E_i p_{H,t}(i) - E_i p_{H,t-1}(i)\right)^2 && (285)\\ &\quad - (E_i\{p_{H,t}(i) - E_i\{p_{H,t-1}(i)\}\})^2 \\ &= \theta var_i\{p_{H,t-1}(i)\} + \frac{\theta}{1-\theta}\left(E_i\{p_{H,t}(i)\} - E_i\{p_{H,t-1}(i)\}\right)^2 \end{aligned}$$

Now we use the loglinear approximation, $E_i\{p_{H,t}(i)\} = p_{H,t} + o\left(\|a\|^2\right)$ to obtain,

$$var_i\{p_{H,t}(i)\} = \theta var_i\{p_{H,t-1}(i)\} + \frac{\theta}{1-\theta}\pi^2_{H,t} + o\left(\|a\|^3\right) \qquad (286)$$

We therefore have a dynamic expression for the evolution of the dispersion of prices which is related to the rate of inflation. Integrating this forward (conditional on some initial distribution of prices which is independent of policy) we obtain,

$$var_i\{p_{H,t}(i)\} = \theta^{t+1} var_i\{p_{H,t-1}(i)\} + \sum_{s=0}^{t} \frac{\theta}{1-\theta}\pi^2_{H,s} + o\left(\|a\|^3\right) \qquad (287)$$

Therefore if we take the discounted value of these terms over all periods $t \geq 0$ we obtain,

$$\begin{aligned} \sum_{t=0}^{\infty} \beta^t var_i\{p_{H,t}(i)\} &= \sum_{t=0}^{\infty} \beta^t \left(\theta^{t+1} var_i\{p_{H,-1}(i)\} + \sum_{s=0}^{t} \frac{\theta}{1-\theta}\pi^2_{H,s} + o\left(\|a\|^3\right)\right) + o\left(\|a\|^3\right) \\ &= \frac{\theta}{(1-\theta)(1-\theta\beta)} \sum_{t=0}^{\infty} \beta^t \pi^2_{H,t} + t.i.p + o\left(\|a\|^3\right) && (288) \end{aligned}$$

where we use the expression of the sum to n terms of a geometric series.

Similarly for wages. $\sum_{t=0}^{\infty} \beta^t var_k\{w_t(k)\} = \frac{1}{\lambda_w}(\pi_t^w)^2$ where $\lambda_w = \frac{(1-\theta_w)(1-\beta\theta_w)}{\theta_w}$

The Calvo price-setting rules imply that the distribution of wages evolve as,

$$\begin{aligned} E_k w_t(k) - E_k w_{t-1}(k) &= \theta_p E_k\{w_{t-1}(k) - E_k\{w_{t-1}(k)\}\} + (1-\theta_w)(\overline{w}_t - E_k\{w_{t-1}(k)\}) \\ &= (1-\theta_w)(\overline{w}_t - E_i\{w_{t-1}(i)\}) \end{aligned} \quad (289)$$

where $\overline{w}_t$ is the optimal price set by all firms who are able to reset their price in period t.(Note that this is an exact relationship which will later be related to the usual Dixit-Stiglitz price indices through a second order approximation)

Now consider the dispersion measure, $var_k\{w_t(k)\}$,

$$\begin{aligned} var_k\{w_t(k)\} &= var_k\{w_t(k) - E_k\{w_{t-1}(k)\}\} \quad (290) \\ &= E_k\{(w_t(k) - E_k\{w_{t-1}(k)\})^2\} - (E_k\{w_t(k) - E_k\{w_{t-1}(l)\}\})^2 \end{aligned}$$

Now by the same logic that applies to price levels, the distribution of prices also evolves as a result of the Calvo assumptions as,

$$\begin{aligned} var_k\{w_t(k)\} &= \theta_w E_k\{(w_{t-1}(k) - E_k\{w_{t-1}(k)\})^2\} + (1-\theta_w)(\overline{w}_t - E_k\{w_{t-1}(k)\})^2 \\ &\quad - (E_k\{w_t(k) - E_k\{w_{t-1}(k)\}\})^2 \end{aligned} \quad (291)$$

From the evolution of prices above, and using the definition of the variance, we can rewrite this

$$\begin{aligned} &= \theta_w var_k\{w_{t-1}(k)\} + \frac{1}{1-\theta_w}\left(E_k w_t(k) - E_k w_{t-1}(k)\right)^2 \quad (292) \\ &\quad - (E_k\{w_t(k) - E_k\{w_{t-1}(k)\}\})^2 \\ &= \theta_w var_k\{w_{t-1}(k)\} + \frac{\theta_w}{1-\theta_w}\left(E_k\{w_t(k)\} - E_k\{w_{t-1}(k)\}\right)^2 \end{aligned}$$

Now we use the loglinear approximation, $E_k\{w_t(k)\} = w_t + o\left(\|a\|^2\right)$to obtain,

$$var_k\{w_t(k)\} = \theta var_k\{p_{t-1}(k)\} + \frac{\theta_w}{1-\theta_w}(\pi_t^w)^2 + o\left(\|a\|^3\right) \quad (293)$$

We therefore have a dynamic expression for the evolution of the dispersion of wages which is related to the rate of wage inflation. Integrating this forward (conditional on some initial distribution of wages which is independent of policy) we obtain,

$$var_k\{w_t(k)\} = \theta_w^{t+1} var_k\{w_{t-1}(k)\} + \sum_{s=0}^{t} \frac{\theta_w}{1-\theta_w}(\pi_{H,s}^w)^2 + o\left(\|a\|^3\right) \quad (294)$$

Therefore if we take the discounted value of these terms over all periods $t \geq 0$ we obtain,

$$\begin{aligned} \sum_{t=0}^{\infty} \beta^t var_k\{w_t(k)\} &= \sum_{t=0}^{\infty} \beta^t \left(\theta_w^{t+1} var_k\{w_{t-1}(k)\} + \sum_{s=0}^{t} \frac{\theta_w}{1-\theta_w}(\pi_{H,s}^w)^2 + o\left(\|a\|^3\right)\right) \\ &= \frac{\theta_w}{(1-\theta_w)(1-\theta_w\beta)} \sum_{t=0}^{\infty} \beta^t (\pi_t^w)^2 + t.i.p + o\left(\|a\|^3\right) \end{aligned} \quad (295)$$

where we use the expression of the sum to n terms of a geometric series.

So we can write

$$\begin{aligned}\Gamma^i &= \sum_{t=0} \beta^t[-\alpha(y_t^g - g_t^g) + \alpha \int_0^1 c_t^{g,j} dj \\ &\quad - \frac{(1+\chi)}{2}((y_t^g)^2(1+\varphi) + \frac{1}{\chi}(g_t^g)^2 + \frac{\epsilon}{\lambda}\pi_{H,t}^2 + \frac{\epsilon_w(1+\varphi\epsilon_w)}{\lambda_w}(\pi_{H,t}^w)^2)] \\ &\quad + tip + o\left(\|a\|^3\right)\end{aligned} \tag{296}$$

Integrating over all economies, and utilising

$$\int (y^{i,g} - c^{i,g} - g^{i,g})di = 0 \tag{297}$$

we obtain

$$\Gamma = -\frac{(1+\chi)}{2}\sum_{t=0}\beta^t \int_0^1 [\frac{\epsilon}{\lambda}\pi_{i,t}^2 + \frac{\epsilon_w(1+\varphi\epsilon_w)}{\lambda_w}(\pi_{i,t}^w)^2 + (y_t^{i,g})^2(1+\varphi) + \frac{1}{\chi}(g_t^{i,g})^2]di + tip + O[3] \tag{298}$$

Welfare is the sum of quadratic terms in inflation (for both wages and prices), the output gap and the government spending gap in each country.

Appendix 4 - Precommitment Policy in the Small Open Economy

Small Open Economy - All Fiscal Instruments

Let us consider the case where the fiscal authorities have access to government spending and both tax instruments in order to stabilise their economy, when operating alongside the national monetary authorities. Here the presence of the national monetary policy implies, $\lambda_t^{y,i} = 0 \forall t$ so that the initial focs reduce to, for sales taxes,

$$\lambda \lambda_t^{\pi,i} = 0 \tag{299}$$

and income taxes,

$$\widetilde{\lambda}_w \lambda_t^{\pi^w,i} = 0 \tag{300}$$

From these it is clear that if the authorities have access to the full set of fiscal instruments, then the sales tax ensures $\lambda \lambda_t^{\pi,i} = 0$ and the income tax implies, $\widetilde{\lambda}_w \lambda_t^{\pi^w,i} = 0$.Imposing this our remaining focs reduce to:

For real wages,

$$\lambda_t^{rw,i} - \beta E_t \lambda_{t+1}^{rw,i} = 0 \tag{301}$$

price inflation,

$$\frac{2\epsilon}{\lambda} \pi_{i,t} + \lambda_t^{rw,i} = 0 \tag{302}$$

wage inflation,

$$\frac{2\epsilon_w}{\widetilde{\lambda}_w} \pi_{i,t}^w - \lambda_t^{rw,i} = 0 \tag{303}$$

the government spending gap,

$$\frac{2}{\chi} g_t^{i,g} = 0 \tag{304}$$

and the output gap,

$$2(1+\varphi) y_t^{i,g} = 0 \tag{305}$$

Combining the focs for price and wage inflation yields the optimal combination of wage and price inflation,

$$\frac{\epsilon}{\lambda} \pi_{i,t} + \frac{\epsilon_w}{\widetilde{\lambda}_w} \pi_{i,t}^w = 0 \tag{306}$$

The foc for real wages also implies,

$$\pi_{i,t}^w - \beta E_t \pi_{i,t+1}^w = 0 \tag{307}$$

which given the New Keynesian Phillips curve for inflation implies,

$$(1+\varphi) y_t^{i,g} - g_t^{i,g} - rw_t^{i,g} - \ln(1-\tau_t^i)^g = 0 \tag{308}$$

and ensures that $\pi_{i,t} = \pi^w_{i,t} = 0$

Therefore, our four target criteria are, for government spending,

$$g_t^{i,g} = 0 \tag{309}$$

for income taxes,

$$\ln(1-\tau_t)^g = -rw_t^{i,g} \tag{310}$$

sales taxes,

$$\ln(1-\tau^v)^g = rw_t^{i,g} \tag{311}$$

and the output gap,

$$y_t^{i,g} = 0 \tag{312}$$

The latter two conditions being achieved through a combination of monetary policy and VAT changes. Here a combination of income tax and VAT changes will achieve the real wage adjustment required to support the flex price equilibrium after monetary policy has eliminated the output gap. Wage and price inflation will be zero, with income taxes achieving the required real wage adjustment.

