CALVO CONTRACTS — OPTIMAL INDEXATION IN GENERAL EQUILIBRIUM

Vo Phuong Mai Le[a]*, Patrick Minford[b]†
[a]Cardiff University, [b]Cardiff University and CEPR

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Abstract

Calvo contracts, which are the basis of the current generation of New Keynesian models, widely include indexation to general inflation. We argue that the indexing formula should be expected inflation rather than lagged inflation. This optimises the welfare of the representative agent in a general equilibrium model of the New Keynesian type. This is shown analytically for a simplified model and by numerical simulation for a full model with price and wage contracts as well as capital. The consequence of such indexation is that monetary policy no longer has any effect on welfare.

1 Introduction

The theoretical basis for nominal rigidity set out by Calvo (1983) has been widely adopted in recent work of the so-called New Keynesian type — also known as the New Neo-Keynesian Synthesis — see Clarida, Gali and Gertler (1999). In the Calvo contract nominal rigidity can last indefinitely in the sense that there is a limited chance for wage- or price-setters to change their setting in any period. Hence once a price or wage is 'out of line' with its equilibrium there is a chance it will continue for ever. This has seemed an attractive set-up for modellers who wanted a basis for nominal rigidity with substantial potential real effects.

Nevertheless recent work has exposed a variety of puzzles arising from this set-up. On the one hand there are the apparently counter-factual implications of the theory noted for example by Mankiw (2001), Mankiw and Reis (2002), Ball (1994), Fuhrer and Moore (1995), Bakhshi et al (2003), Rudd and Whelan (2003) and Eichenbaum and Fisher (2003). On the other hand, a number of articles have pointed to the time-inconsistency problems posed for policy. The essence of these problems lies in that indefiniteness of duration for rigidity; once prices or wages have got out of line there is a strong incentive not to worsen matters by causing yet more prices or wages to get out of line, even if commitments have been made to stabilising prices or inflation along a particular initially-optimal path. A partial list of work that has addressed these issues would include: Goodfriend and King (2001), Khan, King and Wolman (2002), Svensson and Woodford (2003), McCallum (2003), Collard and Dellas (2003), and Woodford (2000).

More recently, it has been recognised (Erceg, Henderson and Levin, 2000; Christiano, Eichenbaum and Evans, 2002 for example) that the uncompromising nominal rigidity in Calvo (1983) ought to be modified to allow for some indexing process whereby general inflation is passed through by wage/price-setters. The argument has been that the chances of changing price identified in the Calvo model relate to the changing of

*Corresponding Author: LeVP@Cardiff.ac.uk

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a relative price, for example on the grounds that some micro menu cost threshold is stochastically disturbed by some micro event. If then there is some general ongoing inflation this would be passed on by all including those who would not on micro grounds wish to change their relative price. Thus the 'menus' outside the restaurants are all uprated for general inflation; some individual menus are then raised more or less than that according to micro shocks. Two specific ways have been widely pursued for doing this: indexing to 'core' inflation and alternatively to lagged inflation.[1]

It is not our intention in this paper to add to the literature of optimal policy under Calvo contracts. Rather our contention is that the indexing modification is correct but has been incorrectly implemented: the Calvo contract should instead be indexed to expected inflation. We argue the case for this in what follows and show that should this be adopted the puzzles we have referred to for optimal policy are eliminated, as the Calvo contract model defaults to something similar to the familiar 'surprise' Phillips Curve model of Lucas (1972) and Sargent and Wallace (1975). Minford and Peel (2004) showed how, in a partial equilibrium context, it was optimal for a price-setter to index to expected inflation rather than to any other index. Here we generalise that argument to a general equilibrium set-up.

We base our analysis on the general equilibrium model proposed by Canzoneri et al. (2004) with optimising agents, staggered wage and price setting, capital formation and an empirically estimated rule for the central bank's interest rate policy. Unlike the indexation proposed by Erceg et al. (ibid.), here we show that the Calvo contract should be instead indexed to (rationally) expected inflation. In this way agents will optimise their expected relative price and wage responses with respect to micro shocks. We show that if this optimal indexation is implemented, the Calvo contract implies rigidity only for relative prices, with the general price level moving flexibly with the pressures of monetary policy and productivity; monetary policy then turns out to be irrelevant to welfare. We show these results at first analytically in a much simplified version of the model; the model's complexity makes analytical treatment intractable beyond this case. We then proceed to show the results generally via numerical simulations on a calibrated version of the full model.

1.1 Preliminaries

We begin heuristically by taking up the discussion of this issue in Woodford (2004). Woodford argues much as above that contracts should be indexed. Indexing, as he points out with the aid of simulations on a basic benchmark model, both raises the persistence of inflation (and output) and lowers the effect of inflation on output. The resulting simulation properties are, he argues, attractive and he supports this sort of indexing accordingly — in this context we should note that Christiano, Eichenbaum and Evans (2002) have also built more elaborate models of this type which appear to replicate the impulse response functions to monetary shocks found in the data.

Woodford argues that indexing to current prices, which would plainly be best for economic agents, is infeasible both because the costs of collecting the data so quickly would be prohibitive and also the costs of implementing the indexation process would be too high. Such an issue is not however new. In market-clearing models agents 'index' their relative price choices to expected inflation available at the time; thus workers for example in the new classical model of labour supply (of Lucas and Sargent and Wallace) supply labour at wages chosen with reference to expected prices, the expectations being formed on the basis of currently generally available information. The point of such expectations is to allow for movements in the general price level in a manner that does not cause excessive costs of information-gathering and processing. Thus it would seem quite plainly that expected prices would also be available as a low-cost instrument for contract-makers to index their price-setting to. Woodford did not consider this alternative. However, it

[1] Further examples are Casares (2002), Ascari (2003); additionally Calvo, Celasun and Kumhof (2003, 2001) and Cespedes, Kumhof and Parrado (2003) have recently suggested further forms of indexing based on rules of thumb based on learning. All these schemes violate the strict natural rate hypothesis (that no monetary policy should be capable of permanently changing output and employment) whose absence from the original Calvo set-up was noted by McCallum (1998) - see also Minford and Peel (2002a) for examples of how monetary policy can 'manipulate' real outcomes.

too has interesting and possibly attractive properties. We show below (last section) the model simulations obtained in the Canzoneri et al model by indexing to expected inflation, side by side with the simulations of the other models- viz none and lagged inflation.

As we will see the simulation properties differ dramatically. Inflation and output persistence fall to zero and a monetary shock causes an inflation and output surprise only. These are well-known 'new classical' features. It may be found surprising that the Calvo model can generate these properties but we explain this further below. It may also be objected that the model cannot in this form fit empirical data; that is a different issue with which we do not grapple in this paper. However, we would argue that this is far from being the case as is shown for example by the performance of new classical models on UK data (Minford, Nowell, Sofat and Srinivasan, 2004 where it is shown that the inflation persistence characteristic of UK data for much of the post-war period can be accounted for with a new classical model and so can its variations across monetary regimes, including its effective disappearance under the inflation targeting regime since 1992).

Our aim in this paper is rather to argue that on purely theoretical grounds this mode of indexation is desirable and thus that if New Keynesian models are to be regarded as the product of optimisation then they should include this 'rational indexation'. We assume that it is feasible on grounds of costs of information-gathering and processing. This does not seem a hard assumption within the general context of these models which assume rational expectations together with necessary current macro information (about interest rates and therefore also output and inflation) in order for agents to form expectations of future inflation and output.

It might conceivably be the case that indexing to lagged inflation would give better welfare results for the representative agent than indexing to expected inflation. It is unlikely since lagged inflation here is acting as a forecast of general inflation for agents unable to current actual general inflation: the rational expectations forecast is unbiased and has the lowest variance of all available forecasts, including in this context lagged inflation. In the rest of this paper we show that, essentially for this reason, indeed it is the case that indexing to expected inflation gives better welfare results than indexing to lagged inflation. In this situation it would be rational for agents to index their contracts to expected, rather than lagged,. inflation.

In what follows we begin by setting out the Canzoneri et al model in its entirety but allowing for full indexation. We then proceed to examine a simplified and loglinearised version to and to show that it is demonstrably optimal for agent's welfare within it to index to rationally expected prices; the alternative we focus on primarily is lagged inflation .We then show the same result numerically within the full model. We conclude briefly with some implications of this result for New Keynesian models.

2 The Model With Indexation

The model used is a nonlinear NNS model (Canzoneri et al, ibid.), characterized by optimizing agents, monopolistic competition, nominal inertial and capital accumulation. It is very closely related to the models of Erceg, Henderson and Levin (2000) and Collard and Dellas (2003). Staggered price setting leads to a dispersion in the firms' prices that creates an inefficient variation in output levels across firms, and staggered wage setting leads to dispersion in the distribution of employment across households. The key equations of the model are presented in this section. However, here we include in the model the indexing variable, $\widetilde{P}_t$; further details are shown in a technical appendix.

2.1 Firms' price setting behaviour

We assume there is a continuum of monopolistically competitive firms indexed by f on the unit interval. They produce differentiated goods. Instead of assuming that household has the problem of choosing the optimal quantity of each differentiated good $Y_t(f)$ for $f \in [0, 1]$, we assume, as in Chari, Kehoe and McGrattan

(2000), the artifice of a competitive bundler: the bundler combines these firms' goods $Y_t(f)$ at the prices $P_t(f)$ into a single product. The bundler acquires the goods in the same proportions as households and the government would choose, and then sells this single product to households and the government, as either a consumption good or capital good. Therefore, the bundler's demand for each differentiated good f is equal to the total demand .

The combined output Y_t is assembled using a constant returns to scale technology of the Dixit and Stiglitz (1977) form:

$$Y_t = \left[\int_0^1 Y_t(f)^{\frac{(\phi_p - 1)}{\phi_p}} \partial f \right]^{\frac{\phi_p}{(\phi_p - 1)}}, \tag{1}$$

where $\phi_p > 1$ is the constant elasticity of substitution.

The output bundler chooses the bundle of goods that minimizes the cost of producing a given quantity of output index Y_t, taking the prices $P_t(f)$ of the goods $Y_t(f)$ as given. The bundler sells units of the output index at their unit cost P_t (aggregate price index):

$$P_t = \left[\int_0^1 P_t(f)^{(1-\phi_p)} \partial f \right]^{\frac{1}{(1-\phi_p)}} \tag{2}$$

The bundler's demand for each good $Y_t(f)$ — or equivalently total household demand for this good — is given by

$$Y_t^d = \left[\frac{P_t}{P_t(f)} \right]^{\phi_p} Y_t \tag{3}$$

Each differentiated good is produced by a single firm that hires capital $K_{t-1}(f)$ [2] and a labour $N_t(f)$ at the rental rate R_t and wage rate W_t respectively. Every firm faces the same Cobb-Douglas production function, with an identical level of total factor productivity Z_t:

$$Y_t(f) = Z_t K_{t-1}(f)^{\nu} N_t(f)^{\nu-1}, \tag{4}$$

where $0 < \nu < 1$. Here, we assume that Z_t follows a simple auto-regressive process: $\log Z_t = \rho \log Z_{t-1} + \varepsilon_{p,t}$. The firm chooses an optimal bundle of capital stock and labour services in order to minimise its cost:

$$\frac{R_t}{W_t} = \frac{\nu}{1-\nu} \frac{N_t(f)}{K_{t-1}(f)} \tag{5}$$

and the firm's marginal cost can be expressed as a function of total productivity, the rental rate and the wage index:

$$MC_t(f) = \frac{1}{\nu^{\nu}(1-\nu)^{(1-\nu)}} \frac{R_t^{\nu} W_t^{1-\nu}}{Z_t} \tag{6}$$

To introduce nominal price stickiness in to the model, we assume that firms set prices according to Calvo (1983) but subject to the ability to change all prices in line with an indexing formula, $\widetilde{P}_t$ The price-setters operate under imperfect competition where if prices were flexible they would be continuously set as a mark-up on marginal cost. However, prices are rigid. That is, forward-looking firms are allowed only periodically reoptimize their prices, so they incorporate higher future expected real marginal costs into their reset prices

[2] $K_{t-1}(f)$ is the firm's demand for capital in period t. The aggregate capital stock is predetermined at the beginning of the period t.

in order to maximise the stream of profits. They do this, because they may not be able to raise prices when the higher marginal costs come. Therefore, the setting is as following. In each period, a firm faces a constant probability, $1-\alpha$, of being able to reset its price level. This provides the average duration of a price contracts is $(1-\alpha)^{-1}$ periods. Whenever the firm is not allowed to reset its price level, it sticks to its old one. For simplicity, it assumes that the firm's ability to reoptimise its price is independent across firms and time. Therefore, a constant fraction $(1-\alpha)$ of firms are allowed to reset their contracts prices each period. However, all prices additionally rise with the general price index formula, $\widetilde{P}_t$.

