

# Targeting Nominal GDP or Prices: Expectation Dynamics and the Interest Rate Lower Bound

Seppo Honkapohja, Bank of Finland

Kaushik Mitra, University of Saint Andrews

*\*Views expressed do not necessarily reflect the views of the Bank of Finland.*

# Introduction

- Two major developments for monetary policy:
  - (1) Inflation targeting is widely considered as the "best practice".
  - (2) First in Japan and now in much of the Western world: the zero (or "effective") lower bound (ZLB) for policy interest rates has become important in practice.

Analytically:

- Taylor-type interest rate rules lead to multiple equilibria (e.g. see Reifschneider & Williams 2000, Benhabib et al 2001, 2002; Evans and Honkapohja 2005, 2010; Evans, Guse and Honkapohja 2008).

- Recently suggestions that price-level or nominal GDP targeting (PLT and NGDP) might be more suitable frameworks than inflation targeting (IT).
  - \* History dependence is a key feature of PLT and NGDP. => Improved guidance to the economy.
- Carney (2012) and C. Evans (2012) discuss ideas for improved guidance.
- Eggertsson & Woodford (2003) suggest that PLT is (nearly) optimal policy under ZLB.

This paper:

- Possibility of multiple steady states is not limited to IT and Taylor rules.
  - Eggertsson & Woodford (2003) note but do not analyze the existence of a deflationary equilibrium.
- We show that the multiplicity problem is also true for (versions of) PLT and NGDP.
- We analyze aspects of global dynamics under adaptive learning in the standard NK model when policy follows either PLT or NGDP.
  - The targeted steady state is locally but not globally E-stable.
  - The low steady state is not E-stable.

- We look different aspects of the adjustment dynamics under PLT and NGDP, robustness with respect to
  - domain of attraction of the target,
  - maximal speed of learning, and
  - volatility of the dynamics.
  
- The form of private agents' learning is very important:
  - (i) If agents forecast in the same way as under IT, then performance of PLT and NGDP targeting are not particularly good.
  - (ii) Performance of PLT and NGDP targeting if agents incorporate additional guidance provided by either regime.
    - In particular, the targeted steady state has a very large domain of attraction (global stability?).

# The Model

- Agent  $s$  solves

$$\text{Max } E_0 \sum_{t=0}^{\infty} \beta^t U_{t,s} \left( c_{t,s}, \frac{M_{t-1,s}}{P_t}, h_{t,s}, \frac{P_{t,s}}{P_{t-1,s}} - 1 \right)$$

$$\text{st. } c_{t,s} + m_{t,s} + b_{t,s} + \Upsilon_{t,s} = m_{t-1,s} \pi_t^{-1} + R_{t-1} \pi_t^{-1} b_{t-1,s} + \frac{P_{t,s}}{P_t} y_{t,s},$$

where  $c_{t,s}$  is the consumption aggregator,  $M_{t,s}$  and  $m_{t,s}$  are nominal and real money balances,  $h_{t,s}$  is the labor input,  $b_{t,s}$  is the real quantity of risk-free one-period nominal bonds,  $\Upsilon_{t,s}$  is the lump-sum tax,  $R_{t-1}$  is the nominal interest rate factor between  $t-1$  and  $t$ ,  $P_{t,s}$  is the price of good  $s$ ,  $y_{t,s}$  is output of good  $s$ ,  $P_t$  is the level, and the inflation rate is  $\pi_t = P_t/P_{t-1}$ .

- The utility function has the parametric form

$$U_{t,s} = \frac{c_{t,s}^{1-\sigma_1}}{1-\sigma_1} + \frac{\chi}{1-\sigma_2} \left( \frac{M_{t-1,s}}{P_t} \right)^{1-\sigma_2} - \frac{h_{t,s}^{1+\varepsilon}}{1+\varepsilon} - \frac{\gamma}{2} \left( \frac{P_{t,s}}{P_{t-1,s}} - 1 \right)^2.$$

The final term is the cost of adjusting prices. There is also the “no Ponzi game” condition.

- Production function for good  $s$  is

$$y_{t,s} = h_{t,s}^\alpha$$

where  $0 < \alpha < 1$ . Each firm faces a demand curve

$$P_{t,s} = \left( \frac{y_{t,s}}{Y_t} \right)^{-1/\nu} P_t.$$

$P_{t,s}$  is the profit maximizing price.  $Y_t$  is aggregate output.

- The government's flow budget constraint is

$$b_t + m_t + \Upsilon_t = g_t + m_{t-1}\pi_t^{-1} + R_{t-1}\pi_t^{-1}b_{t-1},$$

where  $g_t$  is government consumption,  $b_t$  is the real quantity of government debt, and  $\Upsilon_t$  is the real lump-sum tax. Fiscal policy follows a linear tax rule

$$\Upsilon_t = \kappa_0 + \kappa b_{t-1},$$

where usually  $\beta^{-1} - 1 < \kappa < 1$ , i.e. fiscal policy is “**passive**” (Leeper 1991).

- For normal policy  $g_t = \bar{g}$ . From market clearing we have

$$c_t + g_t = y_t.$$



# Optimal decisions

- For simplicity, assume identical expectations and absence of random shocks, log utility and point expectations.
- Optimality conditions become

$$\begin{aligned}m_t &= \chi\beta(1 - R_t^{-1})^{-1}c_t, \\c_t^{-1} &= \beta r_{t+1}^e (c_{t+1}^e)^{-1}, \text{ where } r_{t+1}^e = R_t/\pi_{t+1}^e, \text{ and} \\ \frac{\alpha\gamma}{\nu} (\pi_t - 1) \pi_t &= h_t \left( h_t^\varepsilon - \alpha \left( 1 - \frac{1}{\nu} \right) h_t^{\alpha-1} c_t^{-1} \right) \\ &\quad + \beta \frac{\alpha\gamma}{\nu} \left[ (\pi_{t+1}^e - 1) \pi_{t+1}^e \right].\end{aligned}$$