Small Open Economy - VAT and Government Spending

Now suppose we only have access to VAT and government spending as fiscal instruments, our set of focs become, after imposing $\lambda\lambda_t^{\pi,i} = 0$ from the foc from the sales tax,

Real wages,

$$\widetilde{\lambda}_w \lambda_t^{\pi^w,i} + \lambda_t^{rw,i} - \beta E_t \lambda_{t+1}^{rw,i} = 0 \tag{313}$$

price inflation,

$$\frac{2\epsilon}{\lambda}\pi_{i,t} + \lambda_t^{rw,i} = 0 \tag{314}$$

wage inflation,

$$\frac{2\epsilon_w}{\widetilde{\lambda}_w}\pi^w_{i,t} + \lambda_t^{\pi^w,i} - \lambda_{t-1}^{\pi^w,i} - \lambda_t^{rw,i} = 0 \tag{315}$$

the government spending gap,

$$\frac{2}{\chi} g_t^{i,g} + \widetilde{\lambda}_w \lambda_t^{\pi^w,i} = 0 \tag{316}$$

and the output gap,

$$2(1+\varphi)y_t^{i,g} - \widetilde{\lambda}_w(1+\varphi)\lambda_t^{\pi^w,i} = 0 \tag{317}$$

Combining the focs for the output gap and the government spending gap,

$$2(1+\varphi)y_t^{i,g} + (1+\varphi)\frac{2}{\chi}g_t^{i,g} = 0 \tag{318}$$

which implies the following government spending rule,

$$y_t^{i,g} + \frac{1}{\chi} g_t^{i,g} = 0 \tag{319}$$

which delivers the optimal composition of GDP in the face of shocks.

>From the foc for the output gap we know,

$$2y_t^{i,g} = \widetilde{\lambda}_w \lambda_t^{\pi^w,i} \tag{320}$$

replacing this in the foc for wage inflation,

$$\frac{2\epsilon_w}{\widetilde{\lambda}_w} \pi_{i,t}^w + \frac{2}{\widetilde{\lambda}_w} \Delta y_t^{i,g} = \lambda_t^{rw,i} \tag{321}$$

Eliminating $\lambda_t^{rw,i}$ for the foc for price inflation yields,

$$\frac{\epsilon_w}{\widetilde{\lambda}_w} \pi_{i,t}^w + \frac{\epsilon}{\lambda} \pi_{i,t} + \frac{1}{\widetilde{\lambda}_w} \Delta y_t^{i,g} = 0 \tag{322}$$

Here the loss of the income tax instrument when wages are sticky requires a trade-off between output and inflation stabilisation with the inertia in policy which is typical of precommitment solutions. Note that if we didn't have the government spending instrument, then we would simply drop the fiscal spending rule from this target criterion.

The real wage foc implies,

$$\widetilde{\lambda}_w \lambda_t^{\pi^w,i} + \lambda_t^{rw,i} - \beta E_t \lambda_{t+1}^{rw,i} = 0 \tag{323}$$

substituting for lagrange multipliers,

$$y_t^{i,g} - \frac{\epsilon}{\lambda} (\pi_{i,t} - \beta E_t \pi_{i,t+1}) = 0 \tag{324}$$

which is the additional target crteria. Using the Phillips curve we can rewrite this as,

$$y_t^{i,g} - \epsilon r w_t^{i,g} + \epsilon \ln(1 - \tau_t^{i,s})^g = 0 \tag{325}$$

Therefore, we have the following set of target criteria. The government spending rule,

$$y_t^{i,g} + \frac{1}{\chi} g_t^{i,g} = 0 \tag{326}$$

The output-inflation trade-off to be achieved by monetary policy,

$$\frac{\epsilon_w}{\widetilde{\lambda}_w} \pi_{i,t}^w + \frac{\epsilon}{\lambda} \pi_{i,t} + \frac{1}{\widetilde{\lambda}_w} \Delta y_t^{i,g} = 0 \tag{327}$$

and the VAT tax rule,

$$y_t^{i,g} - \epsilon r w_t^{i,g} + \epsilon \ln(1 - \tau_t^{v})^g = 0 \tag{328}$$

Small Open Economy - Income Tax and Government Spending

Now suppose we have the income tax instrument, but no Sales tax. The focs become, for income taxes,

$$\widetilde{\lambda}_w \lambda_t^{\pi^w,i} = 0 \tag{329}$$

and after imposing this, the remaining focs are, for real wages,

$$-\lambda\lambda_t^{\pi,i} + \lambda_t^{rw,i} - \beta E_t \lambda_{t+1}^{rw,i} = 0 \tag{330}$$

price inflation,

$$\frac{2\epsilon}{\lambda}\pi_{i,t} + \lambda_t^{\pi,i} - \lambda_{t-1}^{\pi,i} + \lambda_t^{rw,i} = 0 \tag{331}$$

wage inflation,

$$\frac{2\epsilon_w}{\widetilde{\lambda}_w}\pi_{i,t}^w - \lambda_t^{rw,i} = 0 \tag{332}$$

the government spending gap,

$$\frac{2}{\chi}g_t^{i,g} + \lambda\lambda_t^{\pi,i} = 0 \tag{333}$$

and the output gap,

$$2(1+\varphi)y_t^{i,g} - (1+\varphi)\lambda\lambda_t^{\pi,i} = 0 \tag{334}$$

Combining the foc for the output gap and government spending gap,

$$2(1+\varphi)y_t^{i,g} + \frac{2}{\chi}g_t^{i,g} - \varphi\lambda\lambda_t^{\pi,i} = 0 \tag{335}$$

The foc for wage inflation can be embedded in the foc for real wages,

$$\lambda\lambda_t^{\pi,i} = \frac{2\epsilon_w}{\widetilde{\lambda}_w}(\pi_{i,t}^w - \beta E_t \pi_{i,t+1}^w) \tag{336}$$

Using the wage inflation Phillips curve,

$$y_t^{i,g} = \epsilon_w((1+\varphi)y_t^{i,g} - g_t^{i,g} - rw_t^{i,g} - \ln(1-\tau_t^i)^g) \tag{337}$$

which is our first target criterion.

This can then be used to eliminate the lagrange multipliers from the foc for price inflation,

$$\frac{\epsilon_w}{\widetilde{\lambda}_w}\pi_{i,t}^w + \frac{\epsilon}{\lambda}\pi_{i,t} + \frac{1}{\lambda\widetilde{\lambda}_w}(\Delta y_t^{i,g}) = 0 \tag{338}$$

which gives us our second. Government Spending rule is given by,

$$\frac{2}{\chi}g_t^{i,g} + 2\epsilon_w((1+\varphi)y_t^{i,g} - g_t^{i,g} - rw_t^{i,g} - \ln(1-\tau_t^i)^g) = 0 \tag{339}$$

Combining gives us our government spending rule, $y_t^{i,g}$

$$\frac{1}{\chi}g_t^{i,g} + y_t^{i,g} = 0 \tag{340}$$

Small Open Economy - No Tax Instruments, Only Government Spending

No tax instruments. Combining the focs for the government spending gap and the output gap yields the familiar fiscal rule,

$$y_t^{i,g} + \frac{1}{\chi}g_t^{i,g} = 0 \tag{341}$$

From the foc for the output gap we have,

$$2y_t^{i,g} = \widetilde{\lambda}_w \lambda_t^{\pi^w,i} \tag{342}$$

Substituting into the foc for wage inflation,

$$\frac{2\epsilon_w}{\widetilde{\lambda}_w}\pi_{i,t}^w + \frac{2}{\widetilde{\lambda}_w}\Delta y_t^{i,g} - \lambda_t^{rw,i} = 0 \tag{343}$$

PLacing in the foc for real wages,

$$-\lambda\lambda_t^{\pi,i} + 2y_t^{i,g} + \frac{2\epsilon_w}{\widetilde{\lambda}_w}(\pi_{i,t}^w - \beta E_t \pi_{i,t+1}^w) + \frac{2}{\widetilde{\lambda}_w}(\Delta y_t^{i,g} - \beta E_t \Delta y_{t+1}^{i,g}) = 0 \tag{344}$$

Then using the foc for price inflation,

$$\frac{2\epsilon}{\lambda}\pi_{i,t} + \lambda_t^{\pi,i} - \lambda_{t-1}^{\pi,i} + \lambda_t^{rw,i} = 0 \tag{345}$$

Eliminating lagrange multipliers,

$$\frac{2\epsilon}{\lambda}\pi_{i,t} + \frac{2\epsilon_w}{\widetilde{\lambda}_w}\pi_{i,t}^w + \frac{2}{\widetilde{\lambda}_w}\Delta y_t^{i,g} + \frac{1}{\lambda}\left(2\Delta y_t^{i,g} + \frac{2\epsilon_w}{\widetilde{\lambda}_w}(\Delta\pi_{i,t}^w - \beta E_t \Delta\pi_{i,t+1}^w) + \frac{2}{\widetilde{\lambda}_w}(\Delta^2 y_t^{i,g} - \beta E_t \Delta^2 y_{t+1}^{i,g})\right) = 0 \tag{346}$$

Can eliminate the dynamics in wage inflation using NKPC for wage inflation.

$$\frac{\pi^w_{i,t} - \beta E_t \pi^w_{i,t+1}}{\widetilde{\lambda}_w} = (1+\varphi) y^{i,g}_t - g^{i,g}_t - rw^{i,g}_t \tag{347}$$

to obtain,

$$\frac{\epsilon}{\lambda}\pi_{i,t} + \frac{\epsilon_w}{\widetilde{\lambda}_w}\pi^w_{i,t} + \frac{1}{\widetilde{\lambda}_w}\Delta y^{i,g}_t + \frac{1}{\lambda}\left(\Delta y^{i,g}_t + \epsilon_w((1+\varphi)y^{i,g}_t - g^{i,g}_t - rw^{i,g}_t) + \frac{1}{\widetilde{\lambda}_w}(\Delta^2 y^{i,g}_t - \beta E_t \Delta^2 y^{i,g}_{t+1})\right) = 0 \tag{348}$$

This describes pre-commitment policy for all cases in the small open economy.

Appendix 5 - Optimal Precommitment Under EMU.