If firm f gets to reset a new contract in period t, it chooses a new price $P_t^*(f)$ to maximize the value of its profit stream over states of nature in which the new price is expected to hold (note that the price prevailing at each period j will be $P_j^*(f)\widetilde{P}_j$):

$$E_t\Sigma_{j=t}^{\infty} (\alpha\beta)^{j-t} \lambda_j \left[P_t^*(f)\widetilde{P}_j Y_j(f) - TC_j(f)\right], \tag{7}$$

where $TC(f)$ is the firm's total cost, β is the household's discount factor , and λ_j is the households' marginal utility of nominal wealth. The first-order condition for a price-setting firm is:

$$P_t^* = \mu_p \frac{PB_t}{PA_t}, \tag{8}$$

where $\mu_p = \frac{\phi_p}{\phi_p - 1}$ is a monopoly mark-up factor and

$$PB_t = E_t\Sigma_{j=t}^{\infty} (\alpha\beta)^{j-t} \lambda_j \frac{MC_j(f)}{P_j} \left(\frac{P_j}{\widetilde{P}_j}\right)^{\phi_p+1} \widetilde{P}_j Y_j = \alpha\beta E_t PB_{t+1} + \lambda_j \frac{MC_j(f)}{P_j} \left(\frac{P_j}{\widetilde{P}_j}\right)^{\phi_p+1} \widetilde{P}_j Y_j \tag{9}$$

$$PA_t = E_t\Sigma_{j=t}^{\infty} (\alpha\beta)^{j-t} \lambda_j \left(\frac{P_j}{\widetilde{P}_j}\right)^{\phi_p} \widetilde{P}_j Y_j = \alpha\beta E_t PA_{t+1} + \lambda_j \left(\frac{P_j}{\widetilde{P}_j}\right)^{\phi_p} \widetilde{P}_j Y_j \tag{10}$$

In the special case of flexible prices (where no indexing is necessary), all firms set their prices every period ($\alpha \to 0$), then $P_t^*(f) \to \mu_p MC_j(f)$. Since $\mu_p > 1$, output will be inefficiently low because the monopolistic competition exists in the market. The expectations operator $E_t x_{t+i} = E(x_{t+i} \mid \Phi_{t-1})$ where Φ_t is information (macro and micro) for period t. Notice that the expressions for PB_t and PA_t involve solely real variables and relative prices, viz. $\lambda_j \widetilde{P}_j Y_j = (\frac{P_j Y_j}{\widetilde{P}_j C_j})$, $\frac{MC_j(f)}{P_j}$ and $(\frac{P_j}{\widetilde{P}_j})$.

2.2 Households' wage setting behaviour and capital accumulation

We assume that a continuum of monopolistically competitive households, indexed by h on the unit interval, who supply differentiated labour services to the production sector. Firms regard each household's labour services $L_t(h)$, $h \in [0,1]$, as an imperfect substitute for the labour services of other households: thus the labour market has a form of monopolistic competition. Again, we assume the artifice of a competitive bundler, who assembles all households' labour supplies $L_t(h)$ at the wages $W_t(h)$ in the same proportions as firms would choose. Thus, the bundler's demand for each household's labour is equal to the sum of firms' demands. The labour combination N_t has the Dixit-Stiglitz form:

$$N_t = \left[\int_0^1 L_t(h)^{\frac{(\phi_w-1)}{\phi_w}} \partial h\right]^{\frac{\phi_w}{\phi_w-1}}, \ \phi_w > 1 \tag{11}$$

The labour bundler is given each household's wage rate $W_t(h)$ and has to choose an optimal amount of labour service so that it minimizes its total cost, and then sells units of the combined labour to the production sector at their unit cost W_t (aggregate wage index):

$$W_t = \left[\int_0^1 W_t(h)^{1-\phi_w} \partial h \right]^{\frac{1}{1-\phi_w}} \tag{12}$$

The bundler's demand for the labour hours of household h (total demand for this household's labour by all goods firms) is given as:

$$L_t^d(h) = \left(\frac{W_t}{W_t(h)} \right)^{\phi_w} N_t \tag{13}$$

The utility of household is:

$$\max U_t(h) = E_t \Sigma_{\tau=t}^{\infty} \beta^{\tau-t} \left[\frac{1}{1-\theta} C_\tau(h)^{1-\theta} - \frac{1}{1+\chi} L_\tau(h)^{1+\chi} \right], \tag{14}$$

where $C_\tau(h)$ is the household's consumption of Y_t, and the second term on the right hind side of the equation reflects the disutility of work[3]. θ is the coefficient of relative risk aversion. Lucas (2003) focused on this parameter, arguing that the welfare cost of fluctuations in consumption are negligible unless θ is very high. However, here the model assumes a log utility function , where $\theta = 1$. $1/\chi$ is the Frish elasticity of labour supply. The household h maximizes (14) subject to its budget constraint, its labour demand curve (13), and its capital accumulation constraint.

Household h's budget constraint in period t states that consumption expenditures plus asset accumulation must equal disposable income:

$$E_t \left[\Delta_{t,t+1} B_{t+1}(h) \right] + P_t \left[C_t(h) \right] + I_t(h) + T_t] = B_t(h) + W_t(h) L_t^d(h) + R_t K_{t-1}(h) + D_t(h) \tag{15}$$

where the first term on the LHS is a portfolio of state-contingent claims (The number of (period $t+1$) dollar claims in the portfolio,contingent on a given state's occurring, times the stochastic discount factor, $\Delta_{t,t+1}$-the price of a dollar claim divided by the probability of the state); I_t, is the household's investment in capital, T_t is a lump sum tax (used in the government constrain to balance every period, so that total lumpsum transfers are equal to seinorage revenue), and the last three terms on the RHS are the household's wage, rental and dividend income. The household's capital accumulation is given by:

$$K_t(h) = (1-\delta) K_{t-1}(h) + I_t(h) - \frac{1}{2} \psi \left[\frac{I_t(h)}{K_{t-1}(h)} - \delta \right]^2 K_{t-1}(h) \tag{16}$$

where δ is the depreciation rate, and the last term of this equation is the cost of adjusting the capital stock.

Households set wages in staggered contracts, under assumptions symmetric to those stated earlier for price contracts. In any given period, each household gets a probability $(1-\omega)$ to reset their wage contract but again subject to the indexation formula, $\widetilde{P}_t$. Whenever the household is not allowed to reset its wage contract, the old contract wage remains in force apart from the indexing formula. The average duration of the wage contract is $(1-\omega)^{-1}$ periods. The probability of reoptimisation of wage contracts is independent across firms and time, so that every period there is a constant number $(1-\omega)$ of households, who are allowed

[3] The utility function and budget constraint should include a term in real money balances, but following much of NNS literature in assuming that this term is negligible. Since an interest rate rule is specified for monetary policy, there is no real need to model money explicitly (Canzoneri et al.).

to reset their wage contracts. If household h gets to announce a new contract in period t, it chooses the new wage $W_t^*(h)$ to maximise its stream of lifetime welfare :

$$W_t^{*(1+\phi_w\chi)} = \mu_w \frac{WB_t}{WA_t} \tag{17}$$

where $\mu_w = \frac{\phi_w}{\phi_w - 1}$ is a monopoly markup factor, and

$$WB_t = E_t\Sigma_{j=t}^{\infty} (\omega\beta)^{j-t} N_j^{1+\chi} \left(\frac{W_j}{\widetilde{P}_j}\right)^{\phi_w(1+\chi)} = \omega\beta E_t WB_{t+1} + N_t^{1+\chi} \left(\frac{W_t}{\widetilde{P}_t}\right)^{\phi_w(1+\chi)} \tag{18}$$

$$WA_t = E_t\Sigma_{j=t}^{\infty} (\omega\beta)^{j-t} \lambda_j \tilde{P}_j N_j \left(\frac{W_j}{\widetilde{P}_j}\right)^{\phi_w} = \omega\beta E_t WA_{t+1} + \lambda_t \widetilde{P}_t N_t \left(\frac{W_t}{\widetilde{P}_t}\right)^{\phi_w} \tag{19}$$

where λ is the household's marginal utility of nominal wealth. In the limiting case where $\omega \to 0$, all households are allowed to reset their wages each period, that is a flexible wage model (again with no indexing required) Then $W_t^*(h) = \mu_w \frac{N_t^{\chi}}{\lambda_t}$, that is, the wage is a markup over the marginal disutility of work. Since the markup is greater than 1, the labour supply will be inefficiently low in the flexible wage solution. When wages are sticky ($\omega > 0$), wage rates will differ across households, and firms will demand more labour from households charging lower wages. Canzoneri et al. set this model so that there are heterogeneous agents, but assume complete contingent claims markets, that means that households are identical in terms of their consumption and investment decisions[4]. So, in equilibrium, aggregate consumption will be equal each household's consumption and to per capita consumption: $C_t = \int_0^1 C_t(h)\partial h = C_t(h)\int_0^1 h = C_t(h)$. The same is true for aggregate capital stock.

So, we can write the equilibrium versions of the households' first order conditions for consumption and investment in terms of aggregate values (where $\theta = 1$, making the welfare function into a log-linear function):

$$(C_t) \qquad \lambda_t = \frac{1}{P_t C_t^{\theta}} \tag{20}$$

$$(B_{t+1}) \quad E_t \frac{\lambda_{t+1}}{\lambda_t} = E_t \Delta_{t,t+1} = \frac{1}{1+i} \tag{21}$$

$$(I_t) \qquad \lambda_t P_t = \xi_t - \xi_t \psi \left[\frac{I_t}{K_{t-1}} - \delta\right] \tag{22}$$

$$(K_t) \quad \xi_t = \beta E_t \left\{ \lambda_{t+1} R_{t+1} + \xi_{t+1} \left([1-\delta] - \frac{1}{2}\psi \left[\frac{I_{t+1}}{K_t} - \delta\right]^2 + \psi \left[\frac{I_{t+1}}{K_t} - \delta\right] \frac{I_{t+1}}{K_t} \right) \right\}, \tag{23}$$

where λ_t and ξ_t are the Lagrangian multipliers for the households' budget and capital accumulation constraints, and i_t is the return on a 'risk free' bond.

[4] The FOC for $B_{t+1}(h)$ is: $\Delta_{t,t+1} = \frac{\lambda_{t+1}(h)}{\lambda_t(h)}$, where $\lambda_t(h)$ is the marginal utility of wealth. All households face the same discount factor, $\Delta_{t,t+1}$; so if all households have the same initial wealth, $\lambda_t(h) = \lambda_t$, for all h. First order condition for $C_t(h), I_t(h)$ and $K_t(h)$ are identical for all h.

2.3 The aggregate price and wage levels, aggregate employment and aggregate output

The aggregate price level is:

$$P_t = \left[\int_{j=0}^{\infty} P_t(f)^{1-\phi_p}\partial f\right]^{\frac{1}{1-\phi_p}} = \left[\sum_{j=0}^{\infty}(1-\alpha)\alpha^j \left(P_{t-j}^*(f)\widetilde{P}_t\right)^{1-\phi_p}\right]^{\frac{1}{1-\phi_p}}, \tag{24}$$

since the law of large numbers implies that $(1-\alpha)\alpha^j$ is the fraction of firms that set their prices t-j periods ago, and have not got to reset them since.

Equation (24) can be rewritten as:

$$\left(\frac{P_t}{\widetilde{P}_t}\right)^{1-\phi_p} = (1-\alpha)P_t^{*(1-\phi_p)} + \alpha\left(\frac{P_{t-1}}{\widetilde{P}_{t-1}}\right)^{1-\phi_p} \tag{25}$$

We may convert this into an inflation equation as follows:

$$\left(\frac{P_t}{P_{t-1}}\right)^{1-\phi_p} = \left[(1-\alpha)\left(\frac{P_t^*\widetilde{P}_t}{P_{t-1}}\right)^{(1-\phi_p)} + \alpha\left(\frac{\widetilde{P}_t}{\widetilde{P}_{t-1}}\right)^{1-\phi_p}\right] \tag{26}$$

Similarly, the aggregate wage (12) can be written as

$$\left(\frac{W_t}{\widetilde{P}_t}\right)^{1-\phi_w} = (1-\omega)W_t^{*(1-\phi_w)} + \omega\left(\frac{W_{t-1}}{\widetilde{P}_{t-1}}\right)^{1-\phi_w} \tag{27}$$

The aggregate output is given by:

$$Y_t = \frac{Z_t K_{t-1}^{\nu} N_t^{1-\nu}}{DP_t}, \tag{28}$$

where $N_t = \int_0^1 N_t(f)\partial f$ is aggregate employment; $K_{t-1} = \int_0^1 K_{t-1}(h)\partial h = \int_0^1 K_{t-1}(f)\partial f$ is the aggregate capital stock; and $DP_t = \int_0^1 \left(\frac{P_t}{P_t(f)\widetilde{P}_t}\right)^{-\phi_p} df$ is a measure of price dispersion across firms and can be written as:

$$DP_t = (1-\alpha)\left(\frac{P_t}{P_t^*(f)\tilde{P}_t}\right)^{\phi_p} + \alpha\left(\frac{P_t}{P_{t-1}}\right)^{\phi_p}\left(\frac{\tilde{P}_t}{\tilde{P}_{t-1}}\right)^{-\phi_p} DP_{t-1} \tag{29}$$

The inefficiency due to price dispersion can be seen in equation (28). Each firm has the same marginal cost; so consumers should choose equal amounts of the firms' products to maximize the consumption good aggregator (4) for a given resource cost. If prices are flexible ($\alpha = 0$), then $P_t(f) = P_t$, for all f, and this efficiency condition will be met; if prices are sticky ($\alpha > 0$), then product prices will differ, and consumption decisions will be distorted. This distortion is illustrated by equation (28).