## The infinite-horizon Phillips curve

- Let  $Q_t = (\pi_t - 1) \pi_t$ , with the appropriate root  $\pi \geq \frac{1}{2}$ . Using  $h_t = y_t^{1/\alpha}$ ,  $c_t = y_t - g_t$  and iterating forward we obtain

$$Q_t = \tilde{K}(y_t, y_{t+1}^e, y_{t+2}^e \dots) \equiv \frac{\nu}{\gamma} y_t^{(1+\varepsilon)/\alpha} - \frac{\nu - 1}{\gamma} \frac{y_t}{y_t - \bar{g}} + \frac{\nu}{\gamma} \sum_{j=1}^{\infty} \alpha^{-1} \beta^j (y_{t+j}^e)^{(1+\varepsilon)/\alpha} - \frac{\nu - 1}{\gamma} \sum_{j=1}^{\infty} \beta^j \left( \frac{y_{t+j}^e}{y_{t+j}^e - \bar{g}} \right).$$

# The consumption function

- Define the asset wealth

$$a_t = b_t + m_t$$

and write the flow budget constraint as

$$a_t + c_t = y_t - \Upsilon_t + r_t a_{t-1} + \pi_t^{-1} (1 - R_{t-1}) m_{t-1},$$

where  $r_t = R_{t-1}/\pi_t$ . Next, iterate forward and impose

$$\lim_{j \rightarrow \infty} (D_{t,t+j}^e)^{-1} a_{t+j}^e = 0 \text{ where } D_{t,t+j}^e = \prod_{i=1}^j r_{t+i}^e$$

with  $r_{t+j}^e = R_{t+j-1}^e / \pi_{t+j}^e$ .

- This yields the life-time budget constraint of the household

$$0 = r_t a_{t-1} + \phi_t - c_t + \sum_{j=1}^{\infty} (D_{t,t+j}^e)^{-1} (\phi_{t+j}^e - c_{t+j}^e),$$

where  $\phi_{t+j}^e = y_{t+j}^e - \Upsilon_{t+j}^e + (\pi_{t+j}^e)^{-1} (1 - R_{t+j-1}^e) m_{t+j-1}^e$ .

- Using the consumption Euler equation, the money-consumption relation, the flow government budget constraint, and the assumption that consumers are Ricardian we obtain **the consumption function**

$$c_t = (1 - \beta) \left( y_t - \bar{g} + \sum_{j=1}^{\infty} (D_{t,t+j}^e)^{-1} (y_{t+j}^e - \bar{g}) \right).$$

# Temporary Equilibrium and Learning

## Equilibrium conditions

- Evolution of expectations: Steady-state learning with point expectations is

$$s_{t+j}^e = s_t^e \text{ for all } j \geq 1, \text{ and } s_t^e = s_{t-1}^e + \omega_t(s_{t-1} - s_{t-1}^e)$$

for  $s = y, z, \pi$ . Here  $\omega_t$  is called the “gain sequence,” and either  $\omega_t = t^{-1}$  (“decreasing gain” learning) or  $\omega_t = \omega$  for  $0 < \omega \leq 1$  and  $\omega$  small (“constant gain” learning).

- **Temporary equilibrium** with steady-state learning:

(1) Aggregate demand (where agents do not know the interest rate rule),

$$y_t = \bar{g} + (\beta^{-1} - 1)(y_t^e - \bar{g}) \left( \frac{\pi_t^e}{R_t} \right) \left( \frac{R_t^e}{R_t^e - \pi_t^e} \right) \equiv Y(y_t^e, \pi_t^e, R_t, R_t^e).$$

- the transparent case is also studied.

(2) Let  $Q(\pi_t) \equiv (\pi_t - 1) \pi_t$ , the nonlinear Phillips curve

$$\begin{aligned} \pi_t &= Q^{-1}[K(y_t, y_t^e)], \text{ where} \\ K(y_t, y_t^e) &\equiv \frac{\nu}{\gamma} \left( \alpha^{-1} y_t^{(1+\varepsilon)/\alpha} - (1 - \nu^{-1}) \frac{y_t}{(y_t - \bar{g})} \right) \\ &\quad + \frac{\nu}{\gamma} \left( \beta(1 - \beta)^{-1} \left( \alpha^{-1} (y_t^e)^{(1+\varepsilon)/\alpha} - (1 - \nu^{-1}) \frac{y_t^e}{y_t^e - \bar{g}} \right) \right), \end{aligned}$$

(3) Equations for bond dynamics and money demand, and (4) Interest rate rule (details to come).

# Monetary Policy Frameworks

- For reference inflation targeting is formulated with a Taylor rule

$$R_t = 1 + \max[\bar{R} - 1 + \psi_\pi[(\pi_t - \pi^*)/\pi^*] + \psi_y[(y_t - y^*)/y^*], 0].$$

where the max operation takes account of the ZLB on the interest rate and  $\bar{R} = \beta^{-1}\pi^*$ . Assume piecewise linear form for convenience.

## Price-level targeting

- Assume a constant growth path for the price level  $\bar{p}_t/\bar{p}_{t-1} = \pi^* \geq 1$ .  
PLT described a **Wicksellian interest rate rule**

$$R_t = 1 + \max[\bar{R} - 1 + \psi_p[(p_t - \bar{p}_t)/\bar{p}_t] + \psi_y[(y_t - y^*)/y^*], 0].$$

# Nominal GDP targeting

- Targeted nominal GDP path  $\bar{z}_t$ :
  - Basic NK model has no trend real growth, so  $\bar{z}_t = \bar{p}_t \bar{y}$  and  $\bar{p}_t / \bar{p}_{t-1} = \pi^*$ .  
Then we have

$$\bar{z}_t / \bar{z}_{t-1} = \Delta \bar{z}_t = \pi^*.$$

- The instrument rule for NGDP targeting is

$$R_t = 1 + \max[\bar{R} - 1 + \psi[(p_t y_t - \bar{z}_t) / \bar{z}_t], 0],$$

where  $\psi > 0$  is a policy parameter. We refer to this rule as **the NGDP rule** below.