EMU - All Fiscal Instruments

With all fiscal instruments available the tax instruments imply, $\widetilde{\lambda}_w \lambda_t^{\pi^w,i} = 0$ and $\lambda_t^{\pi,i} = 0$, such that we can rewrite the focs as, real wages,

$$\lambda_t^{rw,i} - \beta E_t \lambda_{t+1}^{rw,i} = 0 \tag{349}$$

The remaining first-order conditions are for inflation,

$$\frac{2\epsilon}{\lambda}\pi_{i,t} - \beta^{-1}\lambda_{t-1}^{y,i} + \lambda_t^{rw,i} = 0 \tag{350}$$

The foc for wage inflation is given by,

$$\frac{2\epsilon_w}{\widetilde{\lambda}_w}\pi_{i,t}^w - \lambda_t^{rw,i} = 0 \tag{351}$$

All that remains is the foc for the government spending gap,

$$\frac{2}{\chi}g_t^{i,g} - \lambda_t^{y,i} + \beta^{-1}\lambda_{t-1}^{y,i} = 0 \tag{352}$$

and the output gap,

$$2(1+\varphi)y_t^{i,g} + \lambda_t^{y,i} - \beta^{-1}\lambda_{t-1}^{y,i} = 0 \tag{353}$$

Combining the last two conditions yields the fiscal spending rule,

$$(1+\varphi)y_t^{i,g} + \frac{1}{\chi}g_t^{i,g} = 0 \tag{354}$$

which is slightly different from the small open economy case. Using the focs for price and wage inflation,

$$\frac{2\epsilon}{\lambda}\pi_{i,t} - \beta^{-1}\lambda_{t-1}^{y,i} + \frac{2\epsilon_w}{\widetilde{\lambda}_w}\pi_{i,t}^w = 0 \tag{355}$$

Substituting this into the foc for the output gap,

$$2(1+\varphi)y_t^{i,g} + \frac{2\epsilon}{\lambda}(\beta E_t \pi_{i,t+1} - \pi_{i,t}) + \frac{2\epsilon_w}{\widetilde{\lambda}_w}(\beta E_t \pi_{i,t+1}^w - \pi_{i,t}^w) = 0 \tag{356}$$

Quite different dynamics from the open economy case. Why?

The final target criteria isimplied by,

$$\lambda_t^{rw,i} - \beta E_t \lambda_{t+1}^{rw,i} = 0 \tag{357}$$

$$\frac{2\epsilon_w}{\widetilde{\lambda}_w}\pi^w_{i,t} - \lambda^{rw,i}_t = 0 \tag{358}$$

which imply,

$$\pi^w_{i,t} = \beta E_t \pi^w_{i,t+1} \tag{359}$$

which in turn implies the following income tax rule,.

$$(1+\varphi)y^{i,g}_t - g^{i,g}_t - rw^{i,g}_t - \ln(1-\tau^i_t)^g) = 0 \tag{360}$$

As a result the target criterion simplies to,

$$(1+\varphi)y^{i,g}_t + \frac{\epsilon}{\lambda}(\beta E_t \pi_{i,t+1} - \pi_{i,t}) = 0 \tag{361}$$

Using the NKPC to eliminate the dynamics in inflation we get our VAT fiscal rule,

$$(1+\varphi)y^{i,g}_t + \epsilon(\ln(1-\tau^{i,s}_t)^g - rw^{i,g}_t) = 0 \tag{362}$$

Therefore our policy configuration is a government spending rule,

$$(1+\varphi)y^{i,g}_t + \frac{1}{\chi}g^{i,g}_t = 0 \tag{363}$$

the income tax rule,

$$(1+\varphi)y^{i,g}_t - g^{i,g}_t - rw^{i,g}_t - \ln(1-\tau^i_t)^g) = 0 \tag{364}$$

which eliminates wage inflation, and VAT tax rule,

$$(1+\varphi)y^{i,g}_t + \epsilon(\ln(1-\tau^{i,s}_t)^g - rw^{i,g}_t) = 0 \tag{365}$$

Without the national monetary policy instrument we can no-longer offset all shocks completely. Instead the income tax rule will eliminate wage inflation, government spending will adjust to ensure the optimal composition of output and the sales tax will be adjusted to achieve the best trade-off between output and inflation given that competitiveness will need to be restored once any shock has passed.

EMU Case - VAT and Government Spending

Now we start dropping fiscal instruments. Let's suppose we don't have the income tax instrument. The focs become, for the sales tax,

$$\lambda\lambda^{\pi,i}_t = 0 \tag{366}$$

i.e. the price Phillips curve ceases to be a constraint on maximising welfare -VAT tax changes can offset the impact on any other variables driving price inflation. and for real wages,

$$\widetilde{\lambda}_w \lambda^{\pi^w,i}_t + \lambda^{rw,i}_t - \beta E_t \lambda^{rw,i}_{t+1} = 0 \tag{367}$$

The remaining first-order conditions are for inflation,

$$\frac{2\epsilon}{\lambda}\pi_{i,t} - \beta^{-1}\lambda_{t-1}^{y,i} + \lambda_t^{rw,i} = 0 \tag{368}$$

The foc for wage inflation is given by,

$$\frac{2\epsilon_w}{\widetilde{\lambda}_w}\pi_{i,t}^w + \lambda_t^{\pi^w,i} - \lambda_{t-1}^{\pi^w,i} - \lambda_t^{rw,i} = 0 \tag{369}$$

All that remains is the foc for the government spending gap,

$$\frac{2}{\chi}g_t^{i,g} + \widetilde{\lambda}_w\lambda_t^{\pi^w,i} - \lambda_t^{y,i} + \beta^{-1}\lambda_{t-1}^{y,i} = 0 \tag{370}$$

and the output gap,

$$2(1+\varphi)y_t^{i,g} - \widetilde{\lambda}_w(1+\varphi)\lambda_t^{\pi^w,i} + \lambda_t^{y,i} - \beta^{-1}\lambda_{t-1}^{y,i} = 0 \tag{371}$$

Combining the last two conditions,

$$\frac{2}{\chi}g_t^{i,g} + 2(1+\varphi)y_t^{i,g} - \widetilde{\lambda}_w\varphi\lambda_t^{\pi^w,i} = 0 \tag{372}$$

Inserting into the foc for wage inflation,

$$2\epsilon_w\pi_{i,t}^w + \frac{2}{\varphi\chi}\Delta g_t^{i,g} + 2\frac{(1+\varphi)}{\varphi}\Delta y_t^{i,g} - \widetilde{\lambda}_w\lambda_t^{rw,i} = 0 \tag{373}$$

Using the foc for real wages,

$$\begin{aligned} 0 \quad = \quad & \frac{2}{\varphi\chi}g_t^{i,g} + 2\frac{(1+\varphi)}{\varphi}y_t^{i,g} + 2\frac{\epsilon_w}{\widetilde{\lambda}_w}(\pi_{i,t}^w - \beta\pi_{i,t+1}^w) \\ & + \frac{2}{\varphi\chi\widetilde{\lambda}_w}(\Delta g_t^{i,g} - \beta E_t\Delta g_{t+1}^{i,g}) + 2\frac{(1+\varphi)}{\varphi\widetilde{\lambda}_w}(\Delta y_t^{i,g} - \beta E_t\Delta y_{t+1}^{i,g}) \end{aligned} \tag{374}$$

Now consider the foc for price inflation,

$$\frac{2\epsilon}{\lambda}\pi_{i,t} - \beta^{-1}\lambda_{t-1}^{y,i} + \lambda_t^{rw,i} = 0 \tag{375}$$

eliminating, $\lambda_t^{rw,i}$ yields,

$$\frac{2\epsilon}{\lambda}\pi_{i,t} + 2\frac{\epsilon_w}{\widetilde{\lambda}_w}\pi_{i,t}^w + \frac{2}{\varphi\chi\widetilde{\lambda}_w}\Delta g_t^{i,g} + 2\frac{(1+\varphi)}{\varphi\widetilde{\lambda}_w}\Delta y_t^{i,g} = \beta^{-1}\lambda_{t-1}^{y,i} \tag{376}$$

Now consider foc for output gap,

$$2(1+\varphi)y_t^{i,g} - \widetilde{\lambda}_w(1+\varphi)\lambda_t^{\pi^w,i} + \lambda_t^{y,i} - \beta^{-1}\lambda_{t-1}^{y,i} = 0 \quad (377)$$

and eliminate lagrange multipliers,

$$\begin{aligned} 0 \;=\; & 2(1+\varphi)y_t^{i,g} & (378)\\ & -(1+\varphi)\left(\frac{2}{\varphi\chi}g_t^{i,g} + 2\frac{(1+\varphi)}{\varphi}y_t^{i,g}\right) \\ & +\frac{2\epsilon}{\lambda}(\beta E_t\pi_{i,t+1} - \pi_{i,t}) + 2\frac{\epsilon_w}{\widetilde{\lambda}_w}(\beta E_t\pi_{i,t+1}^w - \pi_{i,t}^w) \\ & +\frac{2}{\varphi\chi\widetilde{\lambda}_w}(\beta E_t\Delta g_{t+1}^{i,g} - \Delta g_t^{i,g}) + 2\frac{(1+\varphi)}{\varphi\widetilde{\lambda}_w}(\beta E_t\Delta y_{t+1}^{i,g} - \Delta y_t^{i,g}) \end{aligned}$$

Combining with the first target criterion,

$$\begin{aligned} 0 \;=\; & \frac{2}{\varphi\chi}g_t^{i,g} + 2\frac{(1+\varphi)}{\varphi}y_t^{i,g} + 2\frac{\epsilon_w}{\widetilde{\lambda}_w}(\pi_{i,t}^w - \beta E_t\pi_{i,t+1}^w) & (379)\\ & +\frac{2}{\varphi\chi\widetilde{\lambda}_w}(\Delta g_t^{i,g} - \beta E_t\Delta g_{t+1}^{i,g}) + 2\frac{(1+\varphi)}{\varphi\widetilde{\lambda}_w}(\Delta y_t^{i,g} - \beta E_t\Delta y_{t+1}^{i,g}) & (380) \end{aligned}$$

yields,

$$\begin{aligned} 0 \;=\; & 2(1+\varphi)y_t^{i,g} & (381)\\ & -(1+\varphi)\left(\frac{2}{\varphi\chi}g_t^{i,g} + 2\frac{(1+\varphi)}{\varphi}y_t^{i,g}\right) \\ & +\frac{2\epsilon}{\lambda}(\beta\pi_{i,t+1} - \pi_{i,t}) \\ & \frac{2}{\varphi\chi}g_t^{i,g} + 2\frac{(1+\varphi)}{\varphi}y_t^{i,g} \end{aligned}$$

Simplifying,

$$0 = -\frac{1}{\chi}g_t^{i,g} + \frac{\epsilon}{\lambda}(\beta E_t\pi_{i,t+1} - \pi_{i,t}) \quad (382)$$

Using the NKPC this simplifies to,

$$\frac{1}{\chi}g_t^{i,g} = \epsilon(\ln(1-\tau_t^{i,s})^g - rw_t^{i,g}) \quad (383)$$