2.4 Monetary and fiscal policy

Monetary policy is given by empirical specifications (Canzoneri et al. used the data over the Volcker and Greenspan years 1979:3 -2003:2):

$$i_t = 0.222 + 0.82 i_{t-1} + 0.35552\pi_t + 0.032384 \left(output\ gap\right)_t + \varepsilon_{i,t}, \tag{30}$$

where $\pi_t = \log\left(\frac{P_t}{P_{t-1}}\right)$ and $\varepsilon_{i,t}$ is the interest rate shock. However, it is not optimal in the normative sense, because it is not the one that derived from maximizing the expected utility of the representative household. Rotemberg and Woodford (1998) showed that a simple interest-rate feedback rule, having the response to the inflation and output) does almost as well as a more complicated rule which is optimal in their maximising expected utility framework (this model has only price stickiness, however, we still can base our rule specification on this argument). Canzoneri et al. say that the interest rule and its estimation doesn't show clearly how the rule should be interpreted in this NNS model. The reason is that the model cannot provide estimates of potential output. Therefore, output gap is defined as the difference between actual output under Calvo setting and output that would prevail in the flexible wage/price solution (Neiss and Nelson, 2003).

Unlike the original paper, we omit the fiscal shock:

$$\log G_t = \varepsilon_{g,t}, \tag{31}$$

because we are not so interested in the responses of variables when the fiscal shock happens. Also, Canzoneri et al. concluded that government spending shocks do not have much effect in the model and maybe, government spending shocks have not been modelled correctly.

2.5 Welfare

Our measure of welfare is

$$U_t = E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left[\log C_t - \frac{1}{(1+\chi)} AL_\tau \right], \tag{32}$$

where C_t is per capita consumption, and $AL_t = \int_0^1 L_t(h)^{1+\chi} \partial h$ is the average disutility of work.

If wages are flexible ($\omega = 0$), then $W_t(h) = W_t$ for all h, and firms hire the same amount of work from each household. Therefore, $AL_t = \int_0^1 L_t(h)^{1+\chi}\partial h = L_t(h)^{1+\chi} \int_0^1 \partial h = L_t(h)^{1+\chi}$ for all h because all the households are identical. The measure of welfare $U_t = U_t(h)$ (equation 14).

If there is a nominal wage rigidity ($\omega > 0$), then the dispersion of wages will make firms hire different amount of work from each household. This creates an inefficiency similar to the inefficiency due to price dispersion. This distortion is included in the AL term (appendix):

$$AL_t = N_t^{1+\chi} DW_t \tag{33}$$

$$DW_t = (1-\omega)\left(\frac{W_t^*(h)\tilde{P}_t}{W_t}\right)^{-\phi_w(1+\chi)} + \omega \left(\frac{W_{t-1}}{W_t}\right)^{-\phi_w(1+\chi)} \left(\frac{\tilde{P}_t}{\tilde{P}_{t-1}}\right)^{-\phi_w(1+\chi)} DW_{t-1}, \tag{34}$$

where $DW_t = \int_0^1 \left(\frac{W_t(h)\tilde{P}_t}{W_t}\right)^{\phi_w(1+\chi)} \partial h$ is a measure of wage dispersion.

As is usual in these models we assume that the flexible/wage equilibrium provides the optimum welfare with subsidies (financed by lump sum transfers) offsetting the average monopolistic distortions in the labour and product markets. We now go on to show that the choice of indexing scheme is optimised by indexing to expected price inflation.

3 An analytical treatment of a model with exogenous capital and a competitive labour market

As this model is complex and nonlinear, we investigate a much simplified version in which capital is exogenised and made a non-tradeable endowment resource, while the labour market is assumed to be competitive. Also, all the equations are loglinearised, as follows.

(1) Each firm is a price-setter, which forms the expectation of its price for period t based on micro information of period t and macro information of period $(t-1)$. Each firm f minimises its total cost subject to its production function:

$$\begin{aligned} TC_t(f) &= W_t L_t(f) \\ s.t\ Y_t(f) &= Z_t \bar{K}^{\nu} L_t(f)^{1-\nu} \end{aligned}$$

where $\bar{K}$ is exogenous and $\log Z_t = \rho_1 \log Z_{t-1} + z_t$. The labour demand function is derived from the above production function:

$$L_t(f) = \left(\frac{\bar{Y}_t(f)}{Z_t \bar{K}^{\nu}} \right)^{\frac{1}{1-\nu}}$$

The firm's cost minimising problem implies

$$MC_t = \frac{\partial TC_t(f)}{\partial \bar{Y}_t(f)} = \frac{\partial TC_t(f)}{\partial L_t(f)} \frac{\partial L_t(f)}{\partial \bar{Y}_t(f)} = \frac{1}{1-\nu} W_t L_t(f)^{\nu} \frac{1}{Z_t \bar{K}^{\nu}}$$

The loglinearised real marginal cost is then

$$\log mc_t = \log MC_t - \log P_t = \log W_t - \log P_t + \nu \log L_t(f) - \log Z_t \qquad (35)$$

As regards to households, each of them maximises the life-time expected welfare subject to its budget constraint and labour demand, but without the capital accumulation constraint in this simple set up. However, beside the assumption of fully complete contingent claims that make the households homogeneous in their consumption decisions, the competitive labour market means they are homogeneous in labour supply also. The welfare maximisation implies every household supplies N_t^{χ} units of labour:

$$N_t^{\chi} = \frac{W_t}{P_t C_t}$$

and its log-linear form is given as

$$\chi \log N_t = \log W_t - \log P_t - \log C_t \qquad (36)$$

The competitive labour market also means that in equilibrium the supply of and demand for labour must be equal so that the equation (35) becomes

$$\log mc_t = \log MC_t - \log P_t = \log W_t - \log P_t + \nu \log N_t - \log Z_t$$

(2) The production function is given as:

$$\log Y_t = \log Z_t + (1-\nu) \log N_t \qquad (37)$$

(3) Ignoring government spending, the market clearing condition gives

$$\log Y_t = \log C_t \qquad (38)$$

(4) To represent the role of money, we use a simple cash-in-advance set-up (in which the inflation tax for convenience is refunded to consumers so that its effect on leisure choice is thereby suppressed) so that

$$\log M_t = \log Y_t + \log P_t \tag{39}$$

where $\log M_t = \rho_2 \log M_{t-1} + \mu_{t.}$

(5) the new reset price level, given by equations $(8), (9)$ and (10) is:

$$P_j^*(f) = \mu_p \frac{E_t \sum_{j=t}^{\infty} (\alpha\beta)^{j-t} \lambda_j \frac{MC_j}{P_j} Y_j \left(\frac{P_t}{\tilde{P}_j}\right)^{\phi_p+1} \tilde{P}_j}{E_t \sum_{j=t}^{\infty} (\alpha\beta)^{j-t} \lambda_j \tilde{P}_j Y_j \left(\frac{P_t}{\tilde{P}_j}\right)^{\phi_p}}$$

To loglinearise the renewed price, we use $\log X_t = \log E_t \Sigma_{i=0}^{\infty} \alpha^i Z_{t+i} = \Sigma_{i=0}^{\infty} (1-\alpha)\alpha^i \log Z_{t+i}$. Therefore, the log of price level around its equilibrium is:

$$\begin{aligned}
\log P_t^* &= \log \mu_p + \log \left[\frac{E_t \sum_{j=t}^{\infty} (\alpha\beta)^{j-t} \lambda_j \frac{MC_j}{P_j} Y_j \left(\frac{P_t}{\tilde{P}_j}\right)^{\phi_p+1} \tilde{P}_j}{E_t \sum_{j=t}^{\infty} (\alpha\beta)^{j-t} \lambda_j \tilde{P}_j Y_j \left(\frac{P_t}{\tilde{P}_j}\right)^{\phi_p}} \right] \\
&= \log \mu_p + \log \left(E_t \sum_{j=t}^{\infty} (\alpha\beta)^{j-t} \lambda_j \frac{MC_j}{P_j} Y_j \left(\frac{P_t}{\tilde{P}_j}\right)^{\phi_p+1} \tilde{P}_j \right) + \log \left(E_t \sum_{j=t}^{\infty} (\alpha\beta)^{j-t} \lambda_j \tilde{P}_j Y_j \left(\frac{P_t}{\tilde{P}_j}\right)^{\phi_p} \right) \\
&= \log \mu_p + \log \left(E_t \sum_{j=t}^{\infty} (\alpha\beta)^{j-t} \Lambda_j mc_j Y_j \left(\frac{P_t}{\tilde{P}_j}\right)^{\phi_p} \right) + \log \left(E_t \sum_{j=t}^{\infty} (\alpha\beta)^{j-t} \Lambda_j Y_j \left(\frac{P_t}{\tilde{P}_j}\right)^{\phi_p-1} \right) \\
&= \log \mu_p + E_t \left[\begin{array}{l} \Sigma_{j=t}^{\infty} (1-\alpha\beta)(\alpha\beta)^j \log \left(\Lambda_j mc_j \left(\frac{P_j}{\tilde{P}_j}\right)^{\phi} Y_j \right) \\ -\Sigma_{j=t}^{\infty} (1-\alpha\beta)(\alpha\beta)^j \log \left(\Lambda_j Y_j \left(\frac{P_t}{\tilde{P}_j}\right)^{\phi_p-1} \right) \end{array} \right] \\
&= \log \mu_p + E_t \left[\Sigma_{j=t}^{\infty} (1-\alpha\beta)(\alpha\beta)^j \left(\log \Lambda_j + \log mc_j + \phi_p \left(\log P_j - \log \tilde{P}_j \right) + \log Y_j \right) \right] \\
&\quad - E_t \left[\Sigma_{j=t}^{\infty} (1-\alpha\beta)(\alpha\beta)^j \left(\log \Lambda_j + \log Y_j + \left(\phi_p - 1\right) (\log P_j - \log \tilde{P}_j) \right) \right] \\
&= \log \mu_p + E_t \Sigma_{j=t}^{\infty} (1-\alpha\beta)(\alpha\beta)^j \left(\log mc_j + \log P_j - \log \tilde{P}_j \right)
\end{aligned}$$

$$\log P_t^* = \frac{(1-\alpha\beta) E_t \left(\log mc_t + \log P_t - \log \tilde{P}_t \right)}{1 - \alpha\beta B^{-1}} \tag{40}$$

This can be rewritten using equations $(35), (36), (37)$ and (38) (Appendix 3.1) :

$$\log P_t^* = \frac{(1-\alpha\beta)}{1-\alpha\beta B^{-1}} E_t \left(\frac{1+\chi}{1-\nu} (\log Y_t - \log Z_t) + \log P_t - \log \tilde{P}_t \right) \tag{41}$$

(6) The general price level, given from equation (26) is:

$$P_t^{1-\phi_p} = (1-\alpha) \left(P_t^* \tilde{P}_t \right)^{1-\phi_p} + \alpha \left(\frac{P_{t-1}}{\tilde{P}_{t-1}} \right)^{1-\phi_p} \tilde{P}_t^{1-\phi_p}$$

The first order differential of this is:

$$\begin{aligned}\left(1-\phi_p\right) P_0^{-\phi_p} \partial P_t &= (1-\alpha)\left(1-\phi_p\right) P_t^{*\left(-\phi_p\right)} \tilde{P}_t^{1-\phi_p} \partial P_t^* + (1-\alpha)\left(1-\phi_p\right) P_t^{*\left(1-\phi_p\right)} \tilde{P}_t^{-\phi_p} \partial \tilde{P}_t + \\ &+\alpha\left(1-\phi_p\right)\left(\frac{P_{t-1}}{\tilde{P}_{t-1}}\right)^{1-\phi_p} \tilde{P}_t^{-\phi_p} \partial \tilde{P}_t + \\ &\alpha\left(1-\phi_p\right) \tilde{P}_t^{1-\phi_p}\left[\frac{1}{\tilde{P}_{t-1}^{1-\phi_p}} P_{t-1}^{-\phi_p} \partial P_{t-1} + P_{t-1}^{1-\phi_p} \tilde{P}_{t-1}^{\phi_p-2} \partial \tilde{P}_{t-1}\right]\end{aligned}$$

then we divide the equation by $(1-\phi_p)$ and also, assume that prices at equilibrium equal to 1 and $\frac{\partial \log X_t}{\partial X_t} = \frac{1}{X_t}$:

$$\partial \log P_t = (1-\alpha)\, \partial \log P_t^* + (1-\alpha) \partial \log \tilde{P}_t + \alpha \partial \log \tilde{P}_t + \alpha \left[\partial \log P_{t-1} - \partial \log \tilde{P}_{t-1}\right]$$

We take the integral to find the price:

$$\ln P_t - \ln \tilde{P}_t = \frac{(1-\alpha) \ln P_t^*}{1-\alpha L} \tag{42}$$

It is equivalent to

$$\ln P_t - \ln \tilde{P}_t = \alpha \left(\ln P_{t-1} - \ln \tilde{P}_{t-1}\right) + (1-\alpha) \ln P_t^* \tag{43}$$

(7) To deal with the price dispersion variable DP_t, we will use a conventional second order Taylor expansion of $\log DP$, where DP_t is

$$DP_t = \left[(1-\alpha)\, \Sigma_{i=0}^{\infty} \alpha^i P_{t-i}^{*\gamma}\right]^{\frac{\phi_p}{\gamma}} \left[(1-\alpha)\, \Sigma_{i=0}^{\infty} \alpha^i P_{t-i}^{*\left(-\phi_p\right)}\right]$$

letting $\gamma = 1-\phi_p$.We assume that $DP_t = x_t * y_t$, therefore $\ln DP_t = \ln x_t + \ln y_t$ and $\partial \ln DP_t = \partial \ln x_t + \partial \ln y_t$, where $x_t = \left[(1-\alpha)\, \Sigma_{i=0}^{\infty} \alpha^i P_{t-i}^{*\gamma}\right]^{\frac{\phi_p}{\gamma}}$ and $y_t = \left[(1-\alpha)\, \Sigma_{i=0}^{\infty} \alpha^i P_{t-i}^{*\left(-\phi_p\right)}\right]$.