# Steady States

- Non-stochastic steady state  $(y, \pi, R)$  must satisfy (i) Fisher equation, (ii) the interest rate rule and (iii) steady-state versions of output and inflation equations.
- Two steady states: First, the targeted steady state  $R = R^* = \beta^{-1}\pi^*$ ,  $\pi = \pi^*$  and  $y = y^*$ , where  $y^*$  uniquely solves

$$\pi^* = Q^{-1}[K(Y(y^*, \pi^*, R^*, R^*), y^*)].$$

Moreover, for this steady state  $p_t = \bar{p}_t$  for all  $t$  under PLT or  $\pi^* = \Delta\bar{z}$  under NGDP.

- There is a second steady state in which the ZLB condition is binding:

**Lemma 1** (1) Assume that  $\beta^{-1}\pi^* - 1 < \psi_p$ . Under the Wicksellian PLT rule, there exists a ZLB-constrained steady state in which  $\hat{R} = 1$ ,  $\hat{\pi} = \beta$ , and  $\hat{y}$  solves the equation

$$\hat{\pi} = Q^{-1}[K(Y(\hat{y}, \hat{\pi}, 1, 1), \hat{y})].$$

(2) Assume that  $\beta^{-1}\pi^* - 1 < \psi$ . The ZLB-constrained steady state  $\hat{R}$ ,  $\hat{\pi}$ , and  $\hat{y}$  exists under the NGDP interest rate rule.

In this steady state the price-level target  $\bar{p}_t$  or NGDP target  $\Delta\bar{z}$ , respectively, is not met and the price level  $p_t$  converges toward zero.

# Expectations Dynamics: Theoretical Results

## Price-level targeting

- Theoretical results about E-stability are obtainable when  $\pi^* = 1$ , so that there is no explosive state variable.

**Proposition 2** *Assume  $\pi^* = 1$  and  $\gamma \rightarrow 0$ . Then under PLT the targeted steady state with  $\pi = 1$  and  $R = \beta^{-1}$  is E-stable if  $\psi_p > 0$ .*

- By continuity of eigenvalues, stability obtains also for  $\gamma$  sufficiently small. Simulations show convergence when  $\bar{p}_t$  is increasing and for higher values of  $\gamma$ .

**Proposition 3** *Assume  $\pi^* = 1$ . The steady state with binding ZLB  $(\hat{y}, \beta, 1, \bar{p})$  is not E-stable under PLT.*

## Nominal GDP targeting

**Proposition 4** *Assume  $\gamma \rightarrow 0$ . Then the targeted steady state with  $\pi^* = 1$  and  $R = \beta^{-1}$  is E-stable under the NGDP rule (??).*

**Proposition 5** *Assume  $\pi^* = 1$ . The steady state with binding ZLB  $(\hat{y}, \beta, 1)$  is not E-stable under NGDP targeting.*

# Numerical analysis

## Dynamics under PLT and NGDP

- Calibration:  $\pi^* = 1.02$ ,  $\beta = 0.99$ ,  $\alpha = 0.7$ ,  $\gamma = 350$ ,  $\nu = 21$ ,  $\varepsilon = 1$ ,  $g = 0.2$ , and  $\varepsilon = 1$ .
  - Interest rate expectations  $r_{t+j}^e$  revert to the steady state value  $\beta^{-1}$  for  $j \geq T$ . We use  $T = 28$ .
  - Bound on  $R$  is set at 1.001.
  - The gain parameter is set at  $\omega = 0.002$ .
- The targeted steady state is  $y^* = 0.944025$ ,  $\pi^* = 1.02$  and the low one is  $y_L = 0.942765$ ,  $\pi_L = 0.99$ .
  - For PLT policy parameters we adopt  $\psi_p = 0.25$  and  $\psi_y = 1$ , also used by Williams (2010).

- For NGDP rule we use the value  $\psi = 0.5$  while for IT the Taylor rule parameters are  $\psi_p = 1.5$  and  $\psi_y = 0.5$ .
- The next three figures show mean dynamics for inflation, output and the interest rate using a grid on initial conditions over the region  $\pi_0^e \in [1.01, 1.03]$  and  $y_0^e \in [0.9439, 0.94445]$ .
  - We set  $y_0 = y_0^e + 0.001$ ,  $\pi_0 = \pi_0^e + 0.001$ ,  $R_0^e = R_0 = R^*$ . For PLT initial deviation is  $p_0/\bar{p}_0 = 1.03$ .
  - Simulations run for 1500 periods and the figures use first 500 periods.

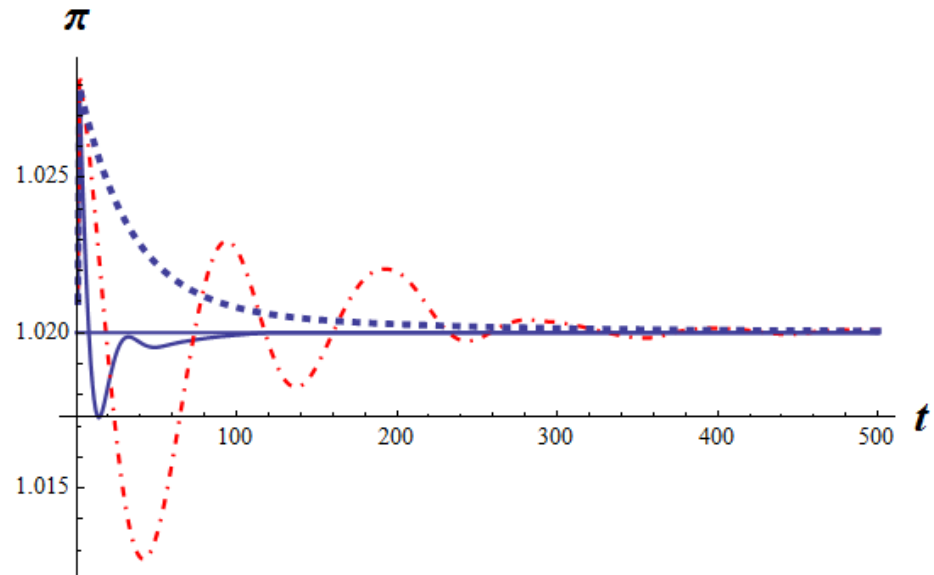


Figure 1: Inflation mean dynamics under IT, PLT, and NGDP without transparency. IT in dashed, PLT in mixed dashed and NGDP in solid line.