This can either be interpreted as a government spending or VAT rule. Now need second criterion function.

$$\begin{aligned} 0 \;=\; & \frac{2}{\varphi\chi}g_t^{i,g} + 2\frac{(1+\varphi)}{\varphi}y_t^{i,g} + 2\frac{\epsilon_w}{\widetilde{\lambda}_w}(\pi_{i,t}^w - \beta E_t\pi_{i,t+1}^w) & (384)\\ & +\frac{2}{\varphi\chi\widetilde{\lambda}_w}(\Delta g_t^{i,g} - \beta E_t\Delta g_{t+1}^{i,g}) + 2\frac{(1+\varphi)}{\varphi\widetilde{\lambda}_w}(\Delta y_t^{i,g} - \beta E_t\Delta y_{t+1}^{i,g}) \end{aligned}$$

Using NKPC for wage inflation,

$$-\frac{2}{\varphi\chi}g_t^{i,g} = 2\frac{(1+\varphi)}{\varphi}y_t^{i,g} + 2\epsilon_w((1+\varphi)y_t^{i,g} - g_t^{i,g} - rw_t^{i,g}) + \frac{2}{\varphi\chi\widetilde{\lambda}_w}(\Delta g_t^{i,g} - \beta E_t \Delta g_{t+1}^{i,g}) + 2\frac{(1+\varphi)}{\varphi\widetilde{\lambda}_w}(\Delta y_t^{i,g} - \beta E_t \Delta y_{t+1}^{i,g}) \tag{385}$$

With only two instruments and four constraints, the precommitment policy implies a degree of both inertial and forward-looking behaviour typical of analysis of monetary policy in the case of sticky wages and prices (see Woodford (2003), Chapter 7).

EMU Case - Income Tax and Government Spending

Now suppose now income tax the only tax instrument, The condition for income taxes is given by,

$$\widetilde{\lambda}_w \lambda_t^{\pi^w,i} = 0 \tag{386}$$

and, after imposing this in the remaining focs, for real wages,

$$-\lambda\lambda_t^{\pi,i} + \lambda_t^{rw,i} - \beta E_t \lambda_{t+1}^{rw,i} = 0 \tag{387}$$

The remaining first-order conditions are for inflation,

$$\frac{2\epsilon}{\lambda}\pi_{i,t} + \lambda_t^{\pi,i} - \lambda_{t-1}^{\pi,i} - \beta^{-1}\lambda_{t-1}^{y,i} + \lambda_t^{rw,i} = 0 \tag{388}$$

The foc for wage inflation is given by,

$$\frac{2\epsilon_w}{\widetilde{\lambda}_w}\pi_{i,t}^w - \lambda_t^{rw,i} = 0 \tag{389}$$

All that remains is the foc for the government spending gap,

$$\frac{2}{\chi}g_t^{i,g} - \lambda_t^{y,i} + \beta^{-1}\lambda_{t-1}^{y,i} = 0 \tag{390}$$

and the output gap,

$$2(1+\varphi)y_t^{i,g} + \lambda_t^{y,i} - \beta^{-1}\lambda_{t-1}^{y,i} = 0 \tag{391}$$

taken together these imply the following government spending rule,

$$\frac{1}{\chi}g_t^{i,g} + (1+\varphi)y_t^{i,g} = 0 \tag{392}$$

Taking real wages and the wage inflation condition together implies,

$$-\lambda\lambda_t^{\pi,i} + \frac{2\epsilon_w}{\widetilde{\lambda}_w}(\pi_{i,t}^w - \beta E_t \pi_{i,t+1}^w) = 0 \tag{393}$$

Using the wage inflation Phillips curve,

$$-\lambda\lambda_t^{\pi,i} + 2\epsilon_w((1+\varphi)y_t^{i,g} - g_t^{i,g} - rw_t^{i,g} - \ln(1-\tau_t)^g) = 0 \quad (394)$$

Using in the price inflation foc,

$$\beta^{-1}\lambda_{t-1}^{y,i} = \frac{2\epsilon}{\lambda}\pi_{i,t} + \frac{2\epsilon_w}{\widetilde{\lambda}_w}\pi_{i,t}^w + \frac{2\epsilon_w}{\lambda}((1+\varphi)y_t^{i,g} - g_t^{i,g} - rw_t^{i,g} - \ln(1-\tau_t)^g) \quad (395)$$

Substituting into the foc for the output gap,

$$\begin{aligned} 0 &= 2(1+\varphi)y_t^{i,g} + \frac{2\epsilon}{\lambda}(\beta E_t\pi_{i,t+1} - \pi_{i,t}) + \frac{2\epsilon_w}{\widetilde{\lambda}_w}(\beta E_t\pi_{i,t+1}^w - \pi_{i,t+1}^w) \quad (396) \\ &+ \frac{2\epsilon_w}{\lambda}((1+\varphi)(\beta E_t y_{t+1}^{i,g} - y_t^{i,g}) - (\beta E_t g_{t+1}^{i,g} - g_t^{i,g}) \\ &- (\beta E_t rw_{t+1}^{i,g} - rw_t^{i,g}) - (\beta E_t \ln(1-\tau_{t+1})^g - \ln(1-\tau_t)^g) \end{aligned}$$

Using the definitions of the wage and price Phillips curves,

$$\begin{aligned} 0 &= (1+\varphi)y_t^{i,g} - \epsilon(rw_t^{i,g}) - \epsilon_w((1+\varphi)y_t^{i,g} - g_t^{i,g} - rw_t^{i,g} - \ln(1-\tau_t)^g) \quad (397) \\ &+ \frac{\epsilon_w}{\lambda}((1+\varphi)(\beta E_t y_{t+1}^{i,g} - y_t^{i,g}) - (\beta E_t g_{t+1}^{i,g} - g_t^{i,g}) \\ &- (\beta E_t rw_{t+1}^{i,g} - rw_t^{i,g}) - (\beta E_t \ln(1-\tau_{t+1})^g - \ln(1-\tau_t)^g) \end{aligned}$$

which is our dynamic income tax rule.

EMU Case - Government Spending the Only Instrument

With only government spending as our available instrument, our focs become, for real wages,

$$-\lambda\lambda_t^{\pi,i} + \widetilde{\lambda}_w\lambda_t^{\pi^w,i} + \lambda_t^{rw,i} - \beta E_t\lambda_{t+1}^{rw,i} = 0 \quad (398)$$

The remaining first-order conditions are for inflation,

$$\frac{2\epsilon}{\lambda}\pi_{i,t} + \lambda_t^{\pi,i} - \lambda_{t-1}^{\pi,i} - \beta^{-1}\lambda_{t-1}^{y,i} + \lambda_t^{rw,i} = 0 \quad (399)$$

The foc for wage inflation is given by,

$$\frac{2\epsilon_w}{\widetilde{\lambda}_w}\pi_{i,t}^w + \lambda_t^{\pi^w,i} - \lambda_{t-1}^{\pi^w,i} - \lambda_t^{rw,i} = 0 \quad (400)$$

All that remains is the foc for the government spending gap,

$$\frac{2}{\chi}g_t^{i,g} + \widetilde{\lambda}_w \lambda_t^{\pi^w,i} - \lambda_t^{y,i} + \beta^{-1}\lambda_{t-1}^{y,i} = 0 \tag{401}$$

and the output gap,

$$2(1+\varphi)y_t^{i,g} - \widetilde{\lambda}_w(1+\varphi)\lambda_t^{\pi^w,i} + \lambda_t^{y,i} - \beta^{-1}\lambda_{t-1}^{y,i} = 0 \tag{402}$$

Combining the focs for government spending and the output gap,

$$2(1+\varphi)y_t^{i,g} + \frac{2}{\chi}g_t^{i,g} - \widetilde{\lambda}_w\varphi\lambda_t^{\pi^w,i} = 0 \tag{403}$$

Using in combination with expression for wage inflation yields,

$$\frac{2\epsilon_w}{\widetilde{\lambda}_w}\pi_{i,t}^w + 2\frac{(1+\varphi}{\widetilde{\lambda}_w\varphi})\Delta y_t^{i,g} + \frac{2}{\chi\widetilde{\lambda}_w\varphi}\Delta g_t^{i,g} = \lambda_t^{rw,i} \tag{404}$$

Using expression for real wages,

$$\begin{aligned}\lambda\lambda_t^{\pi,i} \quad = \quad & 2\frac{(1+\varphi)}{\varphi}y_t^{i,g} + \frac{2}{\varphi\chi}g_t^{i,g} \\ & + \frac{2\epsilon_w}{\widetilde{\lambda}_w}(\pi_{i,t}^w - \beta E_t\pi_{i,t+1}^w) + 2\frac{(1+\varphi}{\widetilde{\lambda}_w\varphi})(\Delta y_t^{i,g} - \beta E_t\Delta y_{t+1}^{i,g}) \\ & + \frac{2}{\chi\widetilde{\lambda}_w\varphi}(\Delta g_t^{i,g} - \beta E_t\Delta g_{t+1}^{i,g})\end{aligned} \tag{405}$$

Now consider expression for price inflation,

$$\frac{2\epsilon}{\lambda}\pi_{i,t} + \lambda_t^{\pi,i} - \lambda_{t-1}^{\pi,i} - \beta^{-1}\lambda_{t-1}^{y,i} + \lambda_t^{rw,i} = 0 \tag{406}$$

Substituting all the elements,

$$\begin{aligned}\beta^{-1}\lambda_{t+1}^{y,i} \quad = \quad & \frac{2\epsilon}{\lambda}\pi_{i,t} \\ & + 2\frac{(1+\varphi)}{\varphi}\Delta y_t^{i,g} + \frac{2}{\varphi\chi}\Delta g_t^{i,g} \\ & + \frac{2\epsilon_w}{\widetilde{\lambda}_w}(\Delta\pi_{i,t}^w - \beta E_t\Delta\pi_{i,t+1}^w) + 2\frac{(1+\varphi}{\widetilde{\lambda}_w\varphi})(\Delta^2 y_t^{i,g} - \beta E_t\Delta^2 y_{t+1}^{i,g}) \\ & + \frac{2}{\chi\widetilde{\lambda}_w\varphi}(\Delta^2 g_t^{i,g} - \beta E_t\Delta^2 g_{t+1}^{i,g}) \\ & + \frac{2\epsilon_w}{\widetilde{\lambda}_w}\pi_{i,t}^w + 2\frac{(1+\varphi}{\widetilde{\lambda}_w\varphi})\Delta y_t^{i,g} + \frac{2}{\chi\widetilde{\lambda}_w\varphi}\Delta g_t^{i,g}\end{aligned} \tag{407}$$

Now turn to the foc for the output gap,

$$2(1+\varphi)y_t^{i,g} - \widetilde{\lambda}_w(1+\varphi)\lambda_t^{\pi^w,i} + \lambda_t^{y,i} - \beta^{-1}\lambda_{t-1}^{y,i} = 0 \tag{408}$$

The can then be solved simultaneously to obatin the target criterion for government spending. However this does not afford any real intuition.