Using $\partial \log x = \frac{\partial x}{x}$ gives

$$
\begin{aligned}
\partial \ln x_t &= \partial \ln \left[(1-\alpha)\,\Sigma_{i=0}^{\infty}\alpha^i P_{t-i}^{*\gamma}\right]^{\frac{\phi_p}{\gamma}} \\
&= \Sigma_{i=0}^{\infty}\frac{\frac{\phi_p}{\gamma}\left[(1-\alpha)\,\Sigma_{i=0}^{\infty}\alpha^i P_{t-i}^{*\gamma}\right]^{\frac{\phi_p}{\gamma}-1}\gamma\,(1-\alpha)\,\alpha^i P_{t-i}^{*(\gamma-1)}}{\left[(1-\alpha)\,\Sigma_{i=0}^{\infty}\alpha^i P_{t-i}^{*\gamma}\right]^{\frac{\phi_p}{\gamma}}}\partial P_{t-i}^{*} \\
&\quad +\frac{1}{2}\Sigma_{i=0}^{\infty}\left[\frac{\phi_p\alpha^i\,(\gamma-1)\,P_{t-i}^{*(\gamma-2)}\left[\Sigma_{i=0}^{\infty}\alpha^i P_{t-i}^{*\gamma}\right]-\phi_p\alpha^i P_{t-i}^{*(\gamma-1)}\alpha^i\gamma P_{t-i}^{*(\gamma-1)}}{\left[\Sigma_{i=0}^{\infty}\alpha^i P_{t-i}^{*\gamma}\right]^2}\right]\left(\partial P_{t-i}^{*}\right)^2+ \\
&\quad +\frac{1}{2}2\Sigma_{j=0}^{\infty}\Sigma_{i=0}^{\infty}\left[-\frac{\phi_p\alpha^i P_{t-i}^{*(\gamma-1)}\gamma\alpha^j P_{t-j}^{*(\gamma-1)}}{\left[\Sigma_{i=0}^{\infty}\alpha^i P_{t-i}^{*\gamma}\right]^2}\right]\partial P_{t-i}^{*}\partial P_{t-j}^{*} \\
&= \Sigma_{i=0}^{\infty}\frac{\phi_p\alpha^i P_{t-i}^{*(\gamma-1)}}{\Sigma_{i=0}^{\infty}\alpha^i P_{t-i}^{*\gamma}}\partial P_{t-i}^{*}+\frac{1}{2}\Sigma_{i=0}^{\infty}\left[\frac{\phi_p\alpha^i P_{t-i}^{*(\gamma-2)}}{\Sigma_{i=0}^{\infty}\alpha^i P_{t-i}^{*\gamma}}\left(\gamma-1-\frac{\gamma\alpha^i P_{t-i}^{*\gamma}}{\Sigma_{i=0}^{\infty}\alpha^i P_{t-i}^{*\gamma}}\right)\right]\left(\partial P_{t-i}^{*}\right)^2 \\
&\quad -\Sigma_{j=0}^{\infty}\Sigma_{i=0}^{\infty}\left[\frac{\phi_p\alpha^i P_{t-i}^{*(\gamma-1)}\gamma\alpha^j P_{t-j}^{*(\gamma-1)}}{\left[\Sigma_{i=0}^{\infty}\alpha^i P_{t-i}^{*\gamma}\right]^2}\right]\partial P_{t-i}^{*}\partial P_{t-j}^{*} \\
&= \Sigma_{i=0}^{\infty}\frac{\phi_p\alpha^i P_{t-i}^{*\gamma}}{\Sigma_{i=0}^{\infty}\alpha^i P_{t-i}^{*\gamma}}\partial \ln P_{t-i}^{*}+\frac{1}{2}\Sigma_{i=0}^{\infty}\left[\frac{\phi_p\alpha^i P_{t-i}^{*\gamma}}{\Sigma_{i=0}^{\infty}\alpha^i P_{t-i}^{*\gamma}}\left(\gamma-1-\frac{\gamma\alpha^i P_{t-i}^{*\gamma}}{\Sigma_{i=0}^{\infty}\alpha^i P_{t-i}^{*\gamma}}\right)\right]\left(\partial \ln P_{t-i}^{*}\right)^2 \\
&\quad -\Sigma_{j=0}^{\infty}\Sigma_{i=0}^{\infty}\left[\frac{\phi_p\alpha^i P_{t-i}^{*\gamma}\gamma\alpha^j P_{t-j}^{*\gamma}}{\left[\Sigma_{i=0}^{\infty}\alpha^i P_{t-i}^{*\gamma}\right]^2}\right]\partial \ln P_{t-i}^{*}\partial \ln P_{t-j}^{*} \qquad (44)
\end{aligned}
$$

and

$$
\begin{aligned}
\partial \ln y_t &= \partial \ln \left[(1-\alpha) \Sigma_{i=0}^{\infty} \alpha^i P_{t-i}^{*\left(-\phi_p\right)} \right] \\
&= \Sigma_{i=0}^{\infty} \frac{1}{\left[(1-\alpha) \Sigma_{i=0}^{\infty} \alpha^i P_{t-i}^{*\left(-\phi_p\right)} \right]} (1-\alpha) \alpha^i \left(-\phi_p\right) P_{t-i}^{*\left(-\phi_p-1\right)} \partial P_{t-i}^{*} \\
&\quad + \frac{1}{2} \Sigma_{i=0}^{\infty} \left[\frac{\left(\begin{array}{c} -\phi_p \alpha^i \left(-\phi_p - 1\right) P_{t-i}^{*\left(-\phi_p-2\right)} \left(\Sigma_{i=0}^{\infty} \alpha^i P_{t-i}^{*\left(-\phi_p\right)} \right) - \\ \left(-\phi_p \alpha^i P_{t-i}^{*\left(-\phi_p-1\right)} \alpha^i \left(-\phi_p\right) P_{t-i}^{*\left(-\phi_p-1\right)} \right) \end{array} \right)}{\left[\Sigma_{i=0}^{\infty} \alpha^i P_{t-i}^{*\left(-\phi_p\right)} \right]^2} \right] \left(\partial P_{t-i}^{*}\right)^2 \\
&\quad + \frac{1}{2} 2 \Sigma_{j=0}^{\infty} \Sigma_{i=0}^{\infty} \left[- \frac{-\alpha^i \phi_p P_{t-i}^{*\left(-\phi_p-1\right)} \alpha^j \left(-\phi_p\right) P_{t-j}^{*\left(-\phi_p-1\right)}}{\left[\Sigma_{i=0}^{\infty} \alpha^i P_{t-i}^{*\left(-\phi_p\right)} \right]^2} \right] \partial P_{t-i}^{*} \partial P_{t-j}^{*} \\
&= \Sigma_{i=0}^{\infty} \frac{-\alpha^i \phi_p P_{t-i}^{*\left(-\phi_p-1\right)}}{\Sigma_{i=0}^{\infty} \alpha^i P_{t-i}^{*\left(-\phi_p\right)}} \partial P_{t-i}^{*} + \frac{1}{2} \Sigma_{i=0}^{\infty} \left[\frac{\phi_p \alpha^i P_{t-i}^{*\left(-\phi_p-2\right)}}{\Sigma_{i=0}^{\infty} \alpha^i P_{t-i}^{*\left(-\phi_p\right)}} \left(\begin{array}{c} \phi_p + 1 - \\ \frac{\alpha^i \phi_p P_{t-i}^{*-\phi_p}}{\Sigma_{i=0}^{\infty} \alpha^i P_{t-i}^{*\left(-\phi_p\right)}} \end{array} \right) \right] \left(\partial P_{t-i}^{*}\right)^2 \\
&\quad - \Sigma_{j=0}^{\infty} \Sigma_{i=0}^{\infty} \left[\frac{\alpha^i \phi_p P_{t-i}^{*\left(-\phi_p-1\right)} \alpha^j \phi_p P_{t-j}^{*\left(-\phi_p-1\right)}}{\left[\Sigma_{i=0}^{\infty} \alpha^i P_{t-i}^{*\left(-\phi_p\right)} \right]^2} \right] \partial P_{t-i}^{*} \partial P_{t-j}^{*} \\
&= -\Sigma_{i=0}^{\infty} \frac{\alpha^i \phi_p P_{t-i}^{*-\phi_p}}{\Sigma_{i=0}^{\infty} \alpha^i P_{t-i}^{*\left(-\phi_p\right)}} \partial \ln P_{t-i}^{*} + \frac{1}{2} \Sigma_{i=0}^{\infty} \left[\frac{\phi_p \alpha^i P_{t-i}^{*-\phi_p}}{\Sigma_{i=0}^{\infty} \alpha^i P_{t-i}^{*\left(-\phi_p\right)}} \left(\begin{array}{c} \phi_p + 1 - \\ \frac{\alpha^i \phi_p P_{t-i}^{*-\phi_p}}{\Sigma_{i=0}^{\infty} \alpha^i P_{t-i}^{*\left(-\phi_p\right)}} \end{array} \right) \right] \left(\partial \ln P_{t-i}^{*}\right)^2 \\
&\quad - \Sigma_{j=0}^{\infty} \Sigma_{i=0}^{\infty} \left[\frac{\alpha^i \phi_p P_{t-i}^{*-\phi_p} \alpha^j \phi_p P_{t-j}^{*-\phi_p}}{\left[\Sigma_{i=0}^{\infty} \alpha^i P_{t-i}^{*\left(-\phi_p\right)} \right]^2} \right] \partial \ln P_{t-i}^{*} \partial \ln P_{t-j}^{*} \qquad (45)
\end{aligned}
$$

Evaluating this at $P_{t-i}^{*} = 1$ gives

$$
\begin{aligned}
\partial \ln x_t &= \Sigma_{i=0}^{\infty} \phi_p \alpha^i (1-\alpha) \partial \ln P_{t-i}^{*} + \frac{1}{2} \Sigma_{i=0}^{\infty} \phi_p \alpha^i (1-\alpha) \left[(1-\phi_p) \left(1 - \alpha^i (1-\alpha)\right) - 1 \right] \left(\partial \ln P_{t-i}^{*}\right)^2 \\
&\quad - \Sigma_{j=0}^{\infty} \Sigma_{i=0}^{\infty} \phi_p (1-\phi_p) \alpha^{i+j} (1-\alpha)^2 \partial \ln P_{t-i}^{*} \partial \ln P_{t-j}^{*} \qquad (46)
\end{aligned}
$$

and

$$
\begin{aligned}
\partial \ln y_t &= -\Sigma_{i=0}^{\infty} \phi_p \alpha^i (1-\alpha) \partial \ln P_{t-i}^{*} + \frac{1}{2} \Sigma_{i=0}^{\infty} \phi_p \alpha^i (1-\alpha) \left[\phi_p + 1 - \phi_p \alpha^i (1-\alpha) \right] \left(\partial \ln P_{t-i}^{*}\right)^2 \\
&\quad - \Sigma_{j=0}^{\infty} \Sigma_{i=0}^{\infty} \phi_p^2 \alpha^{i+j} (1-\alpha)^2 \partial \ln P_{t-i}^{*} \partial \ln P_{t-j}^{*} \qquad (47)
\end{aligned}
$$

therefore,

$$\begin{aligned}
\ln DP_t &= \ln x_t + \ln y_t = \\
&= \frac{1}{2}\Sigma_{i=0}^{\infty}\phi_p\alpha^i(1-\alpha)\left[(1-\phi_p)\left(1-\alpha^i(1-\alpha)\right)+\phi_p-\phi_p\alpha^i(1-\alpha)\right]\left(\ln P_{t-i}^*\right)^2 \\
&\quad -\Sigma_{j=0}^{\infty}\Sigma_{i=0}^{\infty}\phi_p\alpha^{i+j}(1-\alpha)^2\left[1-\phi_p+\phi_p\right]\ln P_{t-i}^*\ln P_{t-j}^* \\
&= \frac{1}{2}\Sigma_{i=0}^{\infty}\phi_p\alpha^i(1-\alpha)\left[1-\alpha^i(1-\alpha)\right]\left(\ln P_{t-i}^*\right)^2-\Sigma_{j=0}^{\infty}\Sigma_{i=0}^{\infty}\phi_p\alpha^{i+j}(1-\alpha)^2\ln P_{t-i}^*\ln P_{t-j}^* \quad (48)
\end{aligned}$$

3.1 Solving the model under rational indexation

Our notation is as follows: $E^{t-1}x_t = E(x_t|\Phi_{t-1}); E_t x_t = E(x_t|\Phi_{t-1},\phi_t); x_t^{UE} = x_t - E^{t-1}x_t$, where Φ_{t-1} is the full information set from period t-1 and ϕ_t is the limited information set available (to the agent forming expectations) for period t.