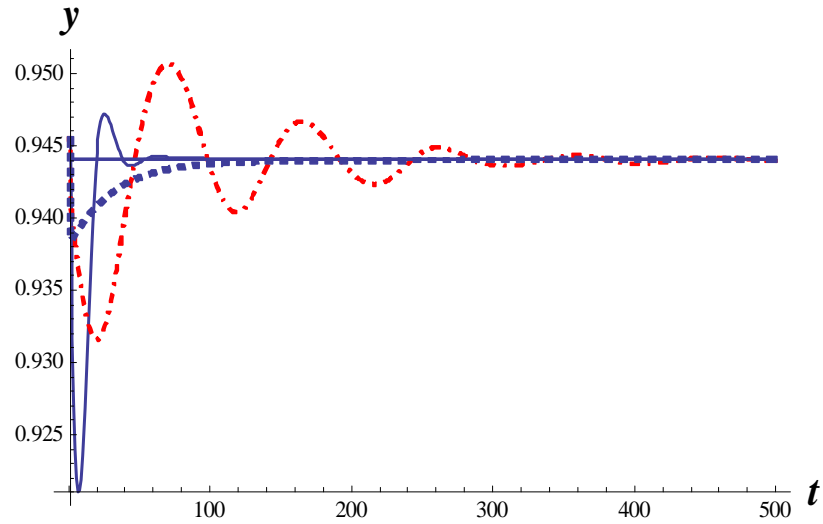


Figure 2: Output mean dynamics under IT, PLT, and NGDP without transparency. IT in dashed, PLT in mixed dashed and NGDP in solid line.



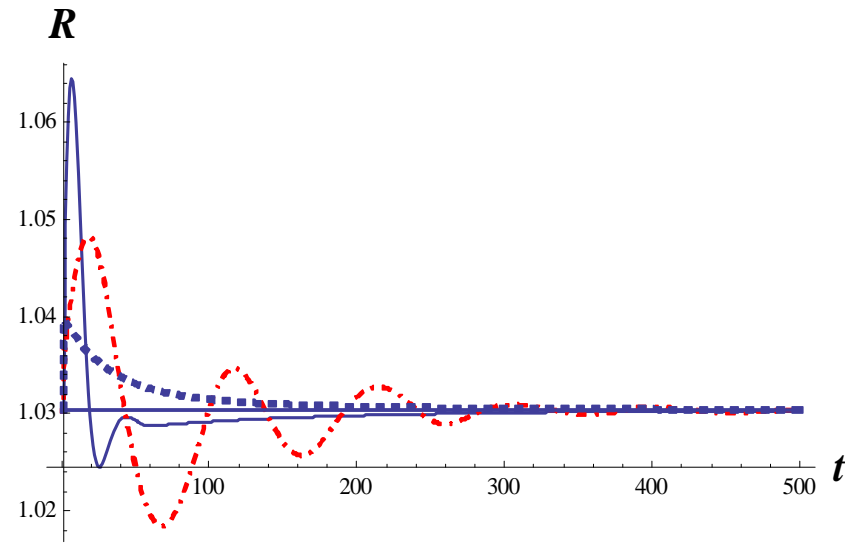


Figure 3: Interest rate mean dynamics under IT, PLT, and NGDP without transparency. IT in dashed, PLT in mixed dashed and NGDP in solid line.

## Robustness of the different rules

- Size of the (partial) domain of attraction,
  - Shown in the next three figures. Grid is over  $\pi_0^e \in [0.95, 1.08]$  and  $y_0^e \in [0.924025, 0.964025]$ .
  - Convergence if  $\pi_t$  and  $y_t$  are within 0.5% of the target.
- Maximal speed of learning for convergence (Table 1)
  - done with initial conditions  $y_0^e = 0.945$ ,  $y_0 = y_0^e + 0.001$ ,  $\pi_0^e = 1.025$ ,  $\pi_0 = \pi_0^e + 0.001$ , and  $R_0^e = R_0 = R^*$ . In PLT the initial deviation for price target is  $p_0/\bar{p}_0 = 1.03$ .
- Volatility of  $\pi$ ,  $y$  and  $R$ , and loss function with  $y$ -weight 0.5 and  $R$ -weight 0.1 (Table 2)
  - Grid as in Figures 1-3.