Appendix 6 - Adding Government Debt

Up till now we have financed any deficit between government spending and distortionary tax revenues with a lump-sum tax. It is, however, interesting to discover how relaxing the assumption that lump-sum taxation balances the budget affects the use of fiscal policy as a stabilisation device.

Recall the home country consumer's budget constraint,

$$P_t C_t + E_t\{Q_{t,t+1} D(k)_{t+1}\} \leq \Pi_t + D(k)_t + W(k)_t N(k)_t (1-\tau_t) - T_t \quad (409)$$

$D(k)_{t+1}$ is a random variable, whose value depends on the state of the world in period $t+1$ i.e. it is the household's planned state-contingent wealth. Note that there is no household index on the household's consumption. This is because the complete set of asset markets implies all households face the same intertemporal budget constraint and will choose the same consumption plan (this is discussed more fully below). We can aggregate these constraints across households, to obtain the private sector's budget constraint in the home economy,

$$P_t C_t + E_t\{Q_{t,t+1} D_{t+1}\} \leq \Pi_t + D_t + W_t N_t (1-\tau_t) - T_t \quad (410)$$

There is a unique stochastic discount factor which has the property,

$$A_t = E_t[Q_{t,t+1} D_{t+1}] \quad (411)$$

where A_t is the end-of period nominal value of the household's portfolio of assets. If the household chooses to hold only risk-less one period bonds then this condition becomes,

$$D_{t+1} = R_t A_t$$

However, households will not only hold government bonds as they will wish to hold a complete set of contingent assets (given the stickiness in wage and price setting). The wealth D_{t+1}being transferred into the next period satisfies the bound,

$$D_{t+1} \geq -\sum_{T=t+1}^{\infty} E_{t+1}[Q_{t+1,T}(\Pi_T + W(k)_T N(k)_T (1-\tau_T) - T_T] \quad (412)$$

with certainty, no matter what state of the world emerges. These series of borrowing constraints and flow budget constraints then defines the intertemporal budget constraint. It is normal to rule out no-Ponzi schemes which amount to,

$$\sum_{T=t}^{\infty} E_t[Q_{t,T}(\Pi_T + W(k)_T N(k)_T (1-\tau_T) - T_T] < \infty \quad (413)$$

at each point in time across all possible states of the world. These can be combined to yield the intertemporal budget constraint (see Woodford, 2003, chapter 2, page 69),

$$\sum_{T=t}^{\infty} E_t[P_T C_T] \leq D_t + \sum_{T=t}^{\infty} E_t[Q_{t,T}(\Pi_T + W(k)_T N(k)_T (1-\tau_T) - T_T)] \quad (414)$$

Note what this implies. For all households to be consuming the same they must have different initial holdings of wealth to compensate for differences in expected incomes caused by stickiness in wage setting. However, this is exactly what complete financial markets are designed to do. To the extent that future incomes are less than other households they have received an insurance premium to compensate for this and allow them to enjoy the same level of consumptuion. Optimisation on the part of households then implies that these constraints hold as equalities (otherwise they are missing out on consumption opportunities by not fully exploiting their intertemporal budget constraints). Aggregating over households would, in a closed economy, allow us to show the equivalence of private and public sector budget constraints.

Noting the equivalence between factor incomes and national output,

$$P_H Y = WN + \Pi - \varkappa WN + \tau^v P_H Y_H \tag{415}$$

we can rewrite the home country's budget constraint as,

$$D_t = -\sum_{T=t}^{\infty} E_t[Q_{t,T}(P_{H,T}Y_T - P_T C_T - W_T N_T(\tau_T - \varkappa) - \tau^v P_{H,T} Y_{H,T} - T_T)] \tag{416}$$

Recall the goods market clearing condition in the home economy,

$$Y = (1-\alpha)\frac{PC}{P_H} + \alpha \int_0^1 (\frac{\varepsilon_i P^i C^i}{P_H})di + G \tag{417}$$

Similar conditions exist in economy j,

$$Y^i = (1-\alpha)\frac{P^j C^j}{P_j} + \alpha \int_0^1 (\frac{\varepsilon_i P^i C^i}{\varepsilon_j P_j})di + G^j \tag{418}$$

This can then be aggregated across member states,

$$\begin{aligned}
\int_0^1 \varepsilon_j P_j Y^j dj &= (1-\alpha)\int_0^1 \varepsilon_j P^j C^j dj + \alpha \int_0^1 \int_0^1 (\varepsilon_i P^i C^i) di dj + \int_0^1 \varepsilon_j P_j G^j dj \\
&= (1-\alpha)\int_0^1 \varepsilon_j P^j C^j dj + \alpha \int_0^1 (\varepsilon_i P^i C^i) di + \int_0^1 \varepsilon_j P_j G^j dj \\
&= \int_0^1 \varepsilon_j P^j C^j dj + \int_0^1 \varepsilon_j P_j G^j dj
\end{aligned} \tag{419}$$

Integrating the budget constraints across economies and using this global market clearing condition yields,

$$\int \varepsilon_i D_t^i di = -\sum_{T=t}^{\infty} E_t[Q_{t,T}(\int_0^1 P_{i,T} G_{i,T} - W_{i,T} N_{i,T}(\tau_{i,T} - \varkappa_i) - \tau_{i,T}^v P_{i,T} Y_T^i - T_{i,T})\varepsilon_i di] \tag{420}$$

with the nominal exchange rate fixed at its normalised value of 1 in monetary union we get the expression in the main text.

Appendix 7 - Optimal Commitment Policy with Government Debt

Open Economy Case

The Lagrangian associated with the open economy case in the presence of a national government budget constraint is given by,

$$
\begin{aligned}
L_t &= E_0 \sum_{t=0}^{\infty} \beta^t [\frac{\epsilon}{\lambda} \pi_{i,t}^2 + \frac{\epsilon_w}{\widetilde{\lambda}_w} (\pi_{i,t}^w)^2 + (y_t^{i,g})^2 (1+\varphi) + \frac{1}{\chi} (g_t^{i,g})^2 \\
&+ \lambda_t^{\pi^w,i} (\pi_{i,t}^w - \beta E_t \pi_{i,t+1}^w - \widetilde{\lambda}_w ((1+\varphi) y_t^{i,g} - g_t^{i,g} - rw_t^{i,g} - \ln(1-\tau_t^i)^g)) \\
&+ \lambda_t^{\pi,i} (\pi_{i,t} - \beta E_t \{\pi_{i,t+1}\} - \lambda [rw_t^{i,g} - \ln(1-\tau_t^{i,s})^g]) \\
&+ \lambda_t^{y,i} (y_t^{i,g} - g_t^{i,g} - E_t \{y_{t+1}^{i,g} - g_{t+1}^{i,g} + \pi_{i,t+1}\} + (r_t^i - r_t^{i,n})) \\
&+ \lambda_t^{rw,i} (rw_t^{i,g} - \pi_{i,t}^w + \pi_{i,t} - rw_{t-1}^{i,g} + \Delta a_t) \\
&+ \lambda_t^{b,i} (b_t^{i,g} - \overline{R} b_{t-1}^{i,g} - \overline{R} (r_{t-1}^{i,g} - \pi_{i,t}) - b_g g_t^{i,g} - b_{\tau^v} \ln(1-\tau_t^{i,s})^g \\
&+ b_y y_t^{i,g} - b_\tau \ln(1-\tau_t^i)^g + b_{rw} rw_t^{i,g})]
\end{aligned}
$$

where $b_g = \frac{\overline{G}^i}{\overline{B}^i} \frac{1-\gamma^{i,n}}{\gamma^{i,n}}$, $b_{\tau^v} = \frac{(1-\overline{\tau}^{i,s})\overline{Y}^i}{\overline{B}^i}$,$b_y = \frac{\overline{\tau}^{i,s}\overline{Y}^i}{\overline{B}^i} + \frac{\overline{\tau rw}^i \overline{N}^i}{\overline{B}^i} - \frac{\overline{G}^i}{\overline{B}^i}$,$b_\tau = \frac{(1-\overline{\tau}^i)\overline{rw}^i\overline{N}^i}{\overline{B}^i}$, and $b_{rw} = \frac{\overline{\tau rw}^i \overline{N}^i}{\overline{B}^i}$. The focs are given by, for the interest rate,

$$\lambda_t^{y,i} - E_t \lambda_{t+1}^{b,i} = 0 \tag{421}$$

Here monetary policy must now take account of its impact on the government's finances.

In terms of national focs, we begin with the foc for the sales tax gap, $\ln(1-\tau^v)^g$,

$$\lambda \lambda_t^{\pi,i} - b_{\tau^v} \lambda_t^{b,i} = 0 \tag{422}$$

Similarly, the condition for income taxes is given by,

$$\widetilde{\lambda}_w \lambda_t^{\pi^w,i} - b_\tau \lambda_t^{b,i} = 0 \tag{423}$$

and for real wages,

$$-\lambda \lambda_t^{\pi,i} + \widetilde{\lambda}_w \lambda_t^{\pi^w,i} + \lambda_t^{rw,i} - \beta E_t \lambda_{t+1}^{rw,i} + b_{rw} \lambda_t^{b,i} = 0 \tag{424}$$

The remaining first-order conditions are for debt,

$$\lambda_t^{b,i} - \beta \overline{R} E_t \lambda_{t+1}^{b,i} = 0 \tag{425}$$

which implies that, $E_0 \lambda_t^{b,i} = \lambda^{b,i}\ \forall t$. In other words policy must ensure that the 'cost' of the government's budget constraint is constant following a shock which is the basis of the random walk result of Schmitt-Grohe and Uribe (2004).