We assume under rational indexation that

$$\log\tilde{P}_t = E\left(\log P_t|\Phi_{t-1}\right) \equiv E^{t-1}\log P_t$$

Applying this assumption to equation (43) , we get:

$$E^{t-1}\log P_t^* = -\frac{\alpha}{1-\alpha}\left(\log P_{t-1} - E^{t-2}\log P_{t-1}\right) \quad (49)$$

and

$$E^{t-1}\log P_{t+i}^* = E^{t-1}E^t\log P_{t+i}^* = 0;\ \forall i \geqslant 1$$

Using the rational expectation assumption and (49) , the new reset price is

$$\log P_t^* = E^{t-1}\log P_t^* + \log P_t^{*UE} = \log P_t^{*UE} - \frac{\alpha}{1-\alpha}\log P_{t-1}^{UE}, \quad (50)$$

where from(43) (Appendix 3.2)

$$logP_t^{UE} = (1-\alpha)\, logP_t^{*UE} \quad (51)$$

and from (41) and the assumption that firms have knowledge of their own micro information (productivity, prices and costs) in period t as well as the macro information of period $(t-1)$:

$$\begin{aligned}
\log P_t^{*UE} &= \log P_t^* - E^{t-1}\log P_t^* \\
&= (1-\alpha\beta)\,\Sigma_{i=0}(\alpha\beta)^i\left(\frac{1+\chi}{1-\nu}\right)\left[E^{t-1}\log Y_{t+i} - E^t\log Z_{t+i}\right] \\
&\quad -(1-\alpha\beta)\,\Sigma_{i=0}(\alpha\beta)^i\left(\frac{1+\chi}{1-\nu}\right)\left[E^{t-1}\log Y_{t+i} - E^{t-1}\log Z_{t+i}\right] \\
&= (1-\alpha\beta)\left(\frac{1+\chi}{1-\nu}\right)\frac{1}{1-\alpha\beta\rho_1}(-z_t) \quad (52)
\end{aligned}$$

Hence, from (50) , (51)and (52) the renewed price is rewritten as

$$\log P_t^* = \log P_t^{*UE} - \alpha\log P_{t-1}^{*UE} = \chi'\frac{(1-\alpha L)}{(1-\alpha)}(-z_t)\,, \quad (53)$$

where $\chi' = (1-\alpha)(1-\alpha\beta)\left(\frac{1+\chi}{1-\nu}\right)\frac{1}{1-\alpha\beta\rho_1}$.

This in turn implies two things:

$$\begin{aligned}(1)\quad E^{t-1}\log P^*_{t+j} &= (1-\alpha\beta)\,\Sigma_{i=0}\,(\alpha\beta)^i\left(\frac{1+\chi}{1-\nu}\right)\left[E^{t-1}\log Y_{t+i} - E^{t-1}\log Z_{t+i}\right]_{+j} = 0 \quad \forall j \geqslant 1\\ &\Leftrightarrow \quad E^{t-1}\log Y_{t+i} = E^{t-1}\log Z_{t+i},\ \forall i \geqslant 1\end{aligned}$$

$$(2)\qquad E^{t-1}\log P^*_t = (1-\alpha\beta)\left(\frac{1+\chi}{1-\nu}\right)\left[E^{t-1}\log Y_t - E^{t-1}\log Z_t\right] = -\frac{\alpha}{1-\alpha}\log P^{UE}_{t-1} \tag{54}$$

Now we look at the output side of the model, where under rational expectations output consists of the expected output and the surprise change in output. First, using equation (39) and (54), we get these two components of the output equation:

$$\log P^{UE}_t = \log M^{UE}_t - \log Y^{UE}_t \tag{55}$$

and

$$E^{t-1}\log Y_t = E^{t-1}\log Z_t - \left(\frac{\alpha}{1-\alpha}\right)\frac{1}{(1-\alpha\beta)}\left(\frac{1-\nu}{1+\chi}\right)\log P^{UE}_{t-1}.$$

The former one states that the surprise change in output is due to the surprise change in the money supply and the surprise change in the price level. While the expected output is a function of expected productivity and some lagged surprise change in price of last period.

The latter equation is in turn written as follows, using (55)

$$E^{t-1}\log Y_t = E^{t-1}\log Z_t - v'\left(\log M^{UE}_{t-1} - \log Y^{UE}_{t-1}\right), \tag{56}$$

where $v' = \left(\frac{\alpha}{1-\alpha}\right)\frac{1}{(1-\alpha\beta)}\left(\frac{1-\nu}{1+\chi}\right)$.

On the other hand, after an algebraic manipulation using (51) and (52), the unexpected component of output is

$$\log Y^{UE}_t = \log M^{UE}_t - \log P^{UE}_t = \mu_t + \chi' z_t, \tag{57}$$

where $\chi' = (1-\alpha)(1-\alpha\beta)\left(\frac{1+\chi}{1-\nu}\right)\frac{1}{1-\alpha\beta\rho_1}$; $\mu_t = \log M_t - \rho_2 \log M_{t-1}$ and $z_t = \log Z_t - \rho_1 \log Z_{t-1}$.

Therefore, combining (56) and (57), output is (appendix 3.3)

$$\begin{aligned}\log Y_t &= E^{t-1}\log Y_t + \log Y^{UE}_t = \\ &= \rho_1 \log Z_{t-1} + v'\chi' z_{t-1} + \mu_t + \chi' z_t\end{aligned} \tag{58}$$

Next, we consider the difference between output and productivity:

$$\log Y_t - \log Z_t = E^{t-1}\left(\log Y_t - \log Z_t\right) + \left(\log Y^{UE}_t - z_t\right) \tag{59}$$

where we know from equation (54) that

$$E^{t-1}\log P^*_t = (1-\alpha\beta)\left(\frac{1+\chi}{1-\nu}\right)\left[E^{t-1}\log Y_t - E^{t-1}\log Z_t\right], \tag{60}$$

and thus (appendix 3.4)

$$E^{t-1}\left(\log Y_t - \log Z_t\right) = \frac{1}{1-\alpha\beta}\left(\frac{1-\nu}{1+\chi}\right)E^{t-1}\log P^*_t = \frac{\alpha}{1-\alpha\beta\rho_1}z_{t-1}. \tag{61}$$

Therefore equation (58) is, based on (57) and (59)

$$\log Y_t - \log Z_t = \frac{\alpha}{1-\alpha\beta\rho_1}z_{t-1} + \mu_t + \chi' z_t - z_t. \tag{62}$$

3.1.1 Welfare

Under the flexible price and wage assumption, the welfare level would be

$$u_t^{FLEX} = \log Z_t \tag{63}$$

However, in the economy of price rigidity and competitive labour market, the welfare function is

$$u_t = \log C_t - \bar{N}^{\chi+1} \log N_t \tag{64}$$

where $\log C_t = \log Y_t = \log Z_t + (1-\nu)\log N_t - \log DP_t$ and $\log \mathrm{N}_t = \frac{1}{1-\nu}(\log Y_t - \log Z_t)$.

We evaluate expected welfare in terms of its deviation from the flex-price optimum:

$$\begin{aligned} E\left(u_t - u_t^{FLEX}\right) &= E\left(\log C_t - \bar{N}^{\chi+1}\log N_t - \log Z_t\right) \\ &= E\left(\log Z_t + (1-\nu)\log N_t - \log DP_t - \bar{N}^{\chi+1}\log N_t - \log Z_t\right) \\ &= E\left[\left(1-\nu-\bar{N}^{\chi+1}\right)\frac{\log Y_t - \log Z_t}{1-\nu} - \log DP_t\right] \\ &= E\left[\frac{1-\nu-\bar{N}^{\chi+1}}{1-\nu}\left(\frac{\alpha}{1-\alpha\beta\rho_1} z_{t-1} + \mu_t + \chi' z_t - z_t\right)\right] - E\log DP_t \end{aligned} \tag{65}$$

Notice that the unconditional mean of the first element in this expected welfare term is 0. So effectively we only consider the second term, where it is known respectively from equations (48) and (53) that

$$\begin{aligned} E\left(u_t - u_t^{FLEX}\right) &= -E\ln DP_t \\ &= -\frac{1}{2}\Sigma_{i=0}^{\infty}\phi_p\alpha^i(1-\alpha)\left[1-\alpha^i(1-\alpha)\right] var\left(\log P_{t-i}^*\right) \\ &\quad + \Sigma_{j=0}^{\infty}\Sigma_{i=0}^{\infty}\phi_p\alpha^{i+j}(1-\alpha)^2 Cov\left(\log P_{t-i}^*, \log P_{t-j}^*\right) \end{aligned}$$

and

$$\log P_t^* = -\chi'' z_t + \alpha\chi'' z_{t-1},$$

where $\chi'' = \frac{\chi'}{1-\alpha}$. With variances and covariances of the renewed price derived in (appendix 3.5), the expected welfare is now

$$E\left(u_t - u_t^{FLEX}\right) = -\phi_p\alpha\left[\chi''^2\frac{\left(1+\alpha^2\right)}{1+\alpha} + (1-\alpha)^2\frac{1}{1-\alpha^2}\right] var(z) \tag{66}$$

3.2 Solving the model with lagged indexation

Assume that

$$\log\tilde{P}_t = E^{t-1}\log P_t + \left(k\log P_{t-1} - E^{t-1}\log P_t\right) \tag{67}$$

where if $k=1$ then there is lagged indexation $\log\tilde{P}_t = \log P_{t-1}$, and if $k=0$ then there is a no indexation and $\log\tilde{P}_t = 0$.

Using this assumption and equation (42), the general price is

$$\log P_t = E^{t-1}\log P_t + v_{t-1} + \frac{(1-\alpha)\log P_t^*}{1-\alpha L},$$

where $v_{t-1} = k \log P_{t-1} - E^{t-1} \log P_t$. This equation can also be written as

$$\left(\log P_t - E^{t-1} \log P_t\right) - v_{t-1} + \alpha v_{t-2} - \alpha \log P_{t-1}^{UE} = (1-\alpha) \log P_t^*$$

and taking the expectation E^{t-1} throughout and manipulating this equation, we get expected renewed price

$$E^{t-1} \log P_t^* = \frac{-v_{t-1} + \alpha v_{t-2} - \alpha \log P_{t-1}^{UE}}{1-\alpha} \tag{68}$$

Notice from equation (41) that

$$\log P_t^* = \frac{(1-\alpha\beta)}{1-\alpha\beta B^{-1}} E_t \left(\frac{1+\chi}{1-\nu} (\log Y_t - \log Z_t) + \log P_t - \log \tilde{P}_t \right) \tag{69}$$

therefore,

$$E^{t-1} \log P_t^* = \frac{(1-\alpha\beta)}{1-\alpha\beta B^{-1}} E^{t-1} E_t \left(\frac{1+\chi}{1-\nu} (\log Y_t - \log Z_t) + \log P_t^{UE} - v_{t-1} \right)$$

and from equation (52) that

$$\log P_t^{*UE} = (1-\alpha\beta) \left(\frac{1+\chi}{1-\nu} \right) \frac{1}{1-\alpha\beta\rho_1} (-z_t) \tag{70}$$

3.2.1 Solve for $\log P_t$ and $\log P_t^*$

Using the cash- in- advance condition (39) and (42) , (69) is rewritten into

$$(1-kL) \log P_t = \frac{1-\alpha}{1-\alpha L} \frac{1-\alpha\beta}{1-\alpha\beta B^{-1}} E_t \left[\chi^* (\log M_t - \log P_t - \log Z_t) + \log P_t - k \log P_{t-1} \right]^5$$

The solution for the general price level has the Wold decomposition form $\log P_t = \Sigma_{i=0} \pi_i \mu_{t-i} + \Sigma_{i=0} \xi_i z_{t-i}$; use undetermined coefficients to solve for $\log P_t$ (appendix 3.6-3.7) . Also, the solution for ν_t is :

$$\begin{aligned} \nu_t &= k \log P_t - E^t \log P_{t+1} \\ &= k \left(\Sigma_{i=1} \pi_i \mu_{t-i} + \Sigma_{i=1} \xi_i z_{t-i} \right) - \left(\Sigma_{i=0} \pi_{i+1} \mu_{t-i} + \Sigma_{i=0} \xi_{i+1} z_{t-i} \right) \\ &= \Sigma_{i=0} \left[(k\pi_i - \pi_{i+1}) \mu_{t-i} + \left(k\xi_i - \xi_{i+1} \right) z_{t-i} \right] \end{aligned} \tag{71}$$

Now we consider the difference between output and productivity once again.

$$\begin{aligned} \log Y_t - \log Z_t &= \log M_t - \log P_t - \log Z_t \\ &= \left(\log M_t^{UE} - \log P_t^{UE} - \log Z_t^{UE} \right) + E^{t-1} (\log M_t - \log P_t - \log Z_t) \\ &= (\mu_t + \chi' z_t - z_t) + \frac{\rho_2 \mu_{t-1}}{1-\rho_2 L} - \frac{\rho_1 z_{t-1}}{1-\rho_{1L}} - \left(\frac{-\nu_{t-1} + \alpha \nu_{t-2} - \alpha \log P_{t-1}^{UE}}{(1-\alpha L)(1-kL)} \right)^6 \end{aligned} \tag{72}$$

3.2.2 Welfare

Expected welfare is

$$E\left(u_t - u_t^{FLEX}\right) = E \left[\begin{array}{c} \frac{1-\nu-\bar{N}^{\chi+1}}{1-\nu} \left(\frac{\alpha}{1-\alpha\beta\rho_1} z_{t-1} + \mu_t + \chi' z_t - z_t \right) + \\ \frac{\rho_2 \mu_{t-1}}{1-\rho_2 L} - \frac{\rho_1 z_{t-1}}{1-\rho_{1L}} - \left(\frac{-\nu_{t-1} + \alpha \nu_{t-2} - \alpha \log P_{t-1}^{UE}}{(1-\alpha L)(1-kL)} \right) \end{array} \right] - E \log DP_t. \tag{73}$$

Again, we only have to consider the second term in this welfare expression, where

$$
\begin{aligned}
-E\log DP_t &= -\frac{1}{2}\Sigma_{i=0}^{\infty}\phi_p\alpha^i(1-\alpha)\left[1-\alpha^i(1-\alpha)\right]var\left(\log P_{t-i}^*\right) \\
&\quad +\Sigma_{j=0}^{\infty}\Sigma_{i=0}^{\infty}\phi_p\alpha^{i+j}(1-\alpha)^2 Cov\left(\log P_{t-i}^*,\log P_{t-j}^*\right)
\end{aligned}
$$

with

$$
\log P_t^* = \frac{\chi'}{1-\alpha}\left(-z_t+\alpha z_{t-1}\right)-\left(\frac{\nu_{t-1}-\alpha\nu_{t-2}}{1-\alpha}\right) \text{ (appendix 3.8)} \tag{74}
$$

and equation (71)

$$
\nu_t = \Sigma_{i=0}\left[\left(k\pi_i-\pi_{i+1}\right)\mu_{t-i}+\left(k\xi_i-\xi_{i+1}\right)z_{t-i}\right]
$$

We have the following expressions: $\nu_{t-1} = k\log P_{t-1} - E^{t-1}\log P_t$ is correlated with z_{t-1}; but $\nu_{t-2} = k\log P_{t-2} - E^{t-2}\log P_{t-1}$ is uncorrelated with z_t and z_{t-1}. Assume that $-\frac{\nu_{t-1}-\alpha\nu_{t-2}}{1-\alpha} = \psi_0 z_{t-1} + q_{t-1}$, where $\psi_0 z_{t-1}$ combines all terms in z_{t-1} and q_{t-1} is uncorrelated with z_{t-1} and z_t. Comparing this with the new reset price under rational indexation, this lagged indexation's renewed price function has the extra term $\frac{\nu_{t-1}-\alpha\nu_{t-2}}{1-\alpha}$. For the task below, we temporarily take the expected welfare under rational expectation as a benchmark. To compare the expected welfare under lagged indexation to the benchmark, we need to investigate whether this extra term in renewed price improves or worsens the expected welfare level in respect to the benchmark. To do this, we break the comparison into two parts.