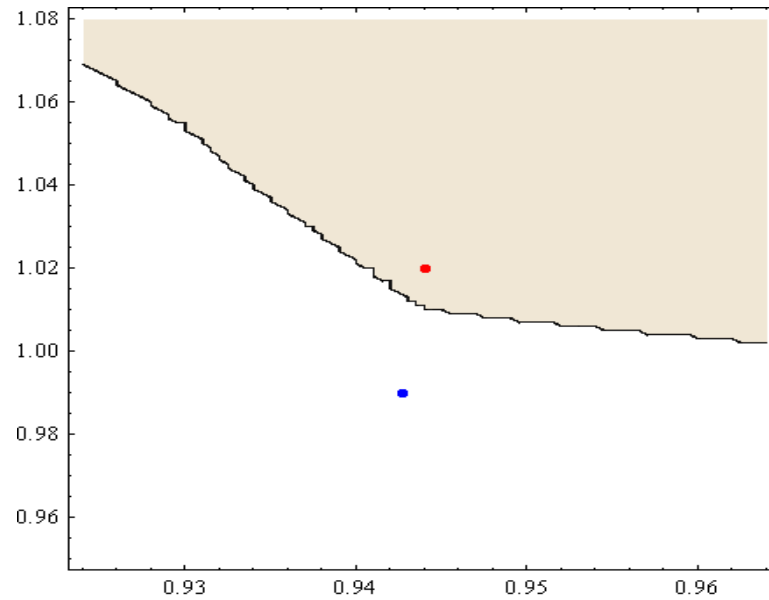


Figure 4: Domain of attraction for IT without transparency. Horizontal axes gives  $y_0^e$  and vertical axis  $\pi_0^e$ . Shaded area indicates convergence. The red circle denotes the intended steady state and the blue circle the unintended one in this and subsequent figures.

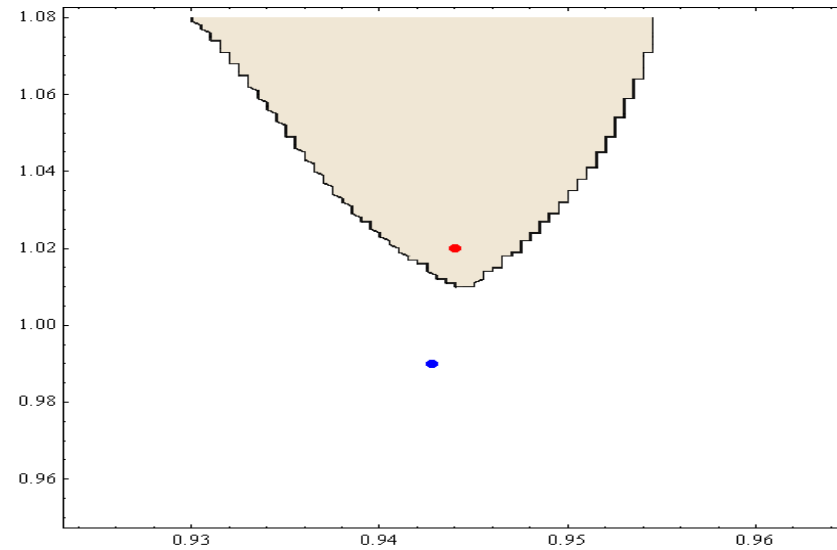


Figure 5: Domain of attraction for NGDP without transparency. Horizontal axes gives  $y_0^e$  and vertical axis  $\pi_0^e$ . Shaded area indicates convergence.

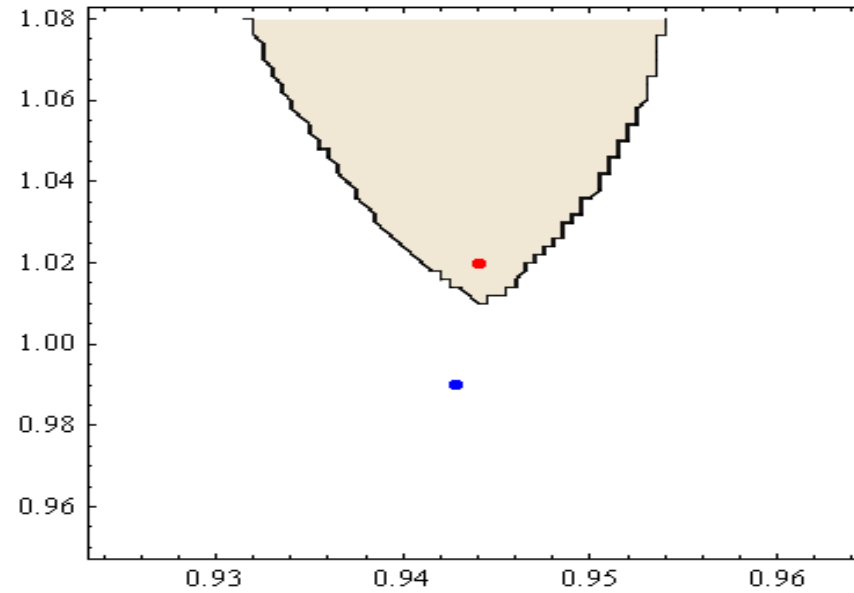


Figure 6: Domain of attraction for PLT without transparency. Horizontal axes gives  $y_0^e$  and vertical axis  $\pi_0^e$ . Shaded area indicates convergence.

- Robustness, domain of attraction: IT performs best, NGDP and PLT are less robust.
- Robustness, speed of learning:

|             |                         |
|-------------|-------------------------|
| <i>NGDP</i> | $0 < \omega \leq 0.028$ |
| <i>IT</i>   | $0 < \omega < 0.126$    |
| <i>PLT</i>  | $0 < \omega < 0.011$    |

Table 1: Robustness with respect to the gain parameter, non-transparent policy

- PLT and NGDP rules are much less robust than IT. The upper bound under PLT and NGDP is at the low end of values used most of the empirical literature.

- Volatility in inflation, output and interest rate during the adjustment.
  - The median volatilities: a run of 1500 periods and  $\omega = 0.002$ .

|             | $var(\pi)$ | $var(y)$ | $var(R)$ | $LOSS$  |
|-------------|------------|----------|----------|---------|
| <i>IT</i>   | 23.9999    | 1.07756  | 52.8747  | 29.8261 |
| <i>NGDP</i> | 1.07151    | 5.2547   | 32.3791  | 6.93676 |
| <i>PLT</i>  | 3.06211    | 6.19571  | 34.9104  | 9.651   |

Table 2: Volatility of inflation, output and interest rate for different policy rules without transparency.

Note: the numbers should be multiplied by  $10^{-6}$ .