This also implies that the lagrange multipliers for the wage and price phillips curves are constant over time too. The remaining focs are for inflation,

$$\frac{2\epsilon}{\lambda}\pi_{i,t} + \lambda_t^{\pi,i} - \lambda_{t-1}^{\pi,i} - \beta^{-1}\lambda_{t-1}^{y,i} + \lambda_t^{rw,i} + \overline{R}\lambda_t^{b,i} = 0 \tag{426}$$

wage inflation,

$$\frac{2\epsilon_w}{\widetilde{\lambda}_w}\pi_{i,t}^w + \lambda_t^{\pi^w,i} - \lambda_{t-1}^{\pi^w,i} - \lambda_t^{rw,i} = 0 \tag{427}$$

the government spending gap,

$$\frac{2}{\chi}g_t^{i,g} + \widetilde{\lambda}_w\lambda_t^{\pi^w,i} - \lambda_t^{y,i} + \beta^{-1}\lambda_{t-1}^{y,i} - b_g\lambda_t^{b,i} = 0 \tag{428}$$

and the output gap,

$$2(1+\varphi)y_t^{i,g} - \widetilde{\lambda}_w(1+\varphi)\lambda_t^{\pi^w,i} + \lambda_t^{y,i} - \beta^{-1}\lambda_{t-1}^{y,i} + b_y\lambda_t^{b,i} = 0 \tag{429}$$

Combining the focs for price and wage inflation,

$$\frac{2\epsilon}{\lambda}\pi_{i,t} + \frac{2\epsilon_w}{\widetilde{\lambda}_w}\pi_{i,t}^w = 0 \tag{430}$$

gives us the optimal combination of wage and price inflation. This essentially describes the balance between wage and price adjustment in achieving the new steady-state real wage consistent with the new steady-state tax rates required to stabilise the debt stock following the shock. Taking the foc for the output gap, we have,

$$2(1+\varphi)y_t^{i,g} + \lambda^{b,i}(-b_\tau(1+\varphi) + (1-\beta^{-1}) + b_y) = 0 \tag{431}$$

which defines the value of the Lagrange multiplier associated with the government's budget constraint which implies that the output gap is constant. Using the focs for the two taxes in conjunction with the foc for real wages implies,

$$-\frac{2\epsilon}{\lambda}(\pi_{i,t} - \beta E_t\pi_{i,t+1}) + (b_{rw} + b_\tau - b_{\tau^v})\lambda_t^{b,i} = 0 \tag{432}$$

and,

$$\frac{2\epsilon_w}{\widetilde{\lambda}_w}(\pi_{i,t}^w - \beta E_t\pi_{i,t+1}^w) + (b_{rw} + b_\tau - b_{\tau^v})\lambda_t^{b,i} = 0 \tag{433}$$

Using the NKPCs for price and wage inflation these can be rewritten as the sales and income tax rules, respectively,

$$-2\epsilon(rw_t^{i,g} - \ln(1-\tau_t^{i,s})^g) + (b_{rw} + b_\tau - b_{\tau^v})\lambda^{b,i} = 0 \tag{434}$$

and,

$$2\epsilon_w((1+\varphi)y_t^{i,g} - g_t^{i,g} - rw_t^{i,g} - \ln(1-\tau_t^i)^g)) + (b_{rw} + b_\tau - b_{\tau^v})\lambda^{b,i} = 0 \tag{435}$$

Finally the government spending rule is given by,

$$\frac{2}{\chi} g_t^{i,g} + (b_\tau - (1 - \beta^{-1}) - b_g)\lambda^{b,i} = 0 \tag{436}$$

which is again constant.

EMU Case:

The Lagrangian associated with the open economy case in the presence of a national government budget constraint is given by,

$$\begin{aligned} L_t &= \int_0^1 \sum_{t=0}^{\infty} \beta^t [\frac{\epsilon}{\lambda}\pi_{i,t}^2 + \frac{\epsilon_w}{\widetilde{\lambda}_w}(\pi_{i,t}^w)^2 + (y_t^{i,g})^2(1+\varphi) + \frac{1}{\chi}(g_t^{i,g})^2 \\ &\quad + \lambda_t^{\pi^w,i}(\pi_{i,t}^w - \beta E_t \pi_{i,t+1}^w - \widetilde{\lambda}_w((1+\varphi)y_t^{i,g} - g_t^{i,g} - rw_t^{i,g} - \ln(1-\tau_t^i)^g)) \\ &\quad + \lambda_t^{\pi,i}(\pi_{i,t} - \beta E_t\{\pi_{i,t+1}\} - \lambda[rw^{i,g} - \ln(1-\tau_t^{i,s})^g]) \\ &\quad + \lambda_t^{y,i}(y_t^{i,g} - g_t^{i,g} - E_t\{y_{t+1}^{i,g} - g_{t+1}^{i,g} + \pi_{i,t+1}\} + (r_t - r_t^{i,n})) \\ &\quad + \lambda_t^{rw,i}(rw_t^{i,g} - \pi_{i,t}^w + \pi_{i,t} - rw_{t-1}^{i,g} + \Delta a_t) \\ &\quad + \lambda_t^{b,i}(b_t^{i,g} - \overline{R} b_{t-1}^{i,g} - \overline{R}(r_{t-1}^g - \pi_{i,t}) - b_g g_t^{i,g} - b_{\tau^v} \ln(1-\tau_t^{i,s})^g \\ &\quad + b_y y_t^{i,g} - b_\tau \ln(1-\tau_t^i)^g + b_{rw} rw_t^{i,g})]di \end{aligned}$$

where $b_g = \frac{\overline{G}^i}{\overline{B}^i}\frac{1-\gamma^{i,n}}{\gamma^{i,n}}$, $b_{\tau^v} = \frac{(1-\overline{\tau}^{i,s})\overline{Y}^i}{\overline{B}^i}$,$b_y = \frac{\overline{\tau}^{i,s}\overline{Y}^i}{\overline{B}^i} + \frac{\overline{\tau rw}^i \overline{N}^i}{\overline{B}^i} - \frac{\overline{G}^i}{\overline{B}^i}$,$b_\tau = \frac{(1-\overline{\tau}^i)\overline{rw}^i\overline{N}^i}{\overline{B}^i}$, and $b_{rw} = \frac{\overline{\tau rw}^i \overline{N}^i}{\overline{B}^i}$. The focs are given by, for the union wide interest rate,

$$\int_0^1 (\lambda_t^{y,i} - E_t \lambda_{t+1}^{b,i}) di = 0 \tag{437}$$

Here monetary policy must now take account of its impact on the union's finances.

In terms of national focs, we begin with the foc for the sales tax gap, $\ln(1 - \tau^v)^g$,

$$\lambda \lambda_t^{\pi,i} - b_{\tau^v} \lambda_t^{b,i} = 0 \tag{438}$$

Similarly, the condition for income taxes is given by,

$$\widetilde{\lambda}_w \lambda_t^{\pi^w,i} - b_\tau \lambda_t^{b,i} = 0 \tag{439}$$

and for real wages,

$$-\lambda\lambda_t^{\pi,i} + \widetilde{\lambda}_w \lambda_t^{\pi^w,i} + \lambda_t^{rw,i} - \beta E_t \lambda_{t+1}^{rw,i} + b_{rw}\lambda_t^{b,i} = 0 \tag{440}$$

The remaining first-order conditions are for debt,

$$\lambda_t^{b,i} - \beta \overline{R} E_t \lambda_{t+1}^{b,i} = 0 \tag{441}$$

which implies that, $E_0\lambda_t^{b,i} = \lambda^{b,i}\ \forall t$. In other words policy must ensure that the 'cost' of the government's budget constraint is constant following a shock which is the basis of the random walk result of Schmitt-Grohe and Uribe (2004). This also implies that the lagrange multipliers for the wage and price phillips curves are constant over time too. The remaining focs are for inflation,

$$\frac{2\epsilon}{\lambda}\pi_{i,t} + \lambda_t^{\pi,i} - \lambda_{t-1}^{\pi,i} - \beta^{-1}\lambda_{t-1}^{y,i} + \lambda_t^{rw,i} + \overline{R}\lambda_t^{b,i} = 0 \tag{442}$$

wage inflation,

$$\frac{2\epsilon_w}{\widetilde{\lambda}_w}\pi_{i,t}^w + \lambda_t^{\pi^w,i} - \lambda_{t-1}^{\pi^w,i} - \lambda_t^{rw,i} = 0 \tag{443}$$

the government spending gap,

$$\frac{2}{\chi}g_t^{i,g} + \widetilde{\lambda}_w\lambda_t^{\pi^w,i} - \lambda_t^{y,i} + \beta^{-1}\lambda_{t-1}^{y,i} - b_g\lambda_t^{b,i} = 0 \tag{444}$$

and the output gap,

$$2(1+\varphi)y_t^{i,g} - \widetilde{\lambda}_w(1+\varphi)\lambda_t^{\pi^w,i} + \lambda_t^{y,i} - \beta^{-1}\lambda_{t-1}^{y,i} + b_y\lambda_t^{b,i} = 0 \tag{445}$$

Combining the last two focs,

$$2(1+\varphi)y_t^{i,g} + \frac{2}{\chi}g_t^{i,g} + (b_y - \varphi b_\tau - b_g)\lambda_t^{b,i} = 0 \tag{446}$$

gives us a definition of the lagrange multiplier associated with the budget constraint, which also implies a constant relationship between the output and government spending gaps following a shock.

Consider the foc for the real wage,

$$\frac{2\epsilon_w}{\widetilde{\lambda}_w}(\pi_{i,t}^w - \beta E_t\pi_{i,t+1}^w) + (b_{rw} + b_\tau - b_{\tau^v})\lambda_t^{b,i} = 0 \tag{447}$$

Using the NKPC for wage inflation we can obtain an income tax rule,

$$2\epsilon_w((1+\varphi)y_t^{i,g} - g_t^{i,g} - rw_t^{i,g} - \ln(1-\tau_t)^g)) + (b_{rw} + b_\tau - b_{\tau^v})\lambda^{b,i} = 0 \tag{448}$$

Combining the wage and price inflation focs,

$$\frac{2\epsilon}{\lambda}\pi_{i,t} + \frac{2\epsilon_w}{\widetilde{\lambda}_w}\pi_{i,t}^w - \beta^{-1}\lambda_{t-1}^{y,i} + \overline{R}\lambda_t^{b,i} = 0 \tag{449}$$

Use in the output gap equation and using the NKPCs to eliminate the inflation dynamics gives us a sales-tax rule,

$$\begin{aligned} 0 &= 2(1+\varphi)y_t^{i,g} + (b_y - \varphi b_\tau + 1 - \beta^{-1} + b_{rw} - b_{\tau^v})\lambda^{b,i} \\ &\quad -2\epsilon(rw_t^{i,g} - \ln(1-\tau_t^v)^g) \end{aligned} \tag{450}$$

Need to get a government spending rule. Foc for output gap gives,

$$
\begin{aligned}
0 &= 2(1+\varphi)y_t^{i,g} + \lambda^{b,i}(-b_\tau(1+\varphi) + (1-\beta^{-1}) + b_y) \\
&\quad -\frac{2\epsilon}{\lambda}(\pi_{i,t} - \beta E_t \pi_{i,t+1}) \\
&\quad -\frac{2\epsilon_w}{\widetilde{\lambda}_w}(\pi_{i,t}^w - \beta E_t \pi_{i,t+1}^w)
\end{aligned} \tag{451}
$$