The first one involves all the elements with q_{t-1} in the expression for $E\log DP_t$.

$$
\begin{aligned}
E\ln DP_t\left(q_{t-1}\right) &= \frac{1}{2}\Sigma_{i=0}^{\infty}\phi_p\alpha^i(1-\alpha)\left[1-\alpha^i(1-\alpha)\right]var\left(\log P_{t-i}^*\right) \\
&\quad -\Sigma_{j=0}^{\infty}\Sigma_{i=0}^{\infty}\phi_p\alpha^{i+j}(1-\alpha)^2 Cov\left(\log P_{t-i}^*,\log P_{t-j}^*\right) \\
&= A(q_{t-1})+B(q_{t-1})
\end{aligned}
$$

Here, we only include in the variance of $\log P_t^*$ the terms in q_{t-1} where q_t follows some autocorrelation process $q_{t-i} = \rho_{i-j}q_{t-j}$, so that $var\left(\log P_t^*[q_{t-1}]\right) = var(q)$. Therefore

$$
A\left(q_{t-1}\right) = \phi_p\frac{\alpha}{1+\alpha}var(q) \tag{75}
$$

We consider $B\left(q_{t-1}\right)$ now. First we assume that $\rho_{i-j}=1$; then

$$
\begin{aligned}
B\left(q_{t-1}\right) &= -\phi_p(1-\alpha)^2\Sigma_{j=0}^{\infty}\Sigma_{i=0}^{\infty}\alpha^{i+j}\rho_{i-j}var\left(q\right)\begin{bmatrix} i=0;\ j=1,2... \\ \alpha+\alpha^2+... \\ i=1;\ j=2,3... \\ \alpha^3+\alpha^4+... \\ . \\ . \end{bmatrix} \\
&= -\frac{\alpha}{1+\alpha}\phi_p var\left(q\right)
\end{aligned} \tag{76}
$$

Therefore

$$
E\ln DP_t(q_{t-1}) = 0 \tag{77}
$$

However, if $\rho_{i-j} < 1$ for any i-j, then this term must be negative. Thus for example suppose that $\rho_{i-j} = \rho^{i-j}$ so that q is a first-order autocorrelation process, then:

$$\begin{aligned} B(q_{t-1}) &= -\phi_p (1-\alpha)^2 \Sigma_{j=0}^{\infty} \Sigma_{i=0}^{\infty} \alpha^{i+j} \rho^{i-j} var(q) \begin{bmatrix} i=0;\ j=1,2... \\ \alpha\rho + \alpha^2\rho^2 + ... \\ i=1;\ j=2,3... \\ \alpha^3\rho + \alpha^4\rho^2 + ... \\ . \\ . \end{bmatrix} \\ &= -\frac{\alpha\rho(1-\alpha)}{(1+\alpha)(1-\alpha\rho)} \phi_p var(q) \end{aligned} \tag{78}$$

As a result, the expected price dispersion is

$$\begin{aligned} E \ln DP_t(q_{t-1}) &= \phi_p \frac{\alpha}{1+\alpha} (var(q)) - \frac{\alpha\rho(1-\alpha)}{(1+\alpha)(1-\alpha\rho)} \phi_p var(q) \\ &= \phi_p \frac{\alpha}{1+\alpha} \left(\frac{1-\rho}{1-\alpha\rho} \right) var(q) \end{aligned} \tag{79}$$

So comparing equations (76) and (79), we find that this term q_{t-1} must raise $E \log DP_{t,}$ this in turn reduces the expected welfare.

The second part involves all other terms that are not q_{t-1}, .that is the term $\psi_0 z_{t-1}$ .Thus it analyses the effect of this term $\psi_0 z_{t-1}$ on expected welfare.. So looking at this part of $\log P_t^*$:

$$\log P_t^*(\psi_0 z_{t-1}) = \frac{\chi'}{1-\alpha} (\alpha z_{t-1} - z_t) + \psi_0 z_{t-1},$$

we find

$$\begin{aligned} E \ln DP_t(\psi_0 z_{t-1}) &= \frac{1}{2} \Sigma_{i=0}^{\infty} \phi_p \alpha^i (1-\alpha) \left[1 - \alpha^i (1-\alpha)\right] var\left(\log P_{t-i}^*\right) \\ &\quad - \Sigma_{j=0}^{\infty} \Sigma_{i=0}^{\infty} \phi_p \alpha^{i+j} (1-\alpha)^2 Cov\left(\log P_{t-i}^*, \log P_{t-j}^*\right) \\ &= A(\psi_0 z_{t-1}) + B(\psi_0 z_{t-1}). \end{aligned}$$

With

$$A(\psi_0 z_{t-1}) = \phi_p \frac{\alpha}{1+\alpha} \left(\chi''^2 + (\psi_0 + \alpha\chi'')^2 \right) var(z)$$

and

$$B(\psi_0 z_{t-1}) = \phi_p \frac{\alpha(1-\alpha)}{1+\alpha} \chi'' (\psi_0 + \alpha\chi'') var(z)$$

this part of expected price dispersion is

$$E \ln DP_t(\psi_0 z_{t-1}) = \phi_p \frac{\alpha}{1+\alpha} \left(\chi''^2 + (\psi_0 + \alpha\chi'')^2 + (1-\alpha)\chi''(\psi_0 + \alpha\chi'') \right) var(z) \tag{80}$$

Because of equation (80) is derived using the part of equation (74), which partly consists equation for $\log P_t^*$ under the rational expectation indexing, we compare this equation (80) and the expected price dispersion under rational indexing (equation (66)) :

$$E \ln DP_t = \phi_p \frac{\alpha}{1+\alpha} \left(\chi''^2 + (\alpha\chi'')^2 + (1-\alpha)\alpha\chi''^2 \right) var(z) \tag{81}$$

It follows that the difference between $E \ln DP_t$ under lagged and rational indexation due to the term in $\psi_0 z_{t-1}$ is:$\phi_p \frac{\alpha}{1+\alpha}\left((\psi_0)^2 [1 + \frac{(1+\alpha)\chi''}{\psi_0}]\right) var(z)$ from which it can be seen that for this term to worsen welfare under lagged indexation requires that $\frac{(1+\alpha)\chi''}{\psi_0}$ should be positive (or if negative should be greater than -1.which can effectively be ruled out). Now by construction $\psi_0 = \frac{-1}{1-\alpha}(k\xi_0 - \xi_1)$ so the (relevant sufficient) condition for the term to worsen welfare is that $\xi_1 \succ k\xi_0$.

We can evaluate ξ_0 readily from the fact that:

$$\log P_t^{UE} = (1-\alpha)\left(\frac{1-\alpha\beta}{1-\alpha\beta\rho_1}\right)\left(\frac{1+\chi}{1-\nu}\right)(-z_t) = \xi_0 z_t \tag{82}$$

Since $\chi'' = \frac{\chi'}{1-\alpha}$ and $\chi' = (1-\alpha)\left(\frac{1-\alpha\beta}{1-\alpha\beta\rho_1}\right)\left(\frac{1+\chi}{1-\nu}\right)$, $\chi' = -\xi_0$ and $\xi_0 = -\chi''(1-\alpha)$. Therefore the first term in ψ_0 is

$$\psi_0(\text{first term}) = -\frac{1}{1-\alpha} k(-\chi'')(1-\alpha) = k\chi'' > 0 \tag{83}$$

This leaves the second term $\frac{\xi_1}{1-\alpha}$.which comes from the subsequent path of prices in the period after the shock; this cannot be solved analytically as it involves solving for all the ξ_i in the equation from appendix 3.6 (involving among other things finding the three stable roots of a fourth order difference equation). Here we find it numerically using the calibration of Canzoneri et al; ξ_1 turns out to be -1.325 while ξ_0 is calibrated at -1.5467. Therefore $\psi_0 = 0.67 > 0$ which implies that the term in $\psi_0 z_{t-1}$ worsens welfare. though this finding is plainly not general, it is shown in appendix 3.7 that it is robust. We conclude here that lagged indexation with $k = 1$ worsens welfare, compared to that of the rational indexation set-up. because both the term $\psi_0 z_{t-1}$ and the term q_{t-1} decrease welfare.

3.3 Conclusions from the analysis of a simplified model

What we have found is that it is robustly optimal within this simplified version of the model to index reset prices to the rational expectation of the price level. This in turn implies that welfare is invariant to monetary policy, as is expected output. in an echo of Sargent and Wallace's (1975) famous irrelevance result three decades ago [7] The intuition behind this result is that rational indexation builds into prices the effect of any shocks known at time t-1. Whatever has happened at t-1 is, in the case of the productivity shock, built into the expected real reset price for next period; this fixes expected real marginal cost and hence expected real output. The expected price index is then calculated as the necessary price increase that will accommodate this and the expected level of money supply. Thus the reset price is optimised apart from the effect of any unanticipated shocks; it is only the latter that reduce welfare, whereas under lagged indexation welfare is also reduced by the effects of previous shocks.

4 Simulation of the full Model

In addition to the analytical part above, we also use numerical methods to prove the point. We compute the impulse response functions of the model's variables under the different indexing processes in the face of temporary productivity, unexpected and expected monetary shocks. Throughout the simulation, we use a

[7] Notice too that in a corrolary of this point, again echoing Sargent and Wallace (ibid.), price level determinacy cannot be produced by an interest-rate rule targeting inflation unless the lagged price level is given; yet the model cannot generate such a lagged price level under such a regime unless again the twice-lagged price level was fixed and so on ad infinitum. It is necessary when using such a rule to specify the lagged price level exogenously via an initial condition, presumably indicating that at some previous point a different rule was being pursued.

discount factor β of 0.99. The Cobb-Douglas capital share parameter, $\nu = 0.25$, implies that the output-labour elasticity is 0.75. The wage and price markup rates are $\mu_w = \mu_p = 1.167$. The constant probability determining the degree of price stickiness is $\alpha = 0.67$, this implies that an average price contract duration is 3 periods, while the probability of wage resetting is assumed to be $1 - \omega = 0.25$ in every period, implying an average contract length of 4 periods. We use for work disutility in the utility function a coefficient of 0.25. These are the calibration parameters used by Canzoneri et al. Our simulations' results are presented graphically below. The impulse response functions show the mean value at each date after the initial shock for each variable.

Firstly, Figure1 shows the impulse response functions produced by a positive temporary productivity shock that occurs in period 1. In all three cases, the rise in productivity initially causes a boom. However, rational expectation indexing brings higher average output, consumption and employment than other two indexation models.

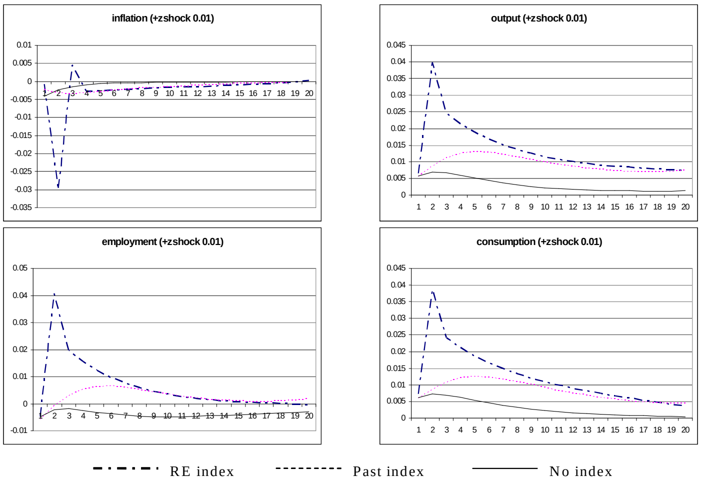

Figure 1: Dynamic paths after an unexpected 0.01 increase in productivity in period 1

While in Figure2 the impulse responses of negative productivity shock of the magnitude of 0.01 on main variables seem to be mirroring what was in Figure1.