- In terms of output fluctuations, IT does clearly best but it is worse in inflation and interest rate fluctuations.
- NGDP does best for the latter two variables. Using the loss function, NGDP rule is the best overall.

# Additional Guidance from PLT or NGDP

- Suppose agents incorporate the target price level path  $\bar{p}_t$  into their learning: agents forecast inflation using  $p_t/\bar{p}_t \equiv rp_t$ , so that

$$\pi_t^e = ((rp_t)^e \times \pi^*)/rp_t.$$

- Note:  $rp_t$  is a natural candidate for incorporating the guidance as it is also the variable in the PLT interest rate rule.
- Agents update the forecasts  $(rp_t)^e$  by

$$(rp_t)^e = (rp_{t-1})^e + \omega(rp_{t-1} - (rp_{t-1})^e).$$

$y_t^e$  and  $R_t^e$  are forecasted as before.

- The temporary equilibrium is as before with the additional relation

$$rp_t = rp_{t-1} \times (\pi_t/\pi^*).$$



- In the NGDP case agents are assumed to forecast future output by making use of  $p_t y_t / z_t \equiv rY_t$ .
  - Given  $(rY_t)^e$ , agents compute  $y_t^e$  from

$$y_t^e = \frac{(rY_t)^e z_{t+1}}{p_t^e} = \frac{(rY_t)^e (\Delta \bar{z}) y_t}{\pi_t^e \times rY_t},$$

where  $\bar{z}_t / \bar{z}_{t-1} = \Delta \bar{z}$ , and do steady-state learning

$$(rY_t)^e = (rY_{t-1})^e + \omega(rY_{t-1} - (rY_{t-1})^e).$$

$\pi_t^e$  and  $R_t^e$  are forecasted as before.

- Actual value of the nominal GDP gap in temporary equilibrium is computed as

$$rY_t = (\Delta \bar{z})^{-1} \pi_t (y_t / y_{t-1}) \times rY_{t-1}.$$

## Dynamics of learning

- Details of specification as before, but set  $rp_0^e = 1.003$  and  $rp_0 = rp_0^e + 0.0001$  for PLT.
- For NGDP case we set  $rY_0^e = 1.003$  and  $rY_0 = rY_0^e + 0.001$ .

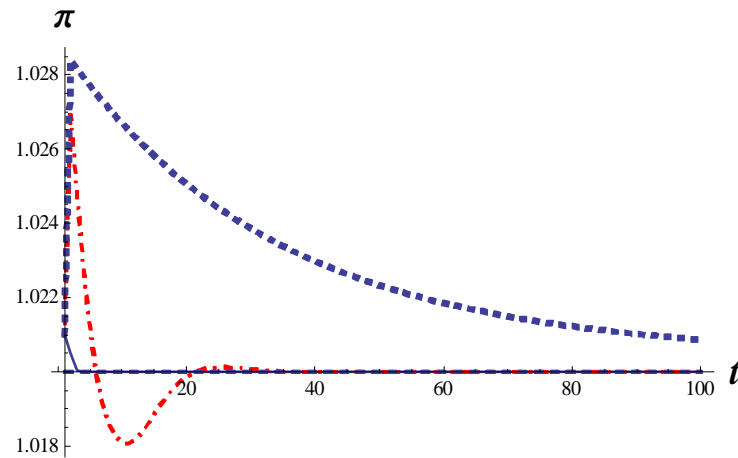


Figure 7: Inflation mean dynamics under IT, PLT, and NGDP without transparency. IT in dashed, NGDP in solid and PLT in mixed dashed line. The horizontal dashed line is the steady state. Note that convergence is very fast under NGDP.

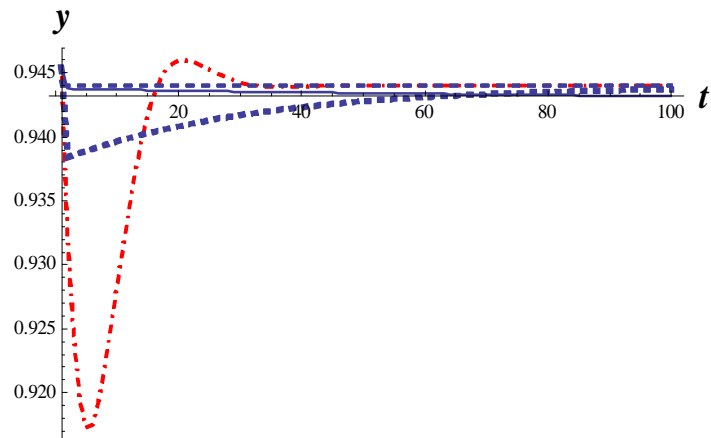


Figure 8: Output mean dynamics under IT, PLT, and NGDP without transparency. IT in dashed, NGDP in solid and PLT in mixed dashed line. The horizontal dashed line is the steady state. For NGDP the curve is barely visible; it stays below the steady state throughout the run of 1,500 periods and converges very slowly towards it from below.

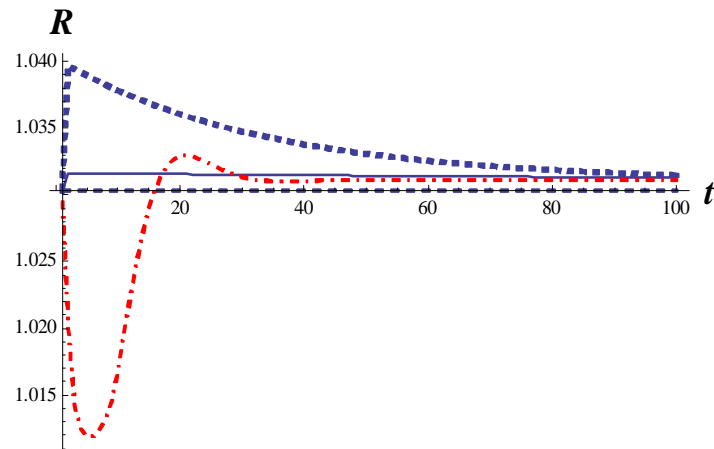


Figure 9: Interest rate mean dynamics under IT, PLT, and NGDP without transparency. IT in dashed, NGDP in solid and PLT in mixed dashed line. The horizontal dashed line is the steady state. For NGDP the curve is barely visible; it stays above the steady state throughout and converges slowly to it. Note that the interest rates are all eventually above the steady state and converge very slowly for all the regimes.