While for government spending we get,

$$
\begin{aligned}
0 &= \frac{2}{\chi} g_t^{i,g} + \lambda^{b,i}(b_\tau - b_g - 1 + \beta^{-1}) \\
&\quad +\frac{2\epsilon}{\lambda}(\pi_{i,t} - \beta E_t \pi_{i,t+1}) \\
&\quad +\frac{2\epsilon_w}{\widetilde{\lambda}_w}(\pi_{i,t}^w - \beta E_t \pi_{i,t+1}^w)
\end{aligned} \tag{452}
$$

Eliminating $\lambda^{b,i}$ we obtain,

$$
\begin{aligned}
0 &= \frac{2}{\chi} g_t^{i,g} - 2(1+\varphi)\frac{(b_\tau - b_g - 1 + \beta^{-1})}{(-b_\tau(1+\varphi) + (1-\beta^{-1}) + b_y)} y_t^{i,g} \\
&\quad +\frac{2\epsilon}{\lambda}(1 + \frac{(b_\tau - b_g - 1 + \beta^{-1})}{(-b_\tau(1+\varphi) + (1-\beta^{-1}) + b_y)})(\pi_{i,t} - \beta E_t \pi_{i,t+1}) \\
&\quad +\frac{2\epsilon_w}{\widetilde{\lambda}_w}(1 + \frac{(b_\tau - b_g - 1 + \beta^{-1})}{(-b_\tau(1+\varphi) + (1-\beta^{-1}) + b_y)})(\pi_{i,t}^w - \beta E_t \pi_{i,t+1}^w)
\end{aligned} \tag{453}
$$

Using the NKPCs for price and wage inflation to eliminate the inflation dynamics gives us our government spending rule,

$$
\begin{aligned}
0 &= \frac{2}{\chi} g_t^{i,g} - 2(1+\varphi)\frac{(b_\tau - b_g - 1 + \beta^{-1})}{(-b_\tau(1+\varphi) + (1-\beta^{-1}) + b_y)} y_t^{i,g} \\
&\quad +2\epsilon(1 + \frac{(b_\tau - b_g - 1 + \beta^{-1})}{(-b_\tau(1+\varphi) + (1-\beta^{-1}) + b_y)})(rw_t^{i,g} - \ln(1-\tau_t^{i,s})^g) \\
&\quad +2\epsilon_w(1 + \frac{(b_\tau - b_g - 1 + \beta^{-1})}{(-b_\tau(1+\varphi) + (1-\beta^{-1}) + b_y)})((1+\varphi)y_t^{i,g} - g_t^{i,g} - rw_t^{i,g} - \ln(1-\tau_t^i)^g)
\end{aligned} \tag{454}
$$

Appendix 8 - Variable Definitions

$A-$ Productivity

$C-$ Aggregate consumption bundle

$C^{*}-$ Aggregate foreign consumption.

C_F- Aggregate of goods produced abroad.

C_H- Bundle of domestically produced consumption goods.

$C_H(j)-$ Good j within bundle of domestically produced consumption goods.

C_i- Bundle of goods produced in country i.

$D-$ Nominal payoff from financial assets (including share of profits in firms)

ε_i- Bilateral nominal exchange rate with country i.

$\varepsilon-$ Effective nominal exchange rate.

$G(j)-$ public good j.

$G-$ Aggregate provision of public goods.

$N(j)-$ domestic labour employed by firm j.

N_k- Labour supplied by houshold k.

$N-$ Aggregate domestic labour input.

$P-$ Aggregate consumer price index associated with C

P_H - Domestic price index associated with C_H

π_H- Rate of inflation in P_H

$P_H(j)-$ Price of good $C_H(j)$

P_i- Index of domestic prices in country i (in home country currency).

P_i^i- Index of domestic prices in country i in country i's currency.

$P_i^i(j)-$Price of country i's good j expressed in terms of country i's currency.

$P^{*}-$ World price level (both consumer and output prices)

$Q_{t,t+1}-$ Stochastic discount factor measuring current certainty equivalent value of an uncertain future payoff.

$\mathcal{Q}_i-$Bilateral real exchange rate.

$\mathcal{Q}$ $-$ Effective real exchange rate.

S_i- Bilateral terms of trade with country i.

$S-$ effective terms of trade.

$\tau-$ Income tax rate

τ^v- Sales tax rate.

$v-$ logged value of employment subsidy $(1-\chi)$

$W(k)-$ Nominal wage charged by household k.

$W-$ Wage index for home country.

π^w- Rate of inflation in W.

In the paper, lower case letters denote logged values of the associated levels variable, n superscipts denote 'natural' values that would occur in the absence of nominal inertia and 'g' denotes 'gap' varaibles - the difference between the logged variable and its logged natural value.

Appendix 9 - Parameter Definitions

$1 - \alpha$ - weight on domestically produced goods in consumption - a measure of home bias.

$\beta-$ Consumers subjective discount factor.

$\epsilon-$ elasticity of substitution between domestically produced goods (= price elasticity of demand for domestically produced goods.

ϵ_w- elasticity of substitution between differentiated labour (= wage elasticity of demand for domestically labour types.

$\eta-$ elasticity of substitution between bundles of goods produced in foreign economies (see equal to 1 for simplicity).

$\chi-$ weight on public goods in utility.

$\varphi-$labour supply parameter.

$1 - \theta_p-$ probability of price adjustment in each period.

$1 - \theta_w-$ probability of wage adjustment in each period.

$\mu-$ steady-state mark-up in domestic goods market.

μ^w- steady-state mark-up in domestic labour market.

Appendix 10 - Matrix Representation of Model

The model can be represented in matrix form as,

$$A0x_{t+1} = A1x_t + B0u_t + \varepsilon_t$$

where x_t is a vector of endogenous variables, u_t are a vector of policy instruments and ε_t a vector of schocks, all of which are defined as follows,

$$x_t = \begin{bmatrix} \ln(1-\tau_t^i)^g \\ \pi_{i,t}^w \\ rw_t^{i,g} \\ g_t^{i,g} \\ \varepsilon_{t+1}^i \\ a_t^i \\ y_t^{i,g} - g_t^{i,g} \\ \pi_{i,t} \\ E_t\pi_{i,t+1} \\ E_t\pi_{i,t+1}^w \end{bmatrix}, u_t = \begin{bmatrix} \ln(1-\tau_{t+1}^{i,s})^g \\ g_{t+1}^{i,g} \\ \ln(1-\tau_{t+1}^i)^g \end{bmatrix} \text{ and } \varepsilon_t = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \varepsilon_{t+2}^i \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$A0 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & \lambda & 0 & 0 & 0 & 0 & 0 & \beta & 0 \\ 0 & 0 & -\widetilde{\lambda}_w & \varphi\widetilde{\lambda}_w & 0 & 0 & (1+\varphi)\widetilde{\lambda}_w & 0 & 0 & \beta \end{bmatrix}$$

$$A1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & \rho_a & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$B0=\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ \lambda & 0 & 0 \\ 0 & 0 & \widetilde{\lambda}_w \end{bmatrix}$$

This can then be solved to obtain the form used in Soderlind (1999),

$$x_{t+1}=Ax_t+Bu_t+\varepsilon_t$$

where $A=(A0)^{-1}A1$ and $B=(A0)^{-1}B0$. The first eight variables in x_t are considered to be predetermined, while the last two are jump variables. The element of this representation which implies this is the EMU case is the dynamic relationship,

$$y_t^{i,g}-g_t^{i,g}=y_t^{i,g}-g_t^{i,g}-\pi_{i,t}-\Delta a_t^i$$

which implies that the system must exhibit the property of price level control. This is obtained from

$$y_t=c_t^*+g_t+s_t \tag{455}$$

and the definition of the terms of trade,

$$\begin{aligned} s_t &= p_{F,t}-p_H,t && (456)\\ &= e_t+p_t^*-p_{H,t} && (457)\end{aligned}$$

after imposing the fixed exchange rate and assuming the shock hits country i only. (Productivity enters by considering the change in the natural level of output).

The open economy case has the same representation, but the output gap can be considered a control variable from the point of view of the monetary authorities. In this case the system would become (note the change in the definition of x_t) ..

$$x_t=\begin{bmatrix} \ln(1-\tau_t^i)^g \\ \pi_{i,t}^w \\ rw_t^{i,g} \\ g_t^{i,g} \\ \varepsilon_{t+1}^i \\ a_t \\ y_t^{i,g} \\ \pi_{i,t} \\ E_t\pi_{i,t+1} \\ E_t\pi_{i,t+1}^w \end{bmatrix},u_t=\begin{bmatrix} y_{t+1}^{i,g} \\ \ln(1-\tau_{t+1}^{i,s})^g \\ g_{t+1}^{i,g} \\ \ln(1-\tau_{t+1})^g \end{bmatrix}\text{ and }\varepsilon_t=\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \varepsilon_{t+2}^i \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$A0 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & \lambda & 0 & 0 & 0 & 0 & 0 & \beta & 0 \\ 0 & 0 & -\widetilde{\lambda}_w & -\widetilde{\lambda}_w & 0 & 0 & (1+\varphi)\widetilde{\lambda}_w & 0 & 0 & \beta \end{bmatrix}$$

$$A1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & \rho_a & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$B0 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \lambda & 0 & 0 \\ 0 & 0 & \widetilde{\lambda}_w & 0 \end{bmatrix}$$

The remaining variants considered in the paper can then be calculated by eliminating the controls no longer in use.