Secondly, we assume an unanticipated negative monetary shock: the interest rate increases in period 1. In all three case, this contractionary policy causes recession, which eventually disappears. While the output, employment and consumption under rational indexation return very quickly to their steady states, the other two cases exhibit a degree of inertia. The response of inflation shows that the effect of the shock under rational indexation dies out quickly, whereas the other two cases show a more gradual responses.

Figure4 shows the dynamic responses to an unanticipated expansionary monetary policy, predicted by the three indexation cases. Again, they are mirroring the effects in Figure3.

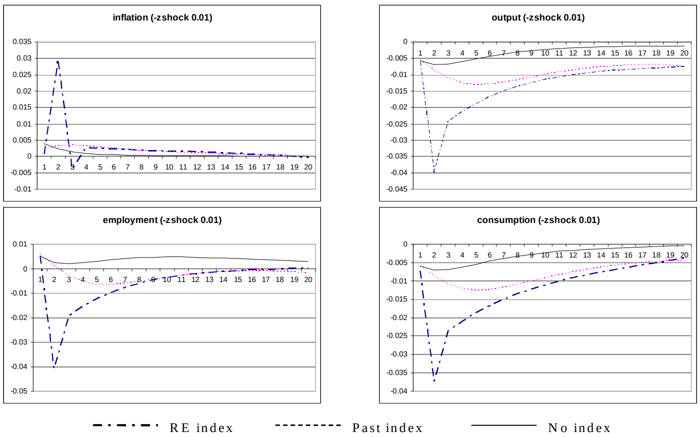

Figure 2: Dynamic paths after an unexpected 0.01 fall in productivity in period 1

We now further investigate within the full model the key result from the simple analytical version: that anticipated monetary policy has no effect on welfare when there is rational indexation. We show in Figure5the effect of an expected future monetary shock; namely the shock (unanticipated at time t) of a monetary shock in time$t+5$. What happens is that there is a shock to output and welfare at the time of the shock's announcement.but when the shock occurs at time $t+5$ there is no effect.

Finally we show the model's computations[8] under stochastic simulation of both monetary and productivity shocks for expected welfare in terms of deviations from the flex-optimum (the numbers are expressed as a flow per quarter — i.e. 'expected permanent welfare'). Table 1 shows that, in line with our analytical version, rational indexation dominates lagged; the latter in turn dominates non-indexation.

Rational indexation	$1.570x10^{-5}$
Lagged indexation	$-0.961x10^{-5}$
Non indexation	$-4.923x10^{-5}$

Table 1: Table of expected welfare for different types of indexation (stochastic simulation of monetary and productivity shocks, each with standard error of 0.01)

5 Conclusion

We conclude that the Calvo contract, which produces a variety of curious and puzzling results in its usually-used form, should be adjusted for expected inflation which is the optimal indexing process for a rational agent to use. The implications of the resulting model for monetary policy are radical: monetary policy ceases to have any effect on welfare, in an echo of Sargent and Wallace's (1975) famous policy irrelevance result. We do not take this to imply that monetary policy necessarily has no such effect; however we do suggest that for those who believe monetary policy does have such an effect, the New Keynesian model of the New NeoKeynesian Synthesis variety is not a suitable vehicle.

[8] These numbers are all produced by Dynare (the slight positive for rational indexation is an anomaly that must be produced by numerical approximation error; we are investigating this by other programming methods).

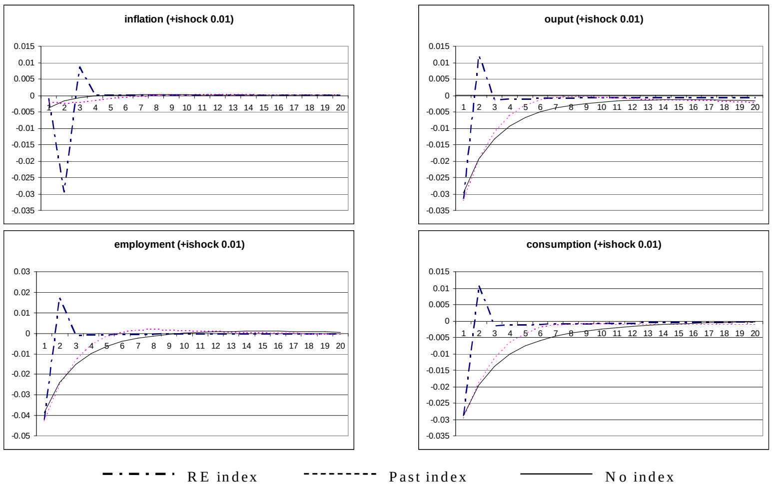

Figure 3: Dynamic paths given an unanticipated monetary constriction in period 1

6 Appendix to Section 3

$$
\begin{aligned}
\log P_t^* &= \frac{(1-\alpha\beta)\, E_t\left(\log mc_t + \log P_t - \log \tilde{P}_t\right)}{1-\alpha\beta B^{-1}} = \\
&= (35)\, \frac{(1-\alpha\beta)}{1-\alpha\beta B^{-1}} E_t\left(\log W_t - \log P_t + \nu \log N_t - \log Z_t + \log P_t - \log \tilde{P}_t\right) \\
&= (36)\, \frac{1-\alpha\beta}{1-\alpha\beta B^{-1}} E_t\left(\chi \log N_t + \log C_t + \nu \log N_t - \log Z_t + \log P_t - \log \tilde{P}_t\right) \\
&= (38)\,(37)\, \frac{1-\alpha\beta}{1-\alpha\beta B^{-1}} E_t\left(\frac{\log Y_t - \log Z_t}{1-\nu}\chi + (\log Y_t - \log Z_t) + \nu\frac{\log Y_t - \log Z_t}{1-\nu} + \log P_t - \log \tilde{P}_t\right) \\
&= \ln P_t^* = \frac{(1-\alpha\beta)}{1-\alpha\beta B^{-1}} E_t\left(\frac{1+\chi}{1-\nu}(\log Y_t - \log Z_t) + \log P_t - \log \tilde{P}_t\right)
\end{aligned}
$$

$$
\begin{aligned}
\ln P_t - \ln \tilde{P}_t &= \alpha\left(\ln P_{t-1} - \ln \tilde{P}_{t-1}\right) + (1-\alpha)\ln P_t^* \Longleftrightarrow \\
\log P_t^{UE} &= \alpha\left(\ln P_{t-1} - \ln \tilde{P}_{t-1}\right) + (1-\alpha)\left(\log P_t^{*UE} - \frac{\alpha}{1-\alpha}\log P_{t-1}^{UE}\right)
\end{aligned}
$$

$$
\begin{aligned}
\log Y_t &= E^{t-1}\log Y_t + \log Y_t^{UE} = \\
&= E^{t-1}\log Z_t - v'\left(\log M_{t-1}^{UE} - \log Y_{t-1}^{UE}\right) + \mu_t + \chi' z_t \\
&= E^{t-1}\log Z_t - v'\left(\mu_{t-1} - \mu_{t-1} - \chi' z_{t-1}\right) + \mu_t + \chi' z_t \\
&= \rho_1 \log Z_{t-1} + v'\chi' z_{t-1} + \mu_t + \chi' z_t
\end{aligned}
$$

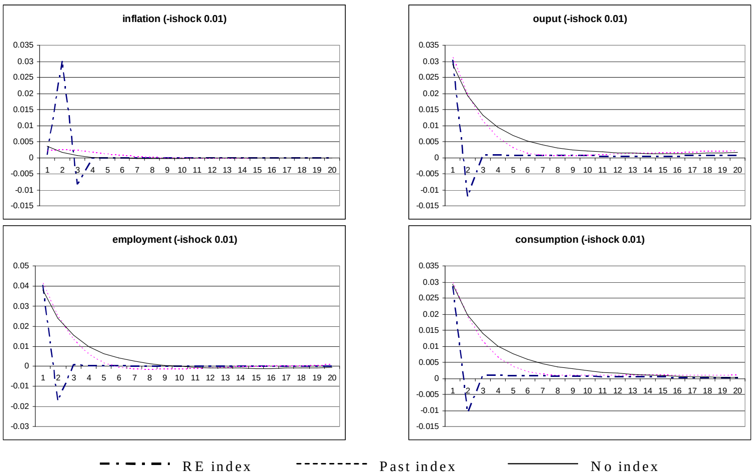

Figure 4: Dynamic paths given an unanticipated monetary expansion in period 1

Manipulating equation (59) with the help of equations (54) and (52), we get

$$\begin{aligned}
E^{t-1}(\log Y_t - \log Z_t) &= \frac{1}{1-\alpha\beta}\left(\frac{1-\nu}{1+\chi}\right)E^{t-1}\log P_t^* \\
&= \frac{1}{1-\alpha\beta}\left(\frac{1-\nu}{1+\chi}\right)\left(-\frac{\alpha}{1-\alpha}\right)\log P_{t-1}^{UE} \\
&= \frac{1}{1-\alpha\beta}\left(\frac{1-\nu}{1+\chi}\right)\left(-\frac{\alpha}{1-\alpha}\right)(1-\alpha)\log P_{t-1}^{*UE} \\
&= \frac{1}{1-\alpha\beta}\left(\frac{1-\nu}{1+\chi}\right)\left(-\frac{\alpha}{1-\alpha}\right)(1-\alpha)(1-\alpha\beta)\left(\frac{1+\chi}{1-\nu}\right)\frac{1}{1-\alpha\beta\rho_1}(-z_{t-1}) \\
&= \frac{\alpha}{1-\alpha\beta\rho_1}z_{t-1}
\end{aligned}$$

$$\begin{aligned}
E\left(u_t - u_t^{FLEX}\right) &= -E\ln DP_t \\
&= -\frac{1}{2}\Sigma_{i=0}^{\infty}\phi_p\alpha^i(1-\alpha)\left[1-\alpha^i(1-\alpha)\right]var\left(\log P_{t-i}^*\right) \\
&\quad +\Sigma_{j=0}^{\infty}\Sigma_{i=0}^{\infty}\phi_p\alpha^{i+j}(1-\alpha)^2 Cov\left(\log P_{t-i}^*, \log P_{t-j}^*\right) \\
&= (1)+(2)
\end{aligned}$$

and

$$\log P_t^* = -\chi'' z_t + \alpha\chi'' z$$

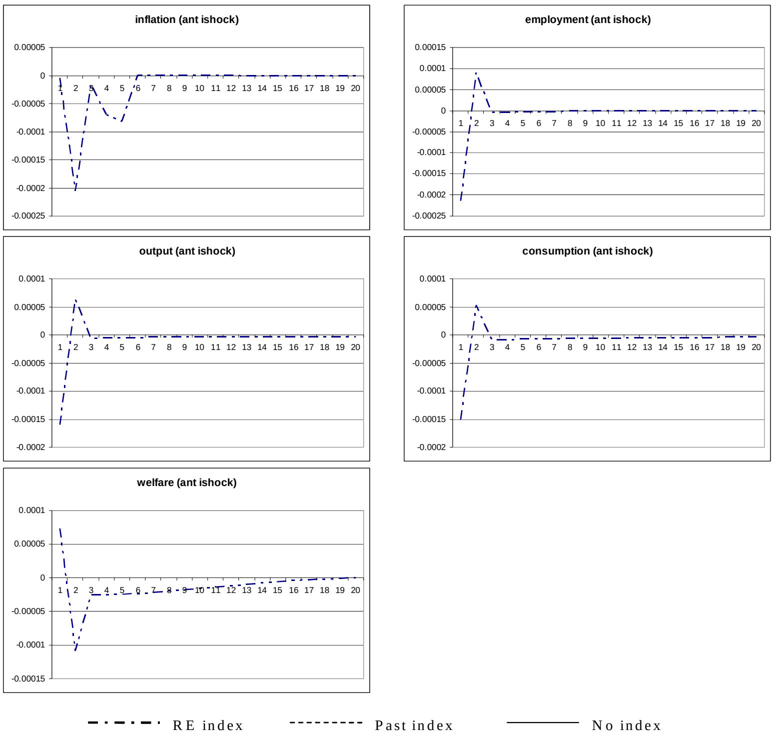

Figure 5: Dynamic paths under an anticipated monetary shock in period 5

Therefore, $var\left(\log P^*_{t-i}\right) = \left(1+\alpha^2\right)\chi''^2 var(z)$ and (1) is

$$
\begin{aligned}
(1) &= -\frac{1}{2}\Sigma_{i=0}^{\infty}\phi_p\alpha^i(1-\alpha)\left[1-\alpha^i(1-\alpha)\right]\left(1+\alpha^2\right)\chi''^2 var(z) \\
&= -\frac{1}{2}\phi_p(1-\alpha)\left(1+\alpha^2\right)\chi''^2 var(z)\Sigma_{i=0}^{\infty}\left[1-\alpha^i(1-\alpha)\right] \\
&= -\frac{1}{2}\phi_p(1-\alpha)\left(1+\alpha^2\right)\chi''^2 var(z)\frac{2\alpha}{1-\alpha^2} \\
&= -\phi_p\chi''^2\frac{\alpha\left(1+\alpha^2\right)}{1+\alpha}var(z)
\end{aligned}
$$

and (2) is

$$
\begin{aligned}
(2) &= \phi_p(1-\alpha)^2\Sigma_{j=0}^{\infty}\Sigma_{i=0}^{\infty}\alpha^{i+j}E\left(\log P^*_{t-i}\log P^*_{t-j}\right) \\
&= \phi_p(1-\alpha)^2\begin{bmatrix} i=0;\ j=1,2... \\ \alpha\left(-\alpha\chi''^2\right)var(z) \\ i=1;\ j=2,3... \\ \alpha^3\left(-\alpha\chi''^2\right)var(z) \\ . \\ . \\ . \end{bmatrix} \\
&= \phi_p(1-\alpha)^2\frac{\alpha}{1-\alpha^2}var(z)
\end{aligned}
$$