- Figures 7-9 show that the dynamics under PLT and NGDP are significantly altered by guidance.
  - Oscillations under PLT are now smaller and die out much faster than without guidance. Convergence of  $R_t$  is fast at first and then it becomes very slow near the steady state.
  - For NGDP case convergence is also much faster with guidance than without. Convergence of  $y_t$  and  $R_t$  is fast but becomes very slow near the steady state.
- Domain of attraction, Figures 10-11: there seems to be global stability.

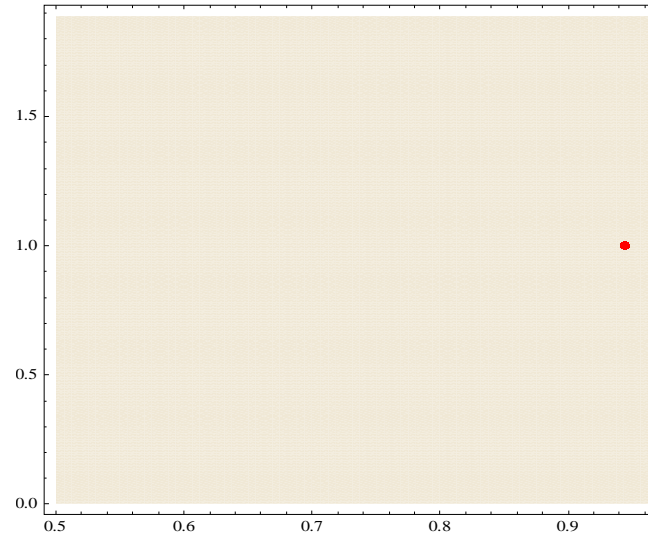


Figure 10: Domain of attraction for PLT with forecasting of gaps. Horizontal axis gives  $y_0^e$  and vertical axis  $rp_0^e$ . Note that the entire state space is in the domain.

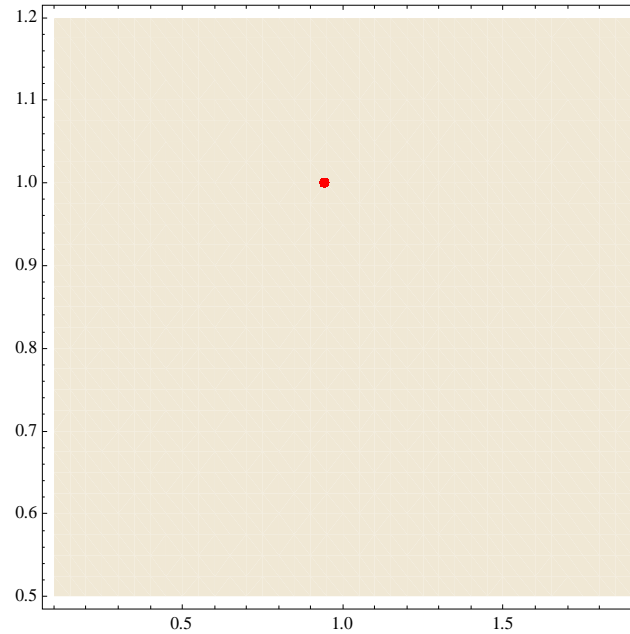


Figure 11: Domain of attraction for NGDP with forecasting of gaps. Horizontal axis gives  $(rY_0)^e$  and vertical axis  $\pi_0^e$ . Convergence obtains from the entire state space.



# Global stability with guidance under PLT and NGDP?

- The low steady state exists but has the feature that  $p_t = 0$ , so that agents cannot use guidance.
  - The low steady state is a singularity for the learning incorporating guidance.
  - The singularity is repelling. Set initial values  $y_0 = \hat{y} = y_0^e$ ,  $\pi_0 = \hat{\pi}$ ,  $R_0 = R_0^e = \hat{R}$  at low steady state.
  - We then set  $rp_0 = rp_0^e$  at a very low positive value 0.0001.
  - Assume gain value  $\omega = 0.05$  to speed up learning and run dynamics for 30 periods.
- **Result:** the system under PLT moves away from the singularity and converges to the targeted steady state.
  - Analogous results holds in the NGDP case.
  - Figures 12-14.

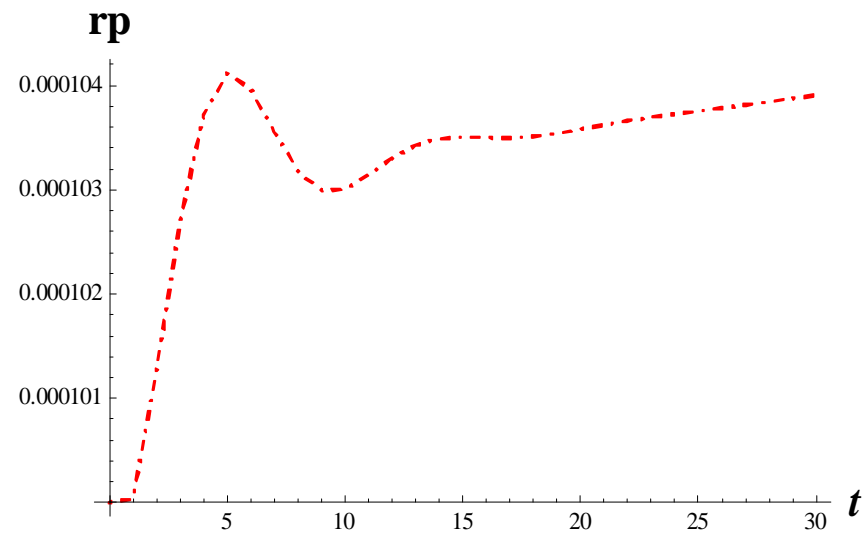


Figure 12: Initial  $r_{p_t}$  dynamics from near low steady state under PLT.