Adding in debt the EMU model becomes,

$$x_t = \begin{bmatrix} b_t^{i,g} \\ \ln(1-\tau_t^i)^g \\ \pi_{i,t}^w \\ rw_t^{i,g} \\ g_t^{i,g} \\ \varepsilon_{t+1}^i \\ a_t^i \\ y_t^{i,g} - g_t^{i,g} \\ \pi_{i,t} \\ E_t\pi_{i,t+1} \\ E_t\pi_{i,t+1}^w \end{bmatrix}, u_t = \begin{bmatrix} \ln(1-\tau_{t+1}^{i,s})^g \\ g_{t+1}^{i,g} \\ \ln(1-\tau_{t+1})^g \end{bmatrix} \text{ and } \varepsilon_t^i = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \varepsilon_{t+2}^i \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$A0 = \begin{bmatrix} 1 & \frac{(1-\overline{\tau}^i)\overline{rw}^i\overline{N}^i}{\overline{B}^i} & 0 & \frac{\overline{\tau}^i\overline{rw}^i\overline{N}^i}{\overline{B}^i} & (\overline{R}-1) - \frac{\overline{G}^i}{\overline{\overline{B}}^i}\frac{1-\gamma^n}{\gamma^n} & 0 & 0 & (\overline{R}-1) & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \lambda & 0 & 0 & 0 & 0 & 0 & \beta & 0 \\ 0 & 0 & 0 & -\widetilde{\lambda}_w & \varphi\widetilde{\lambda}_w & 0 & 0 & (1+\varphi)\widetilde{\lambda}_w & 0 & 0 & \beta \end{bmatrix}$$

$$A1 = \begin{bmatrix} \overline{R} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\overline{R} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & \rho_a & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$B0 = \begin{bmatrix} (1-\overline{\tau}^{i,s})\frac{\overline{Y}^i}{\overline{B}^i} & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ \lambda & 0 & 0 \\ 0 & 0 & \widetilde{\lambda}_w \end{bmatrix}$$

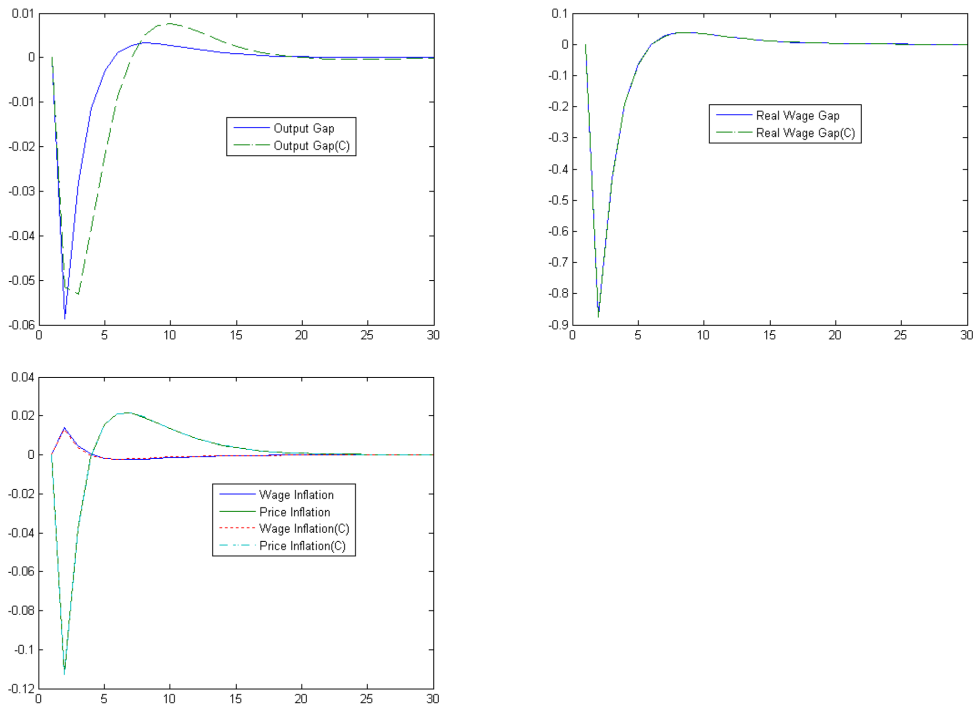

Figure 1: Response to a 1% technology shock in an open economy with only monetary policy as a policy instrument.

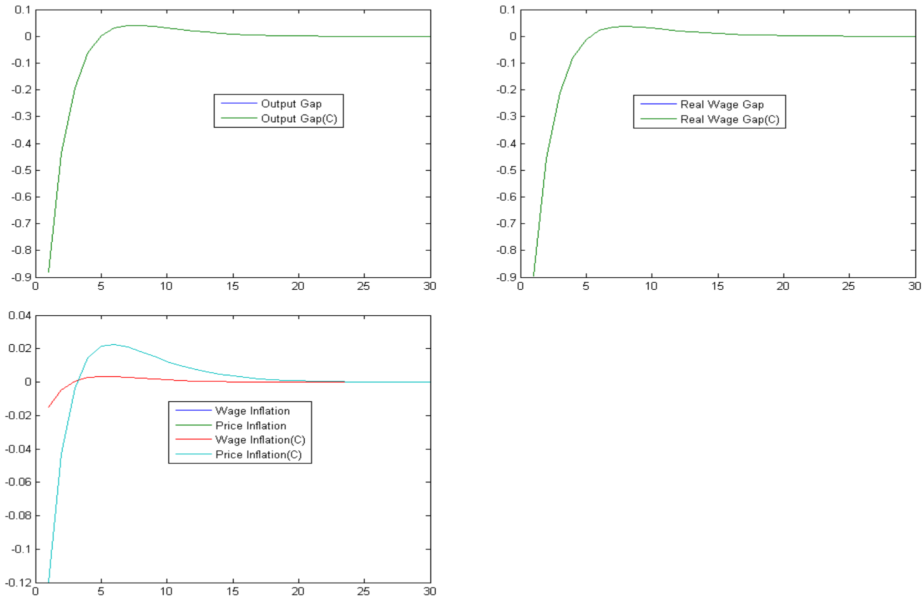

Figure 2: Response to a 1% technology shock under EMU with no policy response.

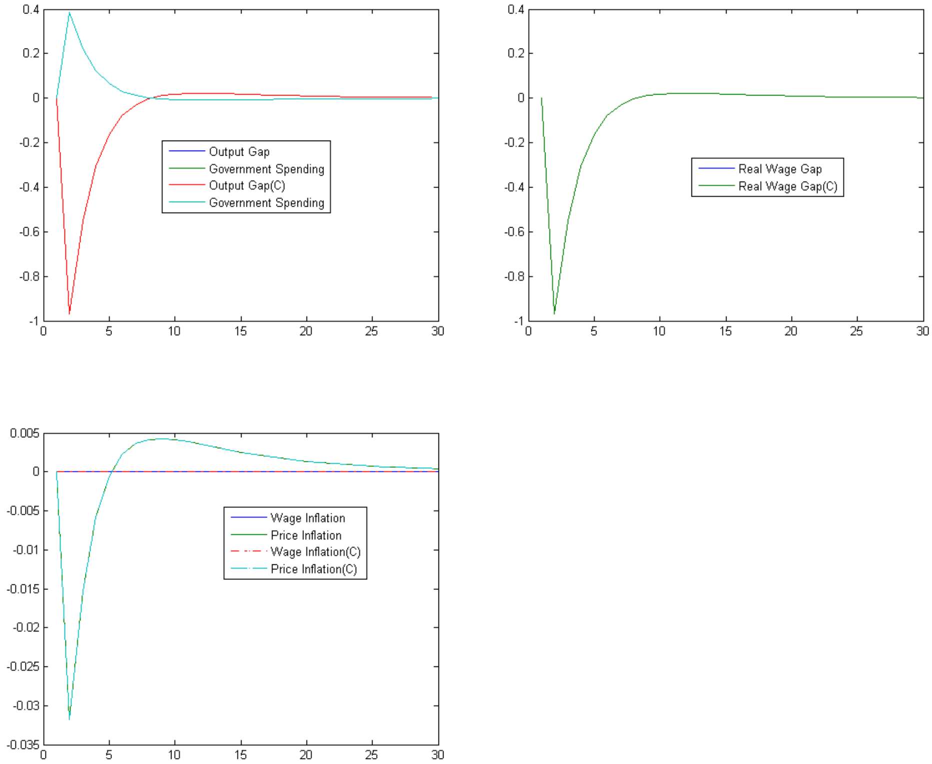

Figure 3: Response to a 1% technology shock under EMU with all fiscal instruments.

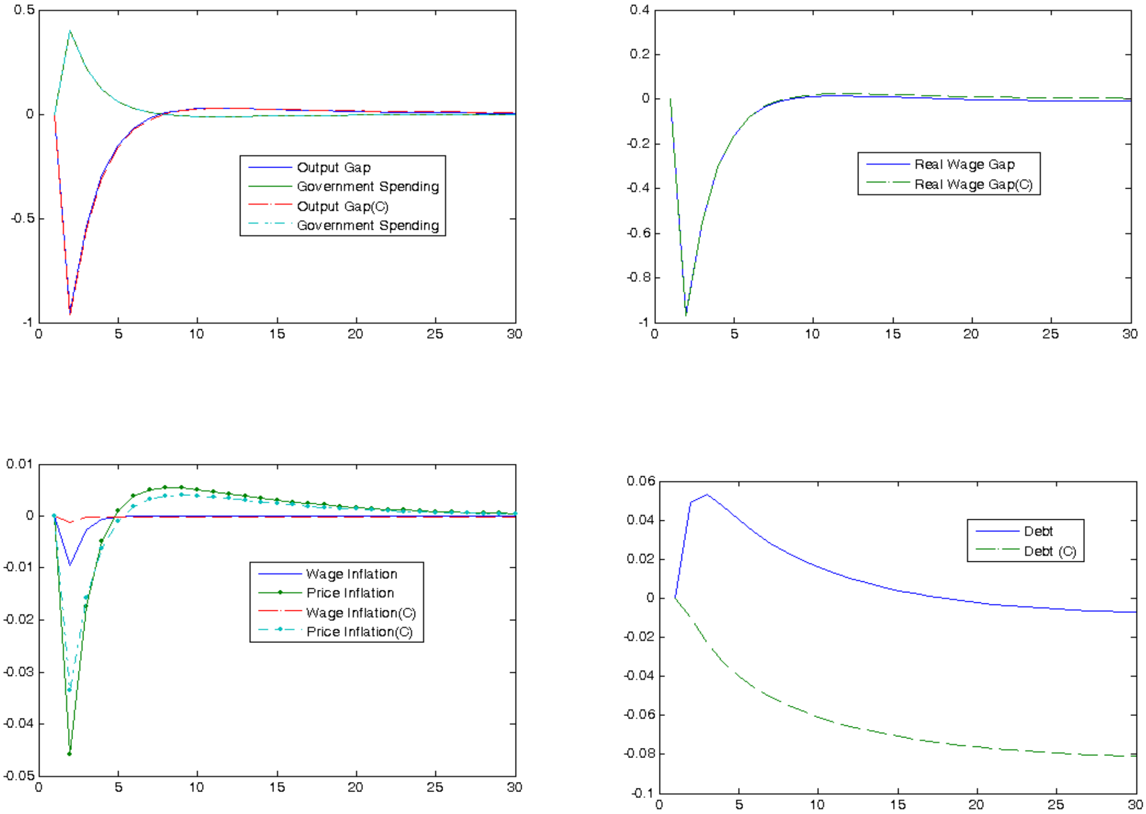

Figure 4: Response to 1% technology shock under EMU with all fiscal instruments and government debt.