Thus,

$$E\left(u_t - u_t^{FLEX}\right) = -\phi_p\alpha\left[\chi''^2\frac{\left(1+\alpha^2\right)}{1+\alpha} + (1-\alpha)^2\frac{1}{1-\alpha^2}\right]var(z)$$

Firstly, under lagged indexation it is necessary to assume some simple money rule that is equivalent to the Taylor rule, accommodating productivity and having the same autocorrelation coefficient $\rho_1 : \log M_t = \rho_1 \log M_{t-1} + dz_t + \varepsilon_t$. (Under rational indexation the money supply rule is irrelevant to welfare as we have seen).Then, using this, equations (39), (42) and (69) to find undetermined coefficients of the price level $\log P_t$:

$$\log P_t = \log \tilde{P}_t + \frac{(1-\alpha)\log P_t^*}{1-\alpha L} = k\log P_{t-1} + \frac{(1-\alpha)\log P_t^*}{1-\alpha L}$$

$\Leftrightarrow$

$$(1-kL)\log P_t = \frac{(1-\alpha)\log P_t^*}{1-\alpha L}$$

$$(1-kL)\log P_t = \frac{1-\alpha}{1-\alpha L}\frac{1-\alpha\beta}{1-\alpha\beta B^{-1}}E_t\left[\chi^*\left(\log M_t - \log P_t - \log Z_t\right) + \log P_t - k\log P_{t-1}\right]$$

$\Leftrightarrow$ Due to the assumption that in period t producers know both macro information from period $(t-1)$ and micro information in period t, the above equation is rewritten as following:

$$\begin{aligned}(1-kL)(1-\alpha L)\log P_t &= (1-\alpha)(1-\alpha\beta)\chi^*\frac{E_t(-\log Z_t)}{(1-\alpha\beta E^t B^{-1})}+(1-\alpha)(1-\alpha\beta)\chi^*\frac{E^{t-1}\log M_t}{(1-\alpha\beta E^{t-1}B^{-1})}\\ &\quad -\frac{(1-\alpha)(1-\alpha\beta)}{(1-\alpha\beta E^{t-1}B^{-1})}E^{t-1}((\chi^*-1)\log P_t+k\log P_{t-1})\\ &= (1-\alpha)(1-\alpha\beta)\chi^*\frac{(-\log Z_t)}{1-\alpha\beta\rho_1}+(1-\alpha)(1-\alpha\beta)\chi^*\frac{E^{t-1}\log M_t}{1-\alpha\beta\rho_1}-\\ &\quad -\frac{(1-\alpha)(1-\alpha\beta)}{(1-\alpha\beta E^{t-1}B^{-1})}E^{t-1}((\chi^*-1)\log P_t+k\log P_{t-1})\\ &= \chi'(-\log Z_t)+\chi' E^{t-1}\log M_t-\frac{(1-\alpha)(1-\alpha\beta)}{(1-\alpha\beta E^{t-1}B^{-1})}E^{t-1}((\chi^*-1)\log P_t+k\log P_{t-1})\end{aligned}$$

$\Leftrightarrow$

$$\begin{aligned}\begin{bmatrix}(1-kL)(1-\alpha L)\left(1-\alpha\beta E^{t-1}B^{-1}\right)\\ +(1-\alpha)(1-\alpha\beta)\chi^*E^{t-1}\\ -(1-\alpha)(1-\alpha\beta)E^{t-1}(1-kL)\end{bmatrix}\log P_t &= \chi'\left[E^{t-1}\log M_t-\log Z_t\right]\left(1-\alpha\beta E^{t-1}B^{-1}\right)\\ \begin{bmatrix}(1-kL)(1-\alpha L)\left(1-\alpha\beta E^{t-1}B^{-1}\right)\\ +(1-\alpha)(1-\alpha\beta)\chi^*E^{t-1}\\ -(1-\alpha)(1-\alpha\beta)E^{t-1}(1-kL)\end{bmatrix}\log P_t &= \chi'\begin{bmatrix}-\log Z_t+\alpha\beta\rho_1\log Z_{t-1}\\ +E^{t-1}\log M_t-\alpha\beta\rho_1E^{t-1}\log M_{t+1}\end{bmatrix}\\ \begin{bmatrix}(1-kL)(1-\alpha L)\left(1-\alpha\beta E^{t-1}B^{-1}\right)\\ +(1-\alpha)(1-\alpha\beta)\chi^*E^{t-1}\\ -(1-\alpha)(1-\alpha\beta)E^{t-1}(1-kL)\end{bmatrix}\log P_t &= \chi'\frac{\begin{bmatrix}-z_t+\alpha\beta\rho_1z_{t-1}+\rho_1(dz_{t-1}+\varepsilon_{t-1})\\ -\alpha\beta\rho_1^2(dz_{t-1}+\varepsilon_{t-1})\end{bmatrix}}{1-\rho_1L}\\ (1-\rho_1L)\begin{bmatrix}(1-kL)(1-\alpha L)\left(1-\alpha\beta E^{t-1}B^{-1}\right)\\ +(1-\alpha)(1-\alpha\beta)\chi^*E^{t-1}\\ -(1-\alpha)(1-\alpha\beta)E^{t-1}(1-kL)\end{bmatrix}\log P_t &= \chi'\begin{bmatrix}-z_t+\alpha\beta\rho_1z_{t-1}+\rho_1(dz_{t-1}+\varepsilon_{t-1})\\ -\alpha\beta\rho_1^2(dz_{t-1}+\varepsilon_{t-1})\end{bmatrix}\end{aligned}$$

Therefore, the equation used to determined the unknown coefficients in $\log P_t=\Sigma_{i=0}\pi_i\mu_{t-i}+\Sigma_{i=0}\xi_iz_{t-i}$ is defined as:

$$\begin{bmatrix}\log P_t-\rho_1\log P_{t-1}-\alpha\beta\left(E^{t-1}\log P_{t+1}-\rho_1E^{t-2}\log P_t\right)\\ +B(\log P_{t-1}-\rho_1\log P_{t-2})+\alpha k(\log P_{t-2}-\rho_1\log P_{t-3})+\\ A\left(E^{t-1}\log P_t-\rho_1E^{t-2}\log P_{t-1}\right)\end{bmatrix}=\chi'\begin{bmatrix}-z_t+\alpha\beta\rho_1z_{t-1}+\rho_1(dz_{t-1}+\varepsilon_{t-1})\\ -\alpha\beta\rho_1^2(dz_{t-1}+\varepsilon_{t-1})\end{bmatrix}$$

given $A=[\alpha\beta(\alpha+k)+(1-\alpha)(1-\alpha\beta)(\chi^*-1)]$ and $B=-(\alpha+k-(1-\alpha\beta-\alpha)k)$. Collect the terms and equalise the RHS and LHS, assuming that $k=1$:
(z_t)

$$\xi_0=-\chi' \tag{84}$$

(z_{t-1})

$$\xi_1(1+A)+\xi_0(B-\rho_1+\alpha\beta\rho_1+\rho_1(1-\alpha\beta\rho_1))-\alpha\beta\xi_2=0 \tag{85}$$

(z_{t-2})

$$\xi_2(1+A+\alpha\beta\rho_1)+\xi_1(B-\rho_1(1+A))+\xi_0(\alpha k-\rho_1B)-\alpha\beta\xi_3=0 \tag{86}$$

$(z_{t-i},\ i \geqslant 3)$

$$\left[\xi_i \left(1 + A\right) - \alpha\beta\xi_{i+1} + B\xi_{i-1} + \alpha k\xi_{i-2}\right] \left(1 - \rho_1 L\right) = 0 \tag{87}$$

For this specific set of calibrated coefficients given in Canzoneri's original paper, this later equation is a fourth order difference equation, consisting of that first bracket- a third order difference equation — and the 2nd bracket — first order difference equation, that gives a stable root of 0.93. To solve the 3rd order difference equation, we consider its characteristic equation:

$$\lambda^3 - 3.923\lambda^2 + 3.03\lambda - 1.015 = 0$$

There are two stable roots $\lambda_{1,2} = 0.44 \pm 0.37i$ and one unstable $\lambda_3 = 3.03$. The equation (87) can be rewritten as

$$\left(1 - \left(0.44 + 0.37i\right) L\right) \left(1 - \left(0.44 - 0.37i\right) L\right) \left(1 - \rho_1 L\right) = 0$$

$$1 - 1.18L + 1.16L^2 - 0.31L^3 = 0$$

and thus,

$$\xi_3 = 1.82\xi_2 - 1.164\xi_1 + 0.312\xi_0 \tag{88}$$

To solve for unknown coefficients, we use equations (84), (85), (86) and (88) that are now defined in term of calibrated coefficients:

$$\begin{aligned} \xi_0 &= -1.547 \\ \xi_1 &= 0.75\xi_0 + 0.254\xi_2 \\ \xi_2 &= 1.4\xi_1 - 0.79\xi_0 + 0.21\xi_3 \\ \xi_3 &= 1.8\xi_2 - 1.16\xi_1 + 0.31\xi_0. \end{aligned}$$

Thus, the solutions are $\xi_0 = -1.547, \xi_1 = -1.325$ and etc. This result ensures that ψ_0 is positive, thus it proves that welfare under lagged indexation is smaller than that under rational indexation. The robustness test to the later conclusion can performed using different values of parameters d, α, χ, ν and ρ_1 (Table 2) :

Equations (70) and (51) give

$$\log P_t^{UE} = (1 - \alpha) \left(\frac{1 - \alpha\beta}{1 - \alpha\beta\rho_1} \right) \left(\frac{1 + \chi}{1 - \nu} \right) (-z_t)$$

Under rational expectations:

$$\log P_t = E^{t-1} \log P_t + \log P_t^{UE}$$

$\Leftrightarrow$

$$\Sigma_{i=0}\pi_i\mu_{t-i} + \Sigma_{i=0}\xi_i z_{t-i} = E^{t-1} \log P_t + (1 - \alpha) \left(\frac{1 - \alpha\beta}{1 - \alpha\beta\rho_1} \right) \left(\frac{1 + \chi}{1 - \nu} \right) (-z_t),$$

therefore

$$\begin{aligned} \pi_0 &= 0 \\ \xi_0 &= -\left(1 - \alpha\right) \left(\frac{1 - \alpha\beta}{1 - \alpha\beta\rho_1} \right) \left(\frac{1 + \chi}{1 - \nu} \right) \end{aligned}$$

$\mathbf{d}$	$\boldsymbol{\alpha}$	$\boldsymbol{\nu}$	$\boldsymbol{\chi}$	ρ_1	ξ_0	ξ_1
1	**0.67**	**0.25**	**3**	**0.93**	$\mathbf{-1.5467}$	$\mathbf{-1.325}$
1	0.73	0.25	3	0.93	-1.2178	-1.2083
1	0.67	0.3	3	0.93	-1.6572	-1.3858
1	0.67	0.25	4	0.93	-1.9334	-1.5286
1	0.67	0.25	3	1	-1.7600	-1.5617
0.446	0.67	0.25	3	0.93	-1.5467	-1.5458
1	0.01	0.25	3	0.93	-5.276	-0.0293
1	0.67	0.2	3	0.93	-1.45	-1.269
1	0.67	0.25	1.41	0.93	-0.9319	-0.9312
1	0.67	0.25	3	0	-0.5926	-0.5282
0.8	0.7	0.25	3	0.93	-1.3817	-1.3476
0.8	0.7	0.28	3	0.93	-1.4392	-1.3854
0.8	0.7	0.28	4	0.93	-1.7991	-1.6046
0.8	0.7	0.28	4	0.80	-1.4353	-1.2243
0.7	0.5	0.24	2	0.5	-1.3245	-0.8333
0.8	0.6	0.27	1.6	0.6	-0.8987	-0.7359
0.7	0.6	0.3	3.6	0.8	-2.0335	-1.3928

Table 2: Robust test results

From equation (51)

$$\log P_t^{UE} = (1-\alpha)\log P_t^{*UE}$$

and equation (68)

$$E^{t-1}\log P_t^* = \frac{v_{t-1} - \alpha v_{t-2} - \alpha \log P_{t-1}^{UE}}{1-\alpha}$$

we get

$$E^{t-1}\log P_t^* = \frac{v_{t-1} - \alpha v_{t-2} - \alpha (1-\alpha) \log P_{t-1}^{*UE}}{1-\alpha} \tag{89}$$

Under the rational expectation and equation (34), the new reset price is

$$\begin{aligned}
\log P_t^* &= E^{t-1}\log P_t^* + \log P_t^{*UE} = \\
&= \frac{v_{t-1} - \alpha v_{t-2}}{1-\alpha} - \alpha \log P_{t-1}^{*UE} + \log P_t^{*UE} \\
&= \frac{v_{t-1} - \alpha v_{t-2}}{1-\alpha} - \alpha \left(\frac{1-\alpha\beta}{1-\alpha\beta\rho_1}\right)\left(\frac{1+\chi}{1-\nu}\right)(-z_{t-1}) + \left(\frac{1-\alpha\beta}{1-\alpha\beta\rho_1}\right)\left(\frac{1+\chi}{1-\nu}\right)(-z_t) \\
&= \frac{v_{t-1} - \alpha v_{t-2}}{1-\alpha} + \frac{\chi'}{1-\alpha}(-z_t + \alpha z_{t-1})
\end{aligned}$$

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