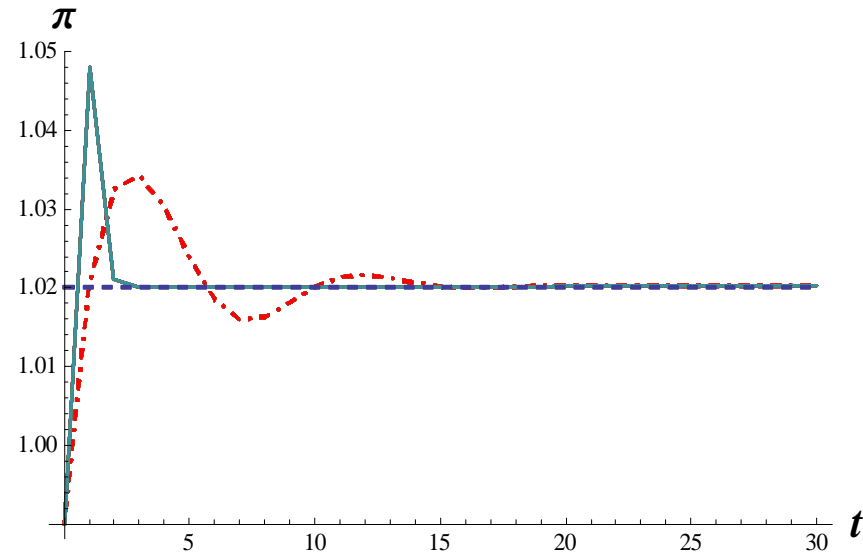


Figure 13: Initial  $\pi_t$  dynamics from near low steady state under PLT and NGDP. Mixed dashed line is PLT and solid line is NGDP. The horizontal dashed line is the targeted steady state.

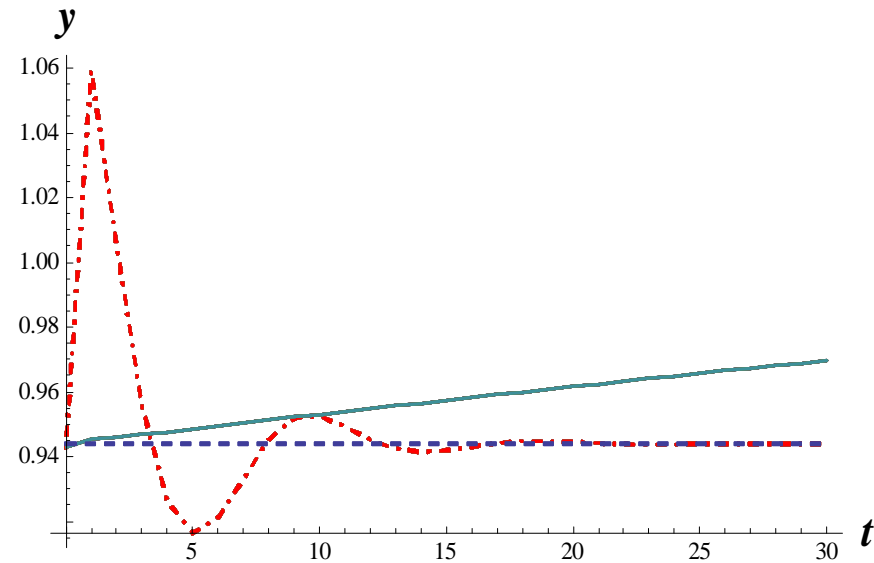


Figure 14: Initial  $y_t$  dynamics from near low steady state under PLT and NGDP. Mixed dashed line is PLT and solid line is NGDP. The horizontal dashed line is the targeted steady state. Under NGDP  $y_t$  does eventually converge to the targeted steady state but this convergence is very slow.

## Other robustness criteria

- Speed of learning:

|             |                         |
|-------------|-------------------------|
| <i>NGDP</i> | $0 < \omega \leq 0.5$   |
| <i>PLT</i>  | $0 < \omega \leq 0.101$ |

Table 3: Robustness with respect to the gain parameter, non-transparent policy, guidance used

- The range of  $\omega$  is much bigger for PLT and NGDP cases under further guidance than with it.

- Volatility in inflation, output and interest rate during the adjustment.
  - Details of specification are as before.

|             | $var(\pi)$ | $var(y)$ | $var(R)$ | $LOSS$  |
|-------------|------------|----------|----------|---------|
| <i>NGDP</i> | 0.0151     | 14.5346  | 2.3424   | 7.5166  |
| <i>PLT</i>  | 0.0693     | 3.035    | 2.704    | 1.8577  |
| <i>IT</i>   | 23.9999    | 1.07756  | 52.8747  | 29.8261 |

Table 4: Volatility of inflation, output and interest rate for NGDP and PLT with guidance but without transparency.

Note: the numbers should be multiplied by  $10^{-6}$ .

- Results for PLT and NGDP for inflation volatility are remarkably improved in comparison to the case without guidance.

# Conclusions

- Starting points:
  - Imperfect knowledge, expectations are not rational.
  - Global nonlinear aspects of the problem are taken on board.
  - We used domain of attraction, maximal speed of learning, volatility of aggregate variables as the robustness criteria for learning transition.
- Major finding: performance of PLT and NGDP depends a great deal on whether private agents include the additional guidance when learning.
  - If a move from IT to either PLT or NGDP targeting is contemplated it is important to ensure that agents use the guidance.
  - If guidance is not adopted, then the comparison if PLT and NGDP to each other and to IT shows that there is not benefit from such a